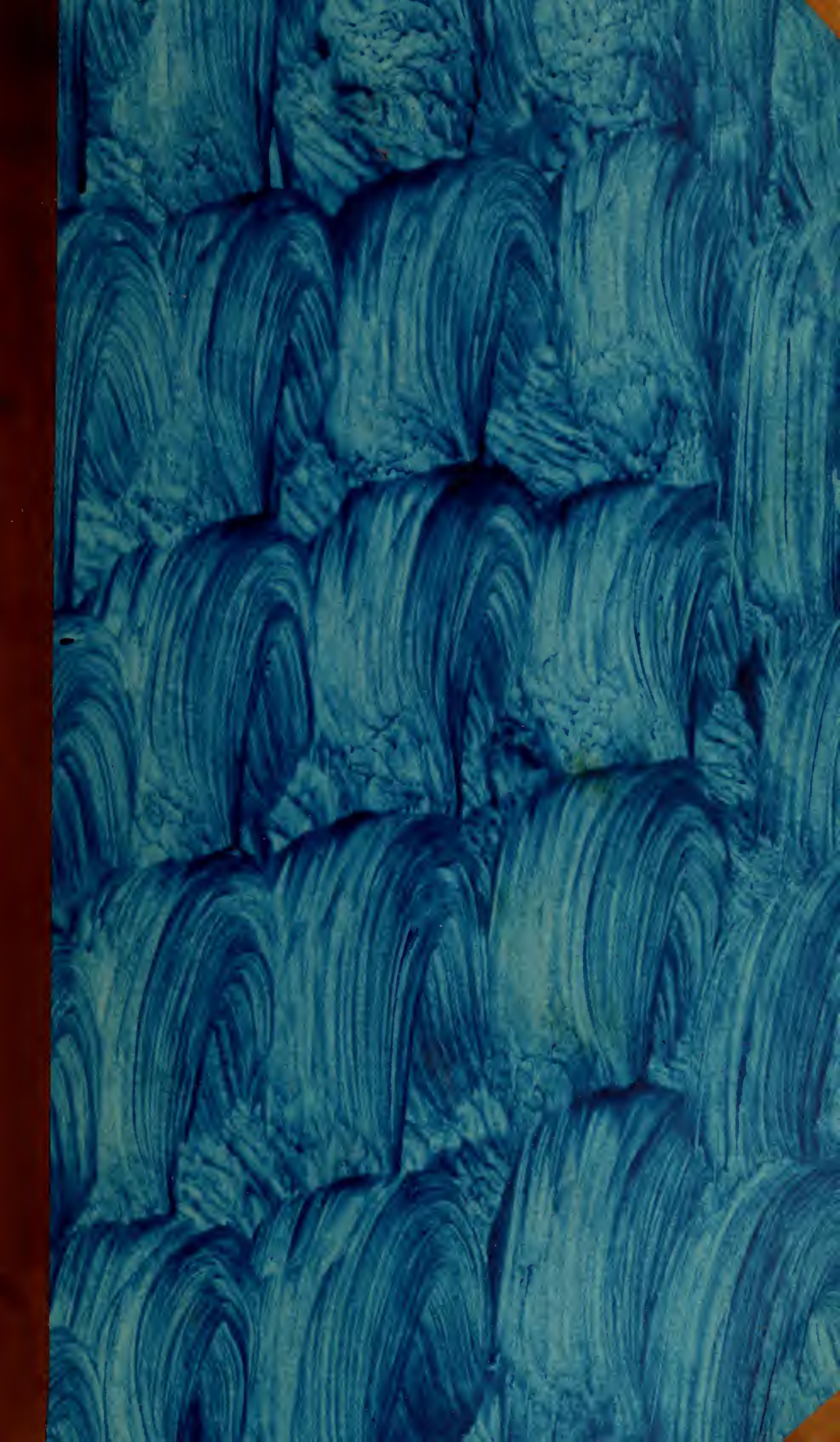

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2

THE

ELEMENTS

OF

TRIGONOMETRY.

CONTAINING

The Properties, Relations, and Calculations of
SINES, TANGENTS, SECANTS, &c.

The DOCTRINE of the SPHERE, and the
PRINCIPLES of PLAIN and SPHERICAL
TRIGONOMETRY.

All plainly and clearly demonſtrated.

The SECOND EDITION, with large Additions:
Together with the TABLES of natural and artificial
SINES, TANGENTS, &c. and LOGARITHMS.

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*Quicquid in Aſtronomicis, Geographicis, vel Nauticis efficiendum,
Trigonometriæ attribuendum eſt.*

LONDON,
Printed for J. NOURSE, Bookſeller in Ordinary
to his MAJESTY.
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Hon. Chas F. Adams,
June 2, 1891.

THE PREFACE.

THE uſe of Trigonometry *is ſo great in all the parts of the mathematics, that he muſt have made a very little progreſs therein, who is not ſenſible of it. Its help is called in upon every occaſion, and its great ſervice is clearly apparent in calculations of all ſorts, both upon the earth, upon the ſeas, and in the heavens. By this, the diſtances of objects upon the earth may be certainly known, if they can but be ſeen; though we cannot come near to meaſure them: likewiſe the geographical diſtances of places on the earth; and their ſeveral poſitions to one another. Navigation depends entirely upon it. Surveying and dialling owe their greateſt exactneſs to it. It is of ſingular ſervice in military affairs: and* Mars, *without this, might live peaceably at home. Upon the wings of Trigonometry (as* Plato *ſays), we mount up from the earth to the heavens, meaſure the diſtances of all the ſtars, and range them in their proper order. And without it* Urania's *ſons may throw aſide their inſtruments, their books and tables; or rather, without this, they would never have had ſuch thing; and conſequently mankind had remained utterly ignorant of this moſt beautiful ſyſtem of the world. In ſhort, the art of Trigonometry is of ſuch univerſal uſe and extent, that it would be an endleſs taſk to enumerate all the various purpoſes to which it is ſubſervient; and the moſt important branches of knowledge would be loſt and uſeleſs if we wanted it.*

This makes me wonder that no body has given us an entire ſyſtem of this art; which ſhall contain all the principles, *as well as the rules of* practice. *Inſtead of that, moſt authors content themſelves with little more than explaining the common caſes of ſolving triangles, and fill up their books with heaps of examples, to ſhew the uſe and application thereof: whilſt they dip very little into the theory, and leave the principles thereof very much in the dark.*

In this treatiſe I have ventured to lay down the whole both theory *and* practice; *and to take in all things of any conſequence that any way belong to the ſubject. And here I account all theſe properties of triangles or other figures that have any relation to the meaſuring their angles by degrees, or by ſines, and tangents, &c. to belong to* Trigonometry, *as their proper ſubject; referring their other properties to* Geometry, *which have no ſuch regard to the meaſures of their angles.*

In purſuance of this deſign, I have in the firſt book laid down in a few propoſitions, the relations of ſines, coſines, &c. of arches; likewiſe of the double arches, and of the ſums and differences of arches. Among which there are a few propoſitions, ſo extenſive and general, as to comprehend a great number of particular propoſitions, evident only by inſpection.

Then I have given the method of calculating natural *and* artificial ſines, tangents, *&c. of any arch: and from thence thoſe of their* multiple arches. *And here I have been obliged to have recourſe to the method of* fluxions, *and the method of* increments; *there being no other poſſible way to effect it. Which if any of my readers happen not to underſtand, they may paſs by the inveſtigations,*

tions, and make equal uſe of the concluſions, nevertheleſs. But for further ſatisfaction, ſee my Method of Increments.

Then you have a few general propoſitions concerning angular ſections, inſcription *of* polygons, *and the* properties of chords *inſcribed in a circle.*

In the ſecond book you have plain Trigonometry, *with the ſolution of all the* caſes, *ſeveral ways. Right-angled triangles are reſolved arithmetically, logarithmically, and algebraically; and oblique ones logarithmically and algebraically. By the firſt method all the caſes of right-angled triangles are reſolved by common arithmetic, without any books or tables whatever. And this comes very near the truth, and is ſufficiently exact for common uſes: and is no further burthenſome to the memory, than getting by heart two or three proportions, with certain fixed numbers; and the* 47. I. Euclid. *And the ſame way may oblique triangles be reſolved, by letting fall a perpendicular.*

In the third book you have all that is material in the doctrine *of the* ſphere; *likewiſe the properties of* ſpherical triangles; *and the principles of* ſpherical Trigonometry; *in which are ſome compound or general propoſitions, including ſeveral particular ones. Among theſe there are ſome that I have taken the liberty to demonſtrate algebraically, to avoid a more prolix method of demonſtration. Then follows the ſolution of all the caſes of ſpherical triangles; in oblique ones both logarithmically and algebraically.*

In the firſt ſection of the firſt book, I have inſerted ſeveral ſcholia *here and there: the uſe of which to Algebraiſts, will be too evident to be further*

ther inſiſted on. And for the ſame reaſon it was; that I inſerted the ſolution of the caſes algebraically.

All theſe things being laid down here in this book, in the ſhorteſt manner poſſible; I think I may venture to entitle it The Elements of Trigonometry. *I won't ſay that I have quite exhauſted the ſubject, but I am of opinion that I have omitted little or nothing of any conſequence.*

I think it needleſs to ſhew the application of Trigonometry to any part of mathematics: that being very eaſy for the reader to do himſelf; eſpecially as all authors abound in theſe things.

If what I have done meet with general approbation, I have my end; if not, I can only ſay,

In magnis voluiſſe ſat eſt.

W. Emerſon.

P. S. *In the firſt edition of this book, having delivered the theory of Trigonometry, with all the practical rules and canons for calculation; I thought it ſufficient to anſwer all the purpoſes that could be deſired. But now finding that a ſet of tables was very much wanted, or rather abſolutely neceſſary, for putting theſe precepts into practice; therefore, to make the treatiſe univerſally uſeful, I have, in this* ſecond edition, *inſerted a table of* natural ſines and tangents, *and a table of* logarithmic ſines and tangents, *and alſo a table of* logarithms, *all extremely correct. And that nothing may be wanting even to the loweſt claſs of readers, I have given the ſolutions of all the caſes of triangles in numbers. I have alſo made many alterations and additions in ſeveral parts of the book, and have added ſeveral propoſitions in ſpherics, which I have put at the end of the laſt ſection; becauſe they could not well be demonſtrated, without the propoſitions in that ſection; all which now makes the book entirely complete.*

W. E.

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Characters uſed in this book.

+	Addition.
—	Subtraction, of the following quantity.
×	Multiplied by.
=	Equal to.
⊥	Perpendicular to.
‖	Parallel to.
: :	Proportion.
>	Greater than.
<	Leſſer than.
∠	Angle.
∽	The difference.

S. ſine; coſ. coſine; T. tangent; cot. cotangent; ſec. ſecant; coſec. coſecant; verſ. verſed ſine; ſup. ſupplement; ſeg. ſegments; diff. difference.

ERRATA.

Nat. tan. 41° 5′, *read* 8435.
Tab. nat. ſines, 24 deg. at the bottom, right-hand; *read* 114, (inſtead of 141.)

THE
ELEMENTS
OF
TRIGONOMETRY.

DEFINITIONS.

1. AN *arch* of a circle is any part of its circumference, as AB. A quarter of the circumference is called a quadrant, as AD; and the half a ſemi-circle, as ADI. FIG. 1.

2. The *radius* is a line drawn from the center to the circumference, as CA.

3. The *complement* of an arch is what it wants of a quadrant; BD is the complement of AB.

4. The *ſupplement* is what it wants of a ſemicircle, as BI is the ſupplement of AB.

Cor. 1. *The difference of two arches is* = *difference of their ſupplements.*

Cor. 2. *The complement of half an arch is equal to half the ſupplement of the whole arch.* For let Q be a quadrant and A an arch; then $Q - \frac{1}{2}A$ is the complement of half the arch, and $2Q - A$ is the ſupplement of the whole, and $Q - \frac{1}{2}A$ is half the ſupplement of the whole.

Cor. 3. *The difference of an arch and its complement is equal to the complement of twice the arch.* $A \backsim \overline{Q - A}$ is the difference of the arch and its complement, and $2A \backsim Q$ is the complement of twice the arch, but $A \backsim \overline{Q - A} = 2A \backsim Q$.

FIG. 1. 5. The *cord* or *ſubtenſe* of an arch AB, is a right line AB draw between the extreme points of the arch.

6. The *ſine* (or right ſine) of an arch AB, is a right line BF drawn from one end of the arch perpendicular to the diameter paſſing thro' the other end of it.

Cor. *The ſine of an arch is half the cord of the double arch.*

7. The *coſine* of an arch is the diſtance between the center and the ſine, as CF.

8. The *verſed ſine* of an arch AB, is the part of the diameter, between the beginning of the arch and the ſine, as AF.

Cor. *The verſed ſine* $+$ *coſine* $=$ *radius.*

9. The *converſed ſine* is the line DE, between the end of the quadrant AD, and BE perpendicular to DC.

10. The *tangent* of an arch BA is the line AG drawn from one end A of the arch perpendicular to the diameter, till it cuts the line CBG drawn thro' the other end of the arch.

11. The *cotangent* of an arch AB is the line DH drawn perpendicular to DC at the end of the quadrant AD, till it meet the line CBH drawn thro' the top of the arch AB.

12. The *ſemitangent* of an arch AB, is the line KC intercepted between the center and the line IK paſſing thro' the end B of the arch.

Cor. *The ſemitangent of an arch is* $=$ *tangent of half the arch.* For if $AT = TB$, then $\angle ACT = \angle AIB$. Whence $CK = AR$.

13. The *ſecant* of an arch AB is the line CG drawn from the center of the circle, through the top of the arch AB, till it meets the tangent AG.

14. The *coſecant* of an arch AB is the line CH, drawn from the center, through the end B of the arch, till it meets the cotangent DH in H.

Cor. 1. *Hence the coſine, cotangent, coſecant, coverſed ſine of an arch, is equal to the ſine, tangent, ſecant, verſed ſine, of its complement reſpectively.*

For the ſine EB of the arch DB (the complement of AB) is = CF, the coſine of AB. Alſo DH, the cotangent of AB, is the tangent of its complement DB. Alſo CH, the coſecant of AB = ſecant of its complement DB. And laſtly DE, the coverſed ſine of AB, is the verſed ſine of its complement DB.

Cor. 2. *A ſine, tangent and ſecant, coſine, cotangent, coſecant, and coverſed ſine, are common to two arches which are the ſupplements of each other.* For theſe lines belong as well to the arch IB, as to AB.

15. A *degree* is the 360th part of the circumference of every circle; and therefore a ſemicircle contains 180 degrees, and a quadrant 90°. Alſo the 60th part of a degree is called a minute, the 60th part of a minute a ſecond, &c.

16. An *angle* ACB is the inclination of two lines AC, CB; and the meaſure of that angle is the degrees in the arch AB.

Note, The middle letter ſtands at the angle.

SCHOLIUM.

Tho' the coſine, tangent, cotangent and ſecant are common to two arches which are the ſupplements to each other; yet in an arch greater than a quadrant, theſe lines are truly negative, and muſt be ſo eſteemed, becauſe they are drawn the contrary way to thoſe in an arch leſs than a quadrant. And therefore the ſine, coverſed ſine, and coſecant, are always affirmative in all arches leſs than a ſemicircle, but negative in greater arches. And in general, the ſines of all arches in the firſt and ſecond quadrants are affirmative, in the third and fourth negative. The coſines in the firſt and fourth quadrants are affirma-

FIG. I. tive, in the ſecond and third negative. The tangents in the firſt and third quadrants are affirmative, in the ſecond and fourth negative. The cotangents in the firſt and third are affirmative, in the ſecond and fourth negative. The ſecants in the firſt and fourth are affirmative, in the ſecond and third negative. The coſecants in the firſt and ſecond are affirmative, in the third and fourth are negative. All verſed ſines are affirmative: moreover, ſines, tangents, &c. of negative arches are contrary in their ſigns to thoſe of affirmative arches.

BOOK I.

The properties of ſines, tangents, ſecants, &c. of arches; the calculation of them, both natural, and artificial; and alſo thoſe of multiple arches; angular ſections, &c.

SECT. I.

The relation of the ſines, tangents and ſecants of arches.

PROP. I.

In any arch, theſe are reſpectively proportional.

FIG. I.

Radius : *Sine* : *Coſine* : :
Secant : *Tangent* : *Radius* : :
Coſecant : *Radius* : *Cotangent.*

THE meaning of the propoſition is, that any four quantities lying in form of a rectangle are proportional. As thus; *Rad* : *Sine* : : *Secant* : *Tangent.* Or *Rad* : *Coſine* : : *Secant* : *Radius.* Or *Tangent* : *Radius* : : *Radius* : *Cotangent.* Or *Cotangent* : *Coſine* : : *Coſecant* : *Radius.* And ſo of others. The ſame is to be underſtood of the following propoſitions. Likewiſe if you have ſeveral quantities placed thus, A : B : C : : D : E : F; the meaning is, that A : B : : D : E, or A : C : : D : F, or B : C : : E : F, &c. or laſtly, if A : B : : C : D : : E : F, &c. it ſignifies, that A : B : : C : D, or A : B : : E : F, or C : D : : E : F; and ſo of others.

For in the ſimilar triangles BFC, GAC, CDH; BC : BF : CF : : GC : GA : AC : : CH : CD : DH.

Q. E. D.

Cor. 1. *Radius ſquare* = *ſine ſquare* + *coſine ſquare* = *ſecant ſquare* — *tangent ſquare* = *coſecant*

FIG. I. *ſquare — cotangent ſquare.* For the triangles BFC, GAC, CDH are right angled.

Cor. 2. *Radius ſquare = tangent × cotangent = coſine × ſecant = ſine × coſecant.*

Cor. 3. *In any arches, the ſine is as the coſine × tangent, or as tangent divided by ſecant, or as coſine divided by cotangent, or reciprocally as the coſecant.*

Cor. 4. *Tangent is as the ſine × ſecant, or as* $\frac{\textit{ſine}}{\textit{coſine}}$, *or as* $\frac{\textit{ſecant}}{\textit{coſecant}}$, *or reciprocally as the cotangent.*

Cor. 5. *Secant is as tangent × coſecant, or as* $\frac{\textit{tangent}}{\textit{ſine}}$, *or as* $\frac{\textit{coſecant}}{\textit{cotangent}}$, *or reciprocally as the coſine.*

Cor. 6. *Coſine is reciprocally as the ſecant, cotangent reciprocally as the tangent, coſecant reciprocally as the ſine.*

Cor. 7. *The ſum of the ſquares of the ſine and verſed ſine of an arch = ſquare of the cord of that arch = 4 times the ſquare of the ſine of half the arch.* For AB = 2AL, and the triangle ABF is right angled.

Cor. 8. *The cord of an arch is a mean proportional between the verſed ſine and diameter.* For AF, AB, AI are continually proportional.

SCHOLIUM.

Let A = any arch.
r = radius.
s = ſine.
c = coſine.
t = tangent.
τ = cotangent.
$ſ$ = ſecant.
σ = coſecant.
v = verſed ſine.
V = verſ. ſ. ſupl.

Then

$$\text{I. } s = \sqrt{rr - cc} = \frac{ct}{r} = \frac{tr}{ſ} = \frac{tr}{\sqrt{rr + tt}} = \frac{rc}{\tau} = \frac{rr}{\sqrt{rr + \tau\tau}} = \frac{rr}{\sigma} = \frac{ſc}{\sigma} = \frac{t\tau}{\sigma} = \frac{r}{ſ}\sqrt{ſſ - rr} = \sqrt{\mathrm{V}v} = \sqrt{2rv - vv}.$$

FIG. I.

II. $c = r - v = V - r = \sqrt{rr - ss} = \frac{rs}{t} = \frac{rr}{\sqrt{rr + tt}} = \frac{rr}{ſ} = \frac{t\tau}{ſ} = \frac{s\sigma}{ſ} = \frac{s\tau}{r} = \frac{r\tau}{\sigma} = \frac{r\tau}{\sqrt{rr + \tau\tau}} = \frac{r}{\sigma}\sqrt{\sigma^2 - rr}.$

III. $t = \frac{rs}{c} = \frac{rs}{\sqrt{rr - ss}} = \frac{r}{c}\sqrt{rr - cc} = \frac{rr}{\tau} = \frac{rr}{\sqrt{\sigma^2 - r^2}} = \frac{cſ}{\tau} = \sqrt{ſ^2 - rr} = \frac{s\sigma}{\tau} = \frac{r\sqrt{2rv - vv}}{r - v} = \frac{r\sqrt{2rV - V^2}}{V - r}.$

IV. $\tau = \sqrt{\sigma\sigma - rr} = \frac{rr}{t} = \frac{s\sigma}{t} = \frac{cſ}{t} = \frac{rc}{s} = \frac{r}{s}\sqrt{rr - ss} = \frac{rc}{\sqrt{rr - cc}} = \frac{rr}{\sqrt{ſſ - rr}} = \frac{r - v}{\sqrt{2rv - vv}}r = \frac{V - r}{\sqrt{2rV - V^2}}r.$

V. $ſ = \sqrt{rr + tt} = \frac{rr}{c} = \frac{rt}{s} = \frac{r^3}{s\tau} = \frac{r\sigma}{\tau} = \frac{t\tau}{c} = \frac{s\sigma}{c} = \frac{rr}{\sqrt{rr - ss}} = \frac{r}{\tau}\sqrt{rr + \tau\tau} = \frac{r\sigma}{\sqrt{\sigma\sigma - rr}} = \frac{rr}{r - v} = \frac{rr}{V - r}.$

VI. $\sigma = \sqrt{rr + \tau\tau} = \frac{rr}{s} = \frac{rſ}{t} = \frac{r\tau}{c} = \frac{r^3}{ct} = \frac{t\tau}{s} = \frac{cſ}{s} = \frac{rr}{\sqrt{rr - cc}} = \frac{r}{t}\sqrt{rr + tt} = \frac{rſ}{\sqrt{ſſ - rr}} = \frac{rr}{\sqrt{2rv - vv}}.$

VII. $v = 2r - V = r - c = r - \sqrt{rr - ss} = r - \frac{rr}{\sqrt{rr + tt}} = r - \frac{r\tau}{\sqrt{rr + \tau\tau}} = r - \frac{rr}{ſ} = r - \frac{r}{\sigma}\sqrt{\sigma\sigma - rr}.$

VIII. $V = 2r - v = r + \sqrt{rr - ss} = r + c = r + \frac{rr}{\sqrt{rr + tt}} = r + \frac{r\tau}{\sqrt{r^2 + \tau^2}} = r + \frac{r}{\sigma}\sqrt{\sigma^2 - rr} = \frac{s + t}{t}r = r + \frac{s\tau}{r} = r + \frac{rr}{ſ} = \frac{\tau + \sigma}{\sigma}r.$

FIG. 1. *The coverſed ſine is eaſily found expreſſed by any of the reſt, by only ſubtracting the ſine from radius.*

PROP. II.

In any arch theſe lines are reſpectively proportional.

Radius : Sine of an arch : Coſine arch : :
2 Sine arch : Verſ. 2 arch : Sine 2 arch : :
2 Coſine arch : Sine 2 arch : Verſ. ſup. 2 arch.

For the triangles CAL, BAF, IBF are ſimilar, and IB = 2CL, therefore CA : AL : CL : : BA : AF : BF : : IB : BF : IF.

Cor. 1. From hence and Prop. I. it is evident, that theſe are reſpectively proportional.

Radius : Sine of an arch : Coſine arch : :
Secant arch : Tangent arch : Radius : :
Coſecant arch : Radius : Cotangent arch : :
2 Sine arch : Verſ. 2 arch : Sine 2 arch : :
2 Coſine arch : Sine 2 arch : Verſ. ſup. 2 arch.

Cor. 2. *In any arch,* $\frac{2\ \text{ſine ſquare}}{\text{radius}}$ = *verſed ſine of the double arch, that is,* $\frac{2ss}{r} = \text{verſ. } 2A$. See Schol. to Prop. I.

Cor. 3. *In any arch,* $\frac{2\ \text{ſine} \times \text{coſine}}{\text{radius}}$ = *ſine of the double arch,* $\frac{2cs}{r} = S.\ 2A$.

Cor. 4. *In any arch,* $\frac{2\ \text{coſine ſquare} - \text{radius ſquare}}{\text{radius}}$ = *coſine of the double arch.* For coſ. $2A = r -$ verſ. $2A = r - \frac{2ss}{r} = \frac{rr - 2ss}{r} = \frac{2cc - rr}{r}$.

FIG. I.

Cor. 5. *In any arch,* $\frac{2 \text{ radius ſquare}}{\text{cotangent} - \text{tangent}} =$ *tangent of twice the arch.* For coſ. : ſine : : rad. : tangent; that is, by Cor. 4, 3. $\frac{2cc-rr}{r}$ or $\frac{cc-ss}{r} : \frac{2cs}{r} : : r :$ $\frac{2csrr}{\overline{cc-ss}.r} = \frac{2rr}{\frac{rc}{s} - \frac{rs}{c}} =$ (by Schol. Pr. I.) $\frac{2rr}{T-t}$.

Cor. 6. *In any arch,* $\frac{1}{2}$ *cotangent* — $\frac{1}{2}$ *tangent* = *cotangent of the double arch.* This follows from Cor. 5. becauſe radius ſquare divided by the tangent is = cotangent.

Cor. 7. *In any arch,* $\frac{\text{ſecant} \times \text{radius}}{2 \text{ coſine} - \text{ſecant}} =$ *ſecant of the double arch. For* $\frac{\text{radius ſquare}}{\text{coſine}} =$ *ſecant; that, is by Cor.* 4. $\frac{rr}{2cr-rr}r = \frac{\frac{rr}{c}}{2c - \frac{rr}{c}}r =$ (by Sch. Pr. I.) $\frac{r\text{ſ}}{2c-\text{ſ}} =$ ſecant of 2 arch.

Cor. 8. *In any arch,* $\frac{\text{ſecant} \times \text{coſecant}}{2 \text{ radius}} =$ *coſecant of the double arch.* For by Cor. 1. of this Prop. $\frac{2rs}{\text{ſ}} =$ S. 2 arch, and (by Sch. Pr. I.) $\frac{\text{radius ſquare}}{\text{ſine}} =$ coſecant; that is, $\frac{rr\text{ſ}}{2rs} = \frac{\text{ſ}\sigma}{2r}$, becauſe $\frac{rr}{s} = \sigma$, by Schol. Pr. I.

Cor. 9. *In any arch,* $\frac{8 \text{ ſine ſquare} \times \text{coſine ſquare}}{\text{radius cube}} =$ *verſed ſine of the quadruple arch.* For, by Cor. 3. $\frac{2cs}{r} =$ S. 2A, and by Cor. 2. $\frac{2 \times 4ccss}{r \times rr} =$ verſed ſine of twice 2A.

FIG. I. Cor. 10. *Hence alſo, the verſed ſines of two arches are as the ſquares of the ſines of their half arches.*

Cor. 11. *The ſines of two arches are as the rectangles of the ſine and coſine of their half arches.*

Cor. 12. *The coſecants of two arches are as the rectangles of the ſecant and coſecant of their half arches.*

Cor. 13. *The verſed ſine of the ſup. of an arch : $\frac{1}{2}$ radius :: ſine ſquare : ſine ſquare of $\frac{1}{2}$ the arch.*

For verſ. ſup. $2A$: coſ. A :: $S.2A : S.A.$
And 2 coſ. A : rad. :: $S.2A : S.A.$ whence
by multiplying, verſ. ſup. $2A$: $\frac{1}{2}$rad. :: $S^2.2A : S^2.A.$

Cor. 14. *What has here been demonſtrated of the relation of ſines and coſines to one another, holds equally true, in reſpect of the cords of arches, and their ſupplements; putting the diameter inſtead of radius.*

Cor. 15. *Hence alſo it follows from Cor. 6. that the difference between the tangent and cotangent of an arch = twice the tangent of the difference between that arch and its complement.*

For the difference between an arch and its complement, is equal to the complement of twice that arch. Hence alſo

Cor. 16. *Let* A *be any arch leſs than* 45°, *then tangent of* $\overline{45+A^\circ}$ = *tangent of* $\overline{45-A^\circ}$ + 2 *tangents of* 2A. For 2A is the difference of the arches $\overline{45+A}$ and $\overline{45-A}$.

SCHOLIUM.

Hence alſo the ſines, coſines, &c. of the double arches may be expreſſed more univerſally thus, (ſee Sch. Prop. I.)

FIG. I.

$$\text{I. } \frac{2cs}{r} = \frac{2ss}{t} = \frac{2cc}{\tau} = \frac{2rs}{ſ} = \frac{2rc}{\sigma} = \frac{2rrt}{rr+tt=ſſ} = \frac{2rr}{\tau+t} = \frac{2r^2\tau}{rr+\tau\tau=\sigma\sigma} = \text{ſine of 2 arch.}$$

$$\text{II. } \frac{cc-ss}{r} = \frac{rr-2ss}{r} = \frac{2cc-rr}{r} = \frac{rr-tt}{rr+tt}r = \frac{\tau-t}{\tau+t}r = \frac{\tau\tau-rr}{\tau\tau+rr}r = \frac{2rr-ſſ}{ſſ}r = \frac{2c-ſ}{ſ}r = \frac{\sigma^2-2r^2}{\sigma^2}r = \frac{\sigma-2s}{\sigma}r = \text{coſine of 2 arch.}$$

$$\text{III. } \frac{2rrt}{rr-tt} = \frac{2rr}{\tau-t} = \frac{2rcs}{rr-2ss} = \frac{2rcs}{2cc-rr} = \frac{2rr\tau}{\tau\tau-rr} = \text{tangent of 2 arch.}$$

$$\text{IV. } \frac{rr-tt}{2t} = \frac{\tau\tau-rr}{2\tau} = \frac{\tau-t}{2} = \frac{rr-2ss}{2cs}r = \frac{2cc-rr}{2cs}r = \text{cotangent of 2 arch.}$$

$$\text{V. } \frac{ſſr}{2rr-ſſ} = \frac{ſr}{2c-ſ} = \frac{r^3}{2cc-rr} = \frac{r^3}{rr-2ss} = \frac{rr+tt}{rr-tt}r = \frac{\tau+t}{\tau-t}r = \frac{\tau\tau+rr}{\tau\tau-rr}r = \frac{r\sigma^2}{\sigma^2-2r^2} = \text{ſecant of 2 arch.}$$

$$\text{VI. } \frac{ſ\sigma}{2r} = \frac{rſ}{2s} = \frac{r\sigma}{2c} = \frac{r3}{2cs} = \frac{rr+tt=ſſ}{2t} = \frac{\tau+t}{2} = \frac{rr+\tau\tau=\sigma\sigma}{2\tau} = \text{coſecant of 2 arch.}$$

$$\text{VII. } \frac{2ss}{r} = \frac{2rr-2cc}{r} = \frac{2vV}{r} = \frac{2v\times\overline{2r-v}}{r} = \frac{2rtt}{rr+tt=ſſ} = \frac{2rt}{\tau+t} = \frac{2r^3}{rr+\tau\tau=\sigma\sigma} = \frac{2rs}{\sigma} = \frac{ſſ-rr}{ſſ}\times 2r = \frac{ſ-c}{ſ}\times 2r = \text{verſed ſine of 2 arch.}$$

$$\text{VIII. } \frac{2rr-2ss}{r} = \frac{2cc}{r} = \frac{2rr}{rr+tt}r = \frac{2\tau\tau}{rr+\tau\tau}r = \frac{2r\tau}{t+\tau} =$$

FIG. $= \frac{2r^3}{ſſ} = \frac{2cr}{ſ} = \frac{2\sigma^2-2r^2}{\sigma^2}r = \frac{2\sigma-2s}{\sigma}r = \frac{\overline{V-r}|^2}{\frac{1}{2}r} = \frac{\overline{r-v}|^2}{\frac{1}{2}r}$ $=$ *verſ. ſup.* 2 A.

The coverſed ſine may be expreſſed by any of the reſt, by ſubtracting the value of the ſine from radius. Alſo other values of each quantity may be inſerted by Sch. Prop. I.

Hence alſo,

I. $r\sqrt{\frac{r-c}{2r}} = r\sqrt{\frac{ſ-r}{2r}} = \sqrt{\frac{rv}{2}} = r\sqrt{\frac{2r-V}{2r}} =$ S. $\frac{1}{2}$ *the arch.*

II. $r\sqrt{\frac{r+c}{2r}} = r\sqrt{\frac{ſ+r}{2ſ}} = \sqrt{\frac{2rr-rv}{2}} = \sqrt{\frac{rV}{2}} =$ *coſ.* $\frac{1}{2}$ *the arch.*

III. $r\sqrt{\frac{r-c}{r+c}} = r\sqrt{\frac{ſ-r}{ſ+r}} = r\sqrt{\frac{v}{2r-v}} = r\sqrt{\frac{2r-V}{V}}$ $= \sigma - \tau =$ *tan.* $\frac{1}{2}$ *the arch.*

IV. $r\sqrt{\frac{r+c}{r-c}} = r\sqrt{\frac{ſ+r}{ſ-r}} = \sqrt{\frac{2rr-rv}{2}} = r\sqrt{\frac{V}{2r-V}}$ $= \sigma + \tau =$ *cotan.* $\frac{1}{2}$ *the arch.*

V. $r\sqrt{\frac{2r}{r+c}} = r\sqrt{\frac{2ſ}{r+ſ}} = r\sqrt{\frac{2r}{2r-v}} = \sqrt{\frac{2r^3}{V}} =$ *ſec.* $\frac{1}{2}$ *the arch.*

VI. $r\sqrt{\frac{2r}{r-c}} = r\sqrt{\frac{2ſ}{ſ-r}} = r\sqrt{\frac{2r}{v}} = r\sqrt{\frac{2r}{2r-V}} =$ *coſec.* $\frac{1}{2}$ *the arch.*

PROP. III.

2. *If three arches* AF, AN, AP *be in arithmetic progreſſion, then theſe are reſpectively proportional.*

Radius

Radius : *S. mean arch* : *Coſ. mean arch* :: FIG.
2 *S. mean arch* : *Verſ. 2 mean arch* : *S. 2 mean arch* :: 2.
2 *Coſ. mean arch* : *S. 2 mean arch* : *Verſ. ſup. 2 mean arch* ::
2 *S. common diff.* : *Diff. coſ. extremes* : *Diff. ſin. extremes* ::
2 *Coſ. com. diff.* : *Sum. ſin. extremes* : *Sum. coſ. extremes.*

Note, *Diff. ſines* = *S. greater arch* — *S. leſſer arch. Diff. coſines* = *coſ. leſſer* — *coſ. greater arch* = *Diff. verſed ſines* = *verſ. greater* — *verſ. leſſer arch, univerſally.*

Draw PCM and PF and MF; and AD ∥ FP, and FS ∥ AB, and CE, MS ⊥ AB; Then MS = PG + FH, and SF = CG + CH. And arch OM = PN = NF. Therefore MF ∥ NC, and MF = 2CR. Whence the triangles CAQ, DAK, BDK, PFI, FMS are ſimilar; and CA : AQ : CQ :: DA : AK : DK :: BD : DK : BK :: PF : FI : PI :: FM : MS : FS.

Cor. 1. *Since the mean* = $\frac{1}{2}$ *ſum of the extremes, therefore in any two arches,*

Radius : *S.* $\frac{1}{2}$ *ſum* : *Coſ.* $\frac{1}{2}$ *ſum* ::
2 *S.* $\frac{1}{2}$ *ſum* : *Verſ. ſum* : *S. ſum* ::
2 *Coſ.* $\frac{1}{2}$ *ſum* : *S. ſum* : *Verſ. ſup. ſum* ::
2 *S.* $\frac{1}{2}$ *difference* : *Diff. coſines* : *Diff. ſines* ::
2 *Coſ.* $\frac{1}{2}$ *diff.* : *Sum ſines* : *Sum coſines.*

Cor. 2. *Let* A, E *be two arches*, A *the greater*, E *the leſſer, then*

$\frac{1}{2}$ *Rad.* : *S.* A. : *Coſ.* A ::
S. E : *Coſ.* $\overline{A-E}$ — *Coſ.* $\overline{A+E}$: *S.* $\overline{A+E}$ — *S.* $\overline{A-E}$ ::
Coſ. E : *S.* $\overline{A+E}$ + *S.* $\overline{A-E}$: *Coſ.* $\overline{A-E}$ + *Coſ.* $\overline{A+E}$.

This will appear by Cor. 1. puting A = half the ſum of 2 arches, and E = half their difference. For then the arches will be $\overline{A+E}$, and $\overline{A-E}$.

FIG. 2. Cor. 3. *As* $\frac{1}{2}$ *rad.* : *S.* $\frac{1}{2}$ *ſum of two arches* : : *S.* $\frac{1}{2}$ *difference* : *difference of their coſines, or of their verſed ſines.*

Cor. 4. *Rectangle of radius and the difference of the verſed ſines (or coſines) of two arches = twice the rectangle of the ſine of half the ſum, and ſine of half the difference of theſe arches.*

Cor. 5. *If three arches* AF, AN, AP *are in arithmetic progreſſion; the rectangle of the ſine of the mean arch, and coſine of the common difference = rectangle of radius, and half the ſum of the ſines of the extreme arches. And if three arches* AF, AN, AP *are in arithmetic progreſſion: rectangle of the coſine of the mean arch, and coſine of the common difference = rectangle of radius and half the ſum of the coſines of the extremes.*

For by Cor. 1. $CA \times \overline{PG + FH} = 2\,CR \times AQ = 2AQ \times \overline{CN - NR}$, whence $2AQ \times NR = 2AQ \times CA - CA \times \overline{PG + FH}$. And $\overline{CA - NR} \times AQ$ or $CR \times AQ = CA \times \frac{PG + FH}{2}$. And if EF, EN, EP be the arches, AQ, PG, FH are coſines.

Cor. 6. *The ſecond difference of the ſines* FH, AQ, PG, × *radius = twice the ſine of the mean arch* × *verſed ſine of the common difference.* For the ſecond difference $= 2AQ - FH - PG$.

Cor. 7. *As radius :*
To twice the coſine of any arch : :
So the ſine of n times that arch :
To ſum of the ſines of n — 1 *and n* + 1 *times that arch : :*
And ſo the coſine of n times that arch :
To the ſum of the coſines of n — 1 *and n* + 1 *times that arch.*

For

For if A be the arch, then $\overline{n-1}\,.\,A$, nA, $\overline{n+1}\,.\,A$ are in arithmetic progreſſion, and A the common difference. FIG. 2.

Cor. 8. *Let* A *be any arch leſs than* 30°, *then ſine of* A $\times \sqrt{3}$ + *ſine of* $\overline{30-A}$ = *ſine of* $\overline{30+A}$.

For the arches 30 − A, 30°, and 30 + A are in arithmetic progreſſion, and coſ. mean 30 = $\frac{1}{2}\sqrt{3}\times$ radius. And by this prop. rad. : coſ. 30° : : 2 ſine of A : S. $\overline{30+A}$ − S. $\overline{30-A}$.

Cor. 9. *If* A *be any arch leſs than* 30, *then* S. $\overline{30+A}$ + S. $\overline{30-A}$ = S. $\overline{90-A}$.

For the S. 30 the mean = $\frac{1}{2}$ rad. and by this prop. rad. : $\frac{1}{2}$ rad. : : 2 coſ. A : S. $\overline{30+A}$ + S. $\overline{30-A}$ = coſ. A.

Cor. 10. *If* A *be any arch leſs than* 30°, *then* S. $\overline{60+A}\times\sqrt{3}$ − S. $\overline{30+A}$ = S. $\overline{90-A}$.

For if the arches be A, 60 + A, 120 + A, and radius × $\frac{1}{2}\sqrt{3}$ = S. 60 the common difference. Then rad. : rad. × $\sqrt{3}$: : S. $\overline{60+A}$: coſ. A — coſ. $\overline{120+A}$ = coſ. A + coſ. $\overline{60-A}$, that is, 1 : $\sqrt{3}$: : S. $\overline{60+A}$ S. $\overline{90-A}$ + S. $\overline{30+A}$.

Cor. 11. *If* A *be any arch leſs than* 45°; *then S.* A × $\sqrt{2}$ + *S.* $\overline{45-A}$ = *S.* $\overline{45+A}$. For in the arches 45 − A, 45, 45 + A; $\frac{1}{2}\sqrt{2}\times$ rad. = coſ. the mean, 45.

Cor. 12. *Let* A *be any arch leſs than* 60; *then S.* A + *S.* $\overline{60-A}$ = *S.* $\overline{60+A}$. For in the arches 60 — A, 60, 60 + A, the coſ. mean, 60° = $\frac{1}{2}$ rad.

Cor. 13. *What is demonſtrated in this prop. and Cor.* 2, 3, 5, 7, *of ſines and coſines, holds in cords of*

FIG. 2. *arches, and their ſupplements, puting diameter for radius.*

PROP. IV.

In any two arches, theſe are reſpectively proportional.

$\frac{1}{2}$ radius : S. $\frac{1}{2}$ ſum : Coſ. $\frac{1}{2}$ ſum : S. $\frac{1}{2}$ diff. : Coſ. $\frac{1}{2}$ diff. : :
S. $\frac{1}{2}$ ſum : Verſ. ſum : S. ſum : Diff. coſ. : Sum ſines : :
Coſ. $\frac{1}{2}$ ſum : S. ſum : V. ſup. ſum : Diff. ſines : Sum coſ. : :
S $\frac{1}{2}$ diff. : Diff. coſ. : Diff. ſines : Verſ. diff. : S. diff. : :
Coſ. $\frac{1}{2}$ diff. : Sum ſines : Sum coſ. : S. diff. : V. ſup. diff.

Note, *Diff. ſines and coſines, is the ſame as in Pr. III.*

The 3 firſt perpendicular rows are evident from the laſt prop. and the firſt, fourth, and fifth terms in the 2 laſt perpendicular rows follow from Prop. II. and the remaining terms are filled up by the proportions in this very prop. thus, $\frac{1}{2}$ rad. : S. $\frac{1}{2}$ diff. : : S. $\frac{1}{2}$ ſum : diff. coſines, which therefore will ſtand in the ſecond place of the fourth perpendicular column, as well as in the fourth place of the ſecond column, &c.

Hence ſeveral corollaries follow of their own accord; as,

Cor. 1. *As verſ. ſupl. ſum of 2 arches : diff. ſines : : diff. ſines : verſed ſine of their difference.*

Cor. 2. *Sine of the ſum of 2 arches : ſum of their ſines : : difference of their ſines : ſine of the difference of the arches.*

Cor. 3. *If three arches are in arithmetical progreſſion,*
As the ſine of one extreme :
Sum of the ſines of the mean arch, and common difference : :
So their difference (S. mean — S. com. diff.)
To the ſine of the other extreme.

For

For let $m=$ mean arch, $d=$ com. difference, and then by Cor. 2. $S.\overline{m+d} : S.m+S.d :: S.m-S.d : S.\overline{m-d}$.

Cor. 4. Let A be any arch; m, n any numbers, then
As ſine of $\overline{m-n}$. A :
$S.\ mA + S.\ nA$ ::
$S.\ mA - S.\ nA$:
Sine of $\overline{m+n}$. A. By this prop. hence

Cor. 5. *As the ſine of an arch* :
Sum of the ſines of the double and ſingle arch ::
So their differences (S. double arch — S. ſingle arch) :
To ſine of the triple arch :

Cor. 6. *As the ſine of an arch* :
Sum of the ſines of triple and double ::
So their difference (S. triple — S. double) :
To ſine of the quintriple arch. &c.

Cor. 7. *Let* $r=$ *radius*, $s=$ *ſine of the arch* 2A, *then the ſine of* $\overline{45+A} = \sqrt{\frac{rr+rs}{2}}$, *and the ſine of* $\overline{45-A} = \sqrt{\frac{rr-rs}{2}}$.

For let $a = S.\ \overline{45+A}$, $e = S.\ \overline{45-A}$. Then, by this prop. as (S. ſum) r : (ſum ſines) $a+e$ :: (diff. ſines) $a-e$: s the ſine of the difference. Therefore,

$$rs = aa - ee$$

but $rr = aa + ee$ by Cor. 1. Prop. I.

therefore $rr + rs = 2aa$

and $rr - rs = 2ee$.

Cor. 8. *What has been demonſtrated in this prop. and firſt 6 collaries, concerning the relation of ſines and coſines, holds equally true with reſpect to their cords; puting the cord of the ſupplement inſtead of coſine, and the diameter inſtead of radius.*

FIG.

SCHOLIUM.

Let there be two arches A, E, and let

$C = \text{coſ.}\ \overline{A+E}.\quad c = \text{coſ.}\ \overline{A-E}.$

$V = \text{verſ.}\ \overline{A+E}.\ v = \text{verſ.}\ \overline{A-E}.$

$Z = \text{ſine}\ \frac{A+E}{2}.\quad z = \text{S.}\ \frac{A-E}{2}.$

$Y = \text{coſ.}\ \frac{A+E}{2}.\quad y = \text{coſ.}\ \frac{A-E}{2}.$

$D = \text{coſ.}\ \overline{2A+2E}.\ d = \text{coſ.}\ \overline{2A-2E}.$ Then

$$\text{S. A} \times \text{S. E} = \overline{Z+z} \times \overline{Z-z} = Z^2 - z^2 = \overline{y+Y} \times \overline{y-Y} = y^2 - Y^2 = \frac{V-v}{2}r = \frac{c-C}{2}r.$$

$$\text{Coſ. A} \times \text{coſ. E} = \frac{c+C}{2}r.$$

$$\text{Tan. A} \times \text{tan. E} = \frac{c-C}{c+C}rr.$$

$$\text{S. A} \times \text{S. E} \times \text{coſ. A} \times \text{coſ. E} = \frac{cc-CC}{4}rr = \frac{d-D}{8}r^3.$$

$$\text{Coſ. A} \times \text{coſ. E} \times \text{tan. A} \times \text{tan. E} = \frac{c-C}{2}r^3.$$

PROP. V.

3. *In any two arches* (AD, DE), *the ſum of the rectangles of the ſine of one into the coſine of the other, is* = *rectangle of the radius, and the ſine of the ſum of theſe arches.*

DEMONSTRATION.

Draw NF ∥ and NR ⊥ CA : then, becauſe the angles ENC, FNR are right, therefore ENF = CNR ; and therefore the triangles ENF, CNR and CDG are ſimilar ; wherefore CD : DG :: CN : $\text{NR} = \frac{\text{DG} \times \text{CN}}{\text{CD}} = \text{FI}$. And CD : CG :: EN : $\text{EF} = \frac{\text{CG} \times \text{EN}}{\text{CD}}$, whence $\text{EI} = \text{EF} + \text{FI} = \frac{\text{CG} \times \text{EN} + \text{DG} \times \text{CN}}{\text{CD}}$.

Q. E. D.

Cor.

Cor. 1. *In any two arches* (AD, DE), *the rectangle of their coſines — the rectangle of their ſines = the rectangle of radius and the coſine of their ſum.* FIG. 3.

For by ſimilar triangles, CD : CG :: CN : CR = $\frac{CG\times CN}{CD}$. And CD : DG :: (CN : NR ::) EN : NF = $\frac{DG\times EN}{CD}$ = IR. And CI = CR — IR = $\frac{CG\times CN-DG\times EN}{CD}$.

Cor. 2. *The ſquare of the ſine of the ſum of two arches is = the ſum of the ſquares of their ſines + twice their rectangle, multiplied by the coſine of the ſum and divided by radius.*

For let the arches A + E = S, their ſines $a+e=s$; and c the coſine of S. Then by this prop. $s=\frac{a\sqrt{rr-ee}+e\sqrt{ra-aa}}{r}$ and $ss=aa+ee-\frac{2aaee}{rr}+\frac{2ae}{rr}\sqrt{rr-aa}\times\sqrt{rr-ee}$. But by Cor. 1. $\frac{\sqrt{rr-aa}\times\sqrt{rr-ee}}{r}-\frac{ae}{r}=c$. Therefore $ss=aa+ee+\frac{2c}{r}ae$.

Cor. 3. *Hence if* A + E = 90°, *then* $aa+ee=rr$.
If A + E = 60, *then* $aa+ee+ae=\frac{3}{4}rr$.
If A + E = 45, *then* $aa+ee+ae\sqrt{2}=\frac{1}{2}rr$.
If A + E = 30, *then* $aa+ee+ae\sqrt{3}=\frac{1}{4}rr$.

SCHOLIUM.

This Prop. and Cor. 1. hold univerſally for all arches, puting negative coſines for arches greater than a quadrant.

PROP. VI.

In two arches (AD, AE), *the rectangle of the ſine of the greater and coſine of the leſſer — the rectangle of*

FIG. 3. *the ſine of the leſſer and coſine of the greater is = rectangle of radius and the ſine of the difference of the arches.*

DEMONSTRATION.

The triangles CDG, CBI, and EBN, are ſimilar; therefore CG : DG :: CI : BI = $\frac{CI\times DG}{CG}$. And CD : CG :: EB or EI $-\frac{CI\times DG}{CG}$: EN = $\frac{CG\times EI-CI\times DG}{CD}$.

Cor. *In any two arches* (AD, AE), *the rectangle of their ſines* + *the rectangle of their coſines is* = *rectangle of the radius and the coſine of the difference of the arches.*

For BI = $\frac{CI\times DG}{CG}$, and by ſimilar triangles CB = $\frac{CI\times CD}{CG}$. And CD : DG :: BE or EI $-\frac{DG\times CI}{CG}$: BN $=\frac{DG\times EI}{CD}-\frac{DG^2\times CI}{CD\times CG}=\frac{DG\times EI}{CD}-\frac{CD^2\times CI}{CD\times CG}+\frac{CG^2\times CI}{CD\times CG}=\frac{DG\times EI}{CD}-CB+\frac{CG\times CI}{CD}$; therefore $\frac{DG\times EI+CG\times CI}{CD}=CB+BN=CN$.

SCHOLIUM I.

This prop. and its cor. hold univerſally for all arches, puting negative coſines for arches greater than a quadrant.

SCHOLIUM II.

From the foregoing propoſitions it follows, that if radius = 1. And

S, C = ſine, and coſine of half the ſum of two arches.

s, c = ſine, and coſine of half their difference; then FIG. 3.

$Sc + Cs$ = ſine of the greater.
$Sc - Cs$ = ſine of the leſſer.
$Cc - Ss$ = coſine of the greater.
$Cc + Ss$ = coſine of the leſſer.

Alſo

If Z = ſecant of the greater arch, z = ſecant of the leſſer.
Y = coſecant of the greater, y = conſecant of the leſſer.

Then $\frac{YyZz}{Yy - Zz}$ = ſecant of the ſum.

$\frac{YyZz}{Zy + Yz}$ = coſecant of the ſum.

$\frac{YyZz}{Yy + Zz}$ = ſecant of the difference.

$\frac{YyZz}{Zy - Yz}$ = coſecant of the difference.

PROP. VII.

If three arches (AF, AN, AP), *be in arithmetic progreſſion,* 4.
The ſum of the ſines of the extreme arches :
To their difference (S. greater — S. leſſer) : :
As tang. mean arch :
To tang. common difference of the arches.

For drawing the lines as in the figure, then by ſimilar triangles, 2OB or PG + FH : 2OD or PI : : OB : OD : : OL : OF : : NM : NK.

Cor. 1. In any two arches (AF, AP ;)
As ſum of their ſines :
To their difference (S. greater — S. leſſer) : :
Tang. half their ſum :
Tang. half their difference.

For the mean arch = $\frac{1}{2}$ ſum of the extremes.

FIG. 4.

Cor. 2. In any two arches (AF, AP),
The ſum of their coſines :
Their diff. (coſ. leſſer — coſ. greater) : :
As cotang. half their ſum :
Tang. half their difference.

For CH + CG or 2CB : CH — CG or 2DF : : CB : BH : : RO : OF : : SN : NK.

Cor. 3. In any two arches (AF, AP), *as* *S. greater : S. leſſer : : T.* $\frac{1}{2}$ *ſum* + *T.* $\frac{1}{2}$ *diff. : T.* $\frac{1}{2}$ *ſum* — *T.* $\frac{1}{2}$ *diff.*

For by the demonſtration of this prop. PG + FH : PG — FH : : MN : NK. And by compoſition and diviſion, PG : FH : : QM : KM.

Cor. 4. In any two arches (AN, NP ;) *as*
The ſum of their tangents :
Their diff. (tan. greater — tan. leſſer) : :
Sine of their ſum :
Sine of their difference.

Let AN be the greater, and make NF = NP, then by ſimilar triangles, QM : KM : : (PL : FL : :) PG : FH.

Cor. 5. In any two arches (AN, NP ;) *as* *Tan. greater : tan. leſſer : : S. ſum* + *S. difference : S. ſum* — *S. difference.*

For let AN be the greater, and make NF = NP, then AP is the ſum, and AF is the difference of the arches ; and by Cor. 4. we have QM : KM : : PG : FH. And by compoſition and diviſion, $\frac{QM + KM}{2}$ or NM : $\frac{QM - KM}{2}$ or NQ : : PG + FH : PG — FH.

Cor. 6. *Let* A *be any arch leſs than* 45° ; *then*
S. $\overline{45 + A}$ + *coſ.* $\overline{45 + A}$:
S. $\overline{45 + A}$ — *coſ.* $\overline{45 + A}$: :
Radius :
Tan. A.

This

This appears by this prop. putting AN = 45°, and NF = NP = A. FIG. 4.

SCHOLIUM.

The propoſitions here delivered hold univerſally, obſerving to take negative coſines and tangents for arches greater than a quadrant.

PROP. VIII.

In any two arches, AE, ED; 5.
Radius ſquare — rectangle of their tangents :
Radius ſquare : :
Sum of their tangents :
Tangent of the Sum of the arches.

For by ſimilar triangles CB or CA — BA : CK or CF — KF : : CF : CA. Whence $CA^2 - BAC = CF^2 - KFC$, and $KFC = CF^2 - CA^2 + BAC = FA^2 + BAC$. Again, CB : BK : : CF : FA, and BK : DG : : KF : FD or FA, therefore *ex equo* CB or CA — AB : DG : : CFK or $FA^2 + BAC$: AF^2; therefore $\overline{AC - AB} \times AF^2 = DG \times FA^2 + DG \times BAC$. And $\overline{AB + DG} \times AF^2 = AC \times \overline{AF^2 - AB \times DG}$. *Q. E. D.*

Cor. 1. *Let* T, *t be the tangents*; X, *x*, *the co-tangents of two arches*; *then* $\frac{rr - Tt}{T + t} = \frac{Xx - rr}{X + x} =$ *co-tangent of the ſum of the arches.*

For let r = radius, τ = cotangent of the ſum. Then $AC = \frac{rr}{\tau}$, and $\overline{AB + DG} \times rr = \frac{rr}{\tau} \times \overline{rr - AB \times DG}$, whence $\tau = \frac{rr - AB \times DG}{AB + DG} = \frac{\textit{rect. cot.} - rr}{\textit{ſum cot.}}$.

FIG. 5. Cor. 2. *If one of the arches be* 45°, $t =$ *tang. other arch, then* $\frac{r+t}{r-t}\times r =$ *tang. ſum of the arches.*

SCHOLIUM.

What belongs in general to the addition of tangents and cotangents, is delivered in this prop. and its cors. All caſes where the arches are greater than a quadrant will eaſily appear by puting negative tangents and cotangents for theſe arches.

PROP. IX.

In any two arches AD, AE; *as*
Radius ſquare + *rectangle of their tangents* :
Radius ſquare : :
Difference (or tangent greater — tangent leſſer) :
Tangent of the difference of the arches.

For you will find as in the laſt prop. CB : DG : : CFK or $FA^2 + BAC : FA^2$. *Q. E. D.*

Cor. 1. *Let* T, *t be the tangents,* X, *x the cotangents of two arches; then* $\frac{rr + Tt}{T - t} = \frac{rr + Xx}{x - X} =$ *cotangent of the difference of the arches.*

This appears by putting $\frac{rr}{T}$ for the tangent of the difference.

Cor. 2. *If one of the arches be* 45°, $t =$ *tangent of the other, then* $\frac{r-t}{r+t}r =$ *tangent of their difference* = *tan.* $\overline{45^\circ - \textit{the other arch}}$, *when the other arch is leſſer than* 45°: *or* $\frac{t-r}{t+r}r =$ *tan.* $\overline{\textit{the other arch} - 45^\circ}$, *when greater.*

SCHO-

FIG. 5.

SCHOLIUM.

For arches greater than a quadrant, put negative tangents and cotangents.

PROP. X.

The ſecant of an arch is equal to the ſum of the tangent of it and the tangent of half its complement.

For let A = arch, T its tangent, ſ its ſecant. $a = \frac{1}{2}$ complement, t its tangent. Then by ſchol. Pr. II. $\text{ſ} = \frac{rr+tt}{2t}$, and $T = \frac{rr-tt}{2t}$, and $\text{ſ} - T = t$, whence ſecant of A = tangent of A + tang. $\frac{1}{2}$ complement of A.

Cor. 1. *The ſecant of an arch = cotangent of half the complement — the tangent of the arch.*

For $\text{ſ} + T = \frac{rr}{t} =$ cot. $a =$ cotang. $\frac{1}{2}$ compl. of A.

Cor. 2. *Half the ſum of the tangent and cotangent of an arch is = ſecant of the difference between that arch and its complement.*

For by ſchol. Pr. II. $\frac{1}{2}$ tangent + $\frac{1}{2}$ cotangent = coſecant of the double arch, and the complement of the double arch is the ſame as the difference between the arch and its complement.

Cor. 3. *As radius + ſecant : radius : : tangent : tangent of half the arch.* For $r + \frac{T+t}{T-t} r$ or $\frac{2rT}{T-t} : r :: \frac{2rr}{T-t} : t$. See ſch. Pr. II.

Cor. 4. *Secant of an arch* A = *tangent of* $\overline{45 + \frac{1}{2}A}^{\circ}$ — *tangent of* A. This follows from Cor. 1.

SCHOLIUM.

From what has been before laid down, it will not be difficult to find the ſines of as many arches as we will, FIG. 5. will, expreſſed in ſurd numbers. As in theſe following, where the prop. in the margin ſhows whence and how they are derived, either immediately, or by help of what ſtep.

Pr. IV, 7. Step 8.	1	S. 15°	$= \frac{1}{2}r\sqrt{2-\sqrt{3}}$.
Pr. IV, 7. and II. 3.	2	S. 18	$= r \times \frac{\sqrt{5}-1}{4}$.
Pr. IV, 7; Step 6.	3	S. 22½	$= \frac{1}{2}r\sqrt{2-\sqrt{2}}$.
Pr. III, Step 12.	4	S. 30	$= \frac{1}{2}r$.
Pr. IV, 7. Step 2.	5	S. 36	$= \frac{1}{4}r\sqrt{10-2\sqrt{5}}$.
Pr. IV, 7.	6	S. 45	$= \frac{1}{2}r\sqrt{2}$.
Pr. IV, 7. Step 2.	7	S. 54	$= r \times \frac{\sqrt{5}+1}{4}$.
Pr. IV, 7. Step 4.	8	S. 60	$= \frac{1}{2}r\sqrt{3}$.
Pr. IV, 7. Step 6.	9	S. 67½	$= \frac{1}{2}r\sqrt{2+\sqrt{2}}$.
Pr. I, 1. Step 2.	10	S. 72	$= \frac{1}{4}r\sqrt{10+2\sqrt{5}}$.
Pr. IV, 7. Step 8.	11	S. 75	$= \frac{1}{2}r\sqrt{2+\sqrt{3}}$.
Def. 2.	12	S. 90	$= r$.

And thus you may find as many ſines as you pleaſe by Prop. III. Cor. 6, 7, 8, 9. And Prop. IV. Cor. 7. and Prop. V. Cor. 4, 5. by the help of theſe already found; but then they will be ſtill more and more compounded with ſurd quantities, except you chuſe to extract them. The ſine of 18 in ſtep the ſecond is not ſo eaſily derived as the reſt; it is had thus. Put $s =$ S. 18. Then by Pr. IV. 7. S. 36 $= \sqrt{\frac{rr-rs}{2}}$, and by Prop. II. 3, the ſame ſine of 36 $= \frac{2s}{r}\sqrt{rr-ss}$; which equated and divided by $\sqrt{r-s}$, and then reduced, you have a cubic equation, $8s^3 + 8rss = r^3$. And the root is $s = \frac{\sqrt{5}-1}{4} \times r$.

SECT. II.

The calculation of natural ſines, tangents, and ſecants of arches.

PROP. XI.

The radius and ſine AB *of an arch* DB *being given; to find the arch.* FIG. 6.

Take the arch Bb infinitely ſmall, and draw Bs ∥ CD, and hsu ∥ BA; and let CB or CD $= r$, AB $= y$, DB $= z$, CA $= \sqrt{rr - yy}$, B$b = \dot{z}$, $sb = \dot{y}$. The triangles CAB, and Bsb are ſimilar: therefore CA $(\sqrt{rr - yy})$: CB (r) : : sb $(\dot{y})$: Bb or $\dot{z} = \frac{r\dot{y}}{\sqrt{rr - yy}}$. And the fluent is $z = y + \frac{y^3}{3.2r^2} + \frac{3.y^5}{5.2.4r^4} + \frac{3.5y^7}{7.2.4.6r^6} + \frac{3.5.7y^9}{9.2.4.6.8.r^8}$ &c. or arch DB $= y + \frac{yy}{2.3rr}\mathrm{A} + \frac{3.3y^2}{4.5rr}\mathrm{B} + \frac{5.5y^2}{6.7r^2}\mathrm{C} + \frac{7.7y^2}{8.9r^2}\mathrm{D}$, &c. putting A, B, C, &c. for the firſt, ſecond, third terms, &c.

Cor. 1. *If* $d =$ *diameter,* $c =$ *cord of an arch, then the arch* $= c + \frac{cc}{2.3dd}\mathrm{A} + \frac{3^2c^2}{4.5d^2}\mathrm{B} + \frac{5^2c^2}{6.7d^2}\mathrm{C}$, &c.

Cor. 2. *Let* Q $=$ *a quadrant, then the arch whoſe coſine is y is* $= \mathrm{Q} - y - \frac{y^3}{2.3r^2} - \frac{3y^5}{2.4.5r^4} - \frac{3.5y^7}{2.4.6.7r^6}$ *&c.*

Or the arch whoſe coſine is y is $= \frac{r - y}{1} + \frac{r^3 - y^3}{2.3r^2} + 3 \times \frac{r^5 - y^5}{2.4.5r^4} + 3 \times 5 \times \frac{r^7 - y^7}{2.4.6.7r^6}$ *&c.*

PROP.

FIG. 6.

PROP. XII.

The arch BD *being given*; *to find the ſine* BA.

Let $CD = r$, $AB = y$, arch $DB = z$. Then by Prop. XI. $\dot{z} = \frac{r\dot{y}}{\sqrt{rr - yy}}$, and $\overline{rr - yy} \times \dot{z}^2 = rr\dot{y}^2$: this put into fluxions, making $\dot{z}$ conſtant, we have $-2y\dot{y}\dot{z}^2 = 2rr\dot{y}\ddot{y}$, or $rr\ddot{y} = -y\dot{z}^2$. Now when z is infinitely ſmall, $z = y$; therefore aſſume,

$$y = z + bz^3 + cz^5 + dz^7 \text{ \&c.}$$

$$\text{then } \dot{y} = \dot{z} + 3bz^2\dot{z} + 5cz^4\dot{z} + 7dz^6\dot{z} \text{ \&c.}$$

$$\text{and } \ddot{y} = 2.3bz\dot{z}^2 + 4.5cz^3\dot{z}^2 + 6.7dz^5\dot{z}^2 \text{ \&c.}$$

then $rr\ddot{y} = -y\dot{z}^2$; that is,

$$2.3brrz\dot{z}^2 + 4.5crrz^3\dot{z}^2 + 6.7rrdz^5\dot{z}^2 + \text{\&c.}$$
$$= -z\dot{z}^2 - bz^3\dot{z}^2 - cz^5\dot{z}^2 - \text{\&c.}$$

Therefore equating the coefficients, $b = \frac{-1}{2.3rr}$, $c = \frac{-b}{4.5rr}$, $d = \frac{-c}{6.7rr}$, $e = \frac{-d}{8.9rr}$, &c. and $y = z - \frac{1}{2.3rr}z^3 - \frac{b}{4.5rr}z^5$ &c. that is, $y = z - \frac{z^3}{2.3rr} + \frac{z^5}{2.3.4.5r^4} - \frac{z^7}{2.3.4.5.6.7r^6} + \text{\&c.}$ that is,

y or ſine $AB = z - \frac{zz}{2.3rr}A - \frac{zz}{4.5rr}B - \frac{zz}{6.7rr}C - \frac{zz}{8.9rr}D$, &c. where A, B, C, &c. are the foregoing terms with their ſigns.

Cor. 1. *If* z *be any arch, its coſine* $CA = r - \frac{zz}{1.2rr}A - \frac{zz}{3.4rr}B - \frac{zz}{5.6rr}C - \frac{zz}{7.8rr}D - \text{\&c.}$

For $CA = \sqrt{rr - yy} = \sqrt{rr - zz + \frac{z^4}{3rr} - \frac{2z^6}{45r^4} + \frac{z}{315r^6} - \text{\&c.}} = r - \frac{zz}{1.2r} + \frac{z^4}{1.2.3.4r^3} + \frac{z^6}{1\ldots\ldots6r^5} + \frac{z^8}{1\ldots\ldots8r^7} - \text{\&c.}$

Cor.

Cor. 2. *If d be the diameter, z any arch; then the cord* $= z - \frac{zz}{2.3dd}A - \frac{zz}{4.5dd}B - \frac{zz}{6.7dd}C - \frac{zz}{8.9dd}D -$ &c. FIG. 6.

PROP. XIII.

The ſine and coſine of an arch A *being given, and if there be given another arch z; to find the ſine and coſine of the ſum of the arches* A + *z, and alſo of the difference* A — *z.*

Let s, c be the ſine and coſine of A. And by Prop. XII. and Cor. 1. the ſine of the arch z is $z - \frac{z^3}{6r^2} + \frac{z^5}{120r^4}$ &c. $=$ B. And its coſine $r - \frac{z^2}{2r} + \frac{z^4}{24r^3}$ &c. $=$ D. Then by Prop. V. and Cor. 1. and Prop. VI. and Cor. $\frac{sD + cB}{r} = S.\overline{A+z}$. And $\frac{cD - sB}{r} =$ coſ. $\overline{A+z}$. And $\frac{sD - cB}{r} = S.\overline{A-z}$. And $\frac{sB + cD}{r} =$ coſ. $\overline{A-z}$; that is,

$$S.\overline{A+z} = s + \frac{cz}{r} - \frac{sz^2}{1.2r^2} - \frac{cz^3}{2.3r^3} + \frac{sz^4}{1.2.3.4r^4} + \frac{cz^5}{1.2.3.4.5r^5} - \&c.$$

$$\text{Coſ. } \overline{A+z} = c - \frac{sz}{r} - \frac{cz^2}{2r^2} + \frac{sz^3}{2.3r^3} + \frac{cz^4}{2.3.4r^4} - \frac{sz^5}{2.3.4.5r^5} - \&c.$$

$$S.\overline{A-z} = s - \frac{cz}{r} - \frac{sz^2}{2r^2} + \frac{cz^3}{2.3r^3} + \frac{sz^4}{2.3.4r^4} - \frac{cz^5}{2.3.4.5r^5} - \&c.$$

Coſ.

FIG.

Coſ. $\overline{A-z} = c + \frac{sz}{r} - \frac{cz^2}{2r^2} - \frac{sz^3}{2.3r^3} + \frac{cz^4}{2.3.4r^4} + \frac{sz^5}{2.3.4.5r^5} - \&c.$

Cor. 1. *If there be given* s, c, *the ſine and coſine of an arch* A; *and* $s + x$ *be the ſine of another arch* $\overline{A+z}$; *then the difference of the ſines* $x = \frac{cz}{r} - \frac{sz^2}{2r^2} - \frac{cz^3}{2.3r^3} + \frac{sz^4}{2.3.4r^4}$ *&c.*

And the difference of the arches $z = \frac{r}{c} \times : x + \frac{sx^2}{2c^2} + \frac{rr+2ss}{6c^4}x^3 + \frac{3r^2+2ss}{8c^6}sx^4$ *&c.*

For by this prop. the ſine of $\overline{A+z}$, or $s + x = s + \frac{cz}{r} - \frac{sz^2}{2rr}$ &c. and $x = \frac{cz}{r} - \frac{sz^2}{2rr}$ &c. and by reverſion of ſeries, z is found as above.

Cor. 2. *The ſecond difference of the ſines of* $A - z$, A, $A + z$, *is* $= 2s \times : \frac{z^2}{1.2r^2} - \frac{z^4}{1.2.3.4r^4} + \frac{z^6}{1.2.3.4.5.6r^6} -$ *&c.* (for in any three things, the ſecond difference is equal to the difference between twice the mean, and the ſum of the extremes.)

PROP. XIV.

6. *The verſed ſine of an arch* DB *being given; to find the arch.*

Let $CD = r$. $DA = v$. $Au = \dot{v}$. $DB = z$. Then $AB = \sqrt{2rv - vv}$. The triangles CAB and B*sb* are ſimilar, therefore $\sqrt{2rv - vv} : r :: \dot{v} : Bb$ or $\dot{z} = \frac{r\dot{v}}{\sqrt{2rv - vv}}$. And the fluent is $z = \sqrt{2rv} \times : 1 + \frac{v}{3.2^2r}$

$\frac{v}{3.2^2r} + \frac{3v^2}{2^3.4.5r^2} + \frac{3.5v^3}{2^4.4.6.7r^3}$, &c. that is, arch DB $= \sqrt{2rv} + \frac{v}{3.4r}A + \frac{3^2v}{5.8r}B + \frac{5^2v}{7.12r}C + \frac{7^2v}{9.16r}D$, &c. Or rather DB $= \sqrt{dv} + \frac{v}{2.3d}A + \frac{3^2v}{4.5d}B + \frac{5^2v}{6.7d}C$, &c. puting $d =$ diameter, where A, B, C are the foregoing terms. FIG. 6.

Cor. 1. *In any arch* a, *the verſed ſine is* $= \frac{aa}{2r} - \frac{aa}{3.4r^2}A - \frac{aa}{5.6r^2}B - \frac{aa}{7.8r^2}C -$ &c. *where* A, B, C *are the foregoing terms with their ſigns.*

For radius — coſine is equal to the verſed ſine, and by Cor. 1. Pr. XII. the coſine is $r - \frac{a^2}{1.2r} + \frac{a^4}{1.2.3.4r^3}$ &c.

Cor. 2. *The coverſed ſine of the arch* a, *is* $= r - a + \frac{a^3}{2.3r^2} - \frac{a^5}{2.3.4.5r^4} + \frac{a^7}{2.3.4.5.6.7r^6}$ &c. for it is $=$ radius — ſine.

Cor. 3. *Verſed ſine of the ſupplement is* $= 2r - \frac{a^2}{1.2r^2}A - \frac{aa}{3.4r^2}B - \frac{aa}{5.6rr}C - \frac{aa}{7.8r^2}D$, &c. for it is $=$ radius $+$ coſine.

P R O P. XV.

The tangent DT *of an arch* DB *being given; to find the arch.* 6.

For draw Ct infinitely near CT, and T$n \perp$ Ct, and let CD $= r$, DT $= t$, T$t = \dot{t}$, DB $= z$, CT $= \sqrt{rr + tt}$.

The triangles CDT and Ttn are ſimilar, as alſo CBb, and CTn, whence CT : r :: $\dot{t}$ :: nT $= \frac{r\dot{t}}{\text{CT}}$,

and

FIG. 6. and CT : nT or $\frac{r\dot{t}}{\mathrm{CT}}$: : r : Bh $= \frac{rr\dot{t}}{\mathrm{CT}^2}$; that is, $\dot{z} = \frac{rr\dot{t}}{rr+tt}$.

And the fluent is $z = t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4} - \frac{t^7}{7r^6} +$ &c. that is, arch DB $= t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4} - \frac{t^7}{7r^6} + \frac{t^9}{9r^8}$ &c.

Cor. 1. *In any arch a, the tangent is* $= a + \frac{a^3}{3rr} + \frac{2a^5}{15r^4} + \frac{17a^7}{315r^6} + \frac{62a^9}{2835r^8} + \frac{1382a^{11}}{155925r^{10}} + \frac{21844a^{13}}{6081075r^{12}} + \frac{929569a^{15}}{638512875r^{14}} +$ &c.

This will appear by reverſion of the ſeries $a = t - \frac{t^3}{3r^2} + \frac{t^5}{5r^4}$ &c. Or rather by multiplying the ſine by the radius, and dividing by the coſine: thus

$$\frac{ra - \frac{a^3}{6r} + \frac{a^5}{120r^3} \text{ \&c.}}{r - \frac{aa}{2r} + \frac{a^4}{24r^3} \text{ \&c.}} = \text{tangent.}$$

Cor. 2. *The cotangent of an arch a is* $= \frac{rr}{a} - \frac{a}{3} - \frac{a^3}{45r^2} - \frac{2a^5}{945r^4} - \frac{a^7}{4725r^6} - \frac{2a^9}{93555r^8} - \frac{1382a^{11}}{638512875r^{10}} - \frac{4a^{13}}{18243225r^{12}} -$ &c. *as will appear by dividing radius ſquare by the value of the tangent in* Cor. 1. *Or the coſine by the ſine, and then multiplying by radius.*

Cor. 3. *Hence alſo if* τ *be the cotangent of an arch, then the arch* $= \frac{rr}{\tau} - \frac{r^4}{3\tau^3} + \frac{r^6}{5\tau^5} - \frac{r^8}{7\tau^7} + \frac{r^{10}}{9\tau^9} -$ &c. For $t = \frac{rr}{\tau}$.

PROP.

PROP. XVI.

FIG. 6.

The ſecant CT *of an arch* DB *being given; to find the arch.*

Let CD $= r$, CT $= ſ$, $nt = \dot{ſ}$, DT $= \sqrt{ſſ - rr}$, DB $= z$, then by ſimilar triangles, DT $: r :: \dot{ſ} : n$T $= \frac{r\dot{ſ}}{\text{DT}}$, and $ſ : r :: n$T or $\frac{r\dot{ſ}}{\text{DT}} : \dot{z} = \frac{rr\dot{ſ}}{ſ \times \text{DT}}$ or $\dot{z} = \frac{rr\dot{ſ}}{ſ\sqrt{ſſ - rr}}$, and the Fluent is $z = \frac{rr}{ſ} - \frac{r^4}{2.3ſ^3} - \frac{3r^6}{2.4.5ſ^5} - \frac{3.5r^8}{2.4.6.7ſ^7}$ &c. but in D, $z = 0$, and $ſ = r$; therefore the Fluent corrected is, arch DB or $z = r \times : \frac{ſ - r}{ſ} + \frac{ſ^3 - r^3}{2.3ſ^3} + \frac{3.\overline{ſ^5 - r^5}}{2.4.5ſ^5} + \frac{3.5.\overline{ſ^7 - r^7}}{2.4.6.7ſ^7}$ + &c.

Or thus, In the point F, $z =$ a quadrant Q, and $ſ =$ infinity. Therefore the whole ſeries $= 0$. Therefore by correction $z - \text{Q} = 0 - \frac{rr}{ſ} - \frac{r^4}{2.3ſ^3}$ &c. whence arch z or DB $= \text{Q} - \frac{rr}{ſ} - \frac{r^4}{2.3.ſ^3} - \frac{3r^6}{2.4.5ſ^5} - \frac{3.5r^8}{2.4.6.7ſ^7}$ — &c.

Cor. 1. *If* σ *be the coſecant of an arch, then the arch is* $= \frac{rr}{\sigma} + \frac{r^4}{2.3\sigma^3} + \frac{3r^6}{2.4.5\sigma^5} + \frac{3.5r^8}{2.4.6.7\sigma^7}$ + &c. *for this is the complement of the arch whoſe ſecant is* σ, *by this prop.*

Cor. 2. *In any arch a, the ſecant* $= r + \frac{aa}{2r} + \frac{5a^4}{24r^3} + \frac{61a^6}{720r^5} + \frac{277a^8}{8064r^7} + \frac{50521a^{10}}{3628800r^9} + \frac{540553a^{12}}{95800320r^{11}}$ + &c.

For radius ſquare (rr), divided by the coſine $(r - \frac{aa}{2} + \frac{a^4}{24r^3})$ &c. is = ſecant.

Cor. 3. *The coſecant of an arch a is* $= \frac{rr}{a} + \frac{a}{6} + \frac{7a^3}{360r^2} + \frac{31a^5}{15120r^4} + \frac{127a^7}{604800r^6} + \frac{73a^9}{3421440r^8} + \frac{1414477a^{11}}{653837184000r^{10}} +$ &c. *for radius ſquare divided by the ſine* $=$ *coſecant.*

SCHOLIUM.

If an arch be given in degrees, its length may be found thus. When the radius of a circle is 1, half the circumference is 3.14159265358979, therefore $\frac{3.14159 \text{ \&c.}}{180} = ,01745329251994 =$ length of 1 degree. Therefore if r be the *radius* of any circle, then $r \times ,01745$ &c. $=$ length of the arch of 1 degree in that circle. Conſequently $r \times ,01745329$ &c. $\times$ *number of degrees* and decimal parts, gives the length of the arch. And this muſt be taken for a or z in the foregoing propoſitions.

* * *

SECT. III.

The calculation of logarithmic ſines, tangents, and ſecants.

Tho' the finding the logarithmic ſines, tangents, &c. is no more than finding the logarithms of the natural ones by a table of logarithms: yet theſe logarithmic ſines, tangents, &c. may be found without the table of logarithms, or elſe without the natural ſines or tangents, and ſometimes without either, by having only the arch given.

The logarithmic or artificial ſines, tangents and ſecants are calculated to the radius 1 with 10 cyphers annext, viz. 10000000000; ſo that the log. radius will therefore be 10. But in calculating theſe ſines, tangents, or ſecants, we can more eaſily compute them for the radius 1, and then adding 10, gives the log. ſine, tangent, or ſecant, to the radius of the tables. Therefore,

In a circle whoſe radius is 1, the log. of that radius is 0, and the length of an arch of 1 degree $= \frac{3.1415926536}{180} =$,0174532925, and this number multiplied into any number of degrees, gives the length of the arch of theſe degrees, for the radius 1. and this length muſt be uſed in the following propoſitions, when the degrees are given.

Now if you put M = ,4342944819, and having any quantity given (expreſſing a ſine, tangent, &c.) you have no more to do but to divide the fluxion of that quantity by the quantity itſelf, and find the fluent by infinite ſeries, which multiplied by M is the logarithm of the quantity required.

PROP. XVII.

To find the logarithmic or artificial ſine of the arch x.

Let $y =$ nat. ſine of x, then by Prop. XII. $y = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ &c. where $r = 1$. And $\dot{y} = \dot{x} - \frac{x^2\dot{x}}{2} + \frac{x^4\dot{x}}{24} - \frac{x^6\dot{x}}{720}$ &c. then by diviſion $\frac{\dot{y}}{y} = \frac{\dot{x}}{x} - \frac{x\dot{x}}{3} - \frac{x^3\dot{x}}{45} - \frac{2x^5\dot{x}}{945} - \frac{x^7\dot{x}}{4725}$ &c. and Fluent of $M\frac{\dot{y}}{y}$, or the log. $y =$ log. $x - M \times : \frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \frac{x^8}{37800}$ + &c. to which add 10, and you have the logarithmic ſine in the tables; and that without knowing the natural ſine.

Otherwiſe.

Let $z = \frac{1-y}{1+y}$, where $y =$ nat. ſine; then will $y = \frac{1-z}{1+z}$, and $\frac{\dot{y}}{y} = \frac{-2\dot{z}}{1-zz} = -2 \times : \dot{z} + z^2\dot{z} + z^4\dot{z} + z^6\dot{z}$ &c. whence Fl. $M\frac{\dot{y}}{y}$, or log. $y = -2M \times : z + \frac{1}{3}z^3 + \frac{1}{5}z^5 + \frac{1}{7}z^7$ &c. And the log. ſine of the tables $= 10 - 2M \times : z + \frac{1}{3}z^3 + \frac{1}{5}z^5 + \frac{1}{7}z^7$ &c. which is had without the table of logarithms.

Or thus.

Let $y =$ nat. ſine, $z =$ coſine. Then $y = \sqrt{1-zz}$, and $\frac{\dot{y}}{y} = \frac{-z\dot{z}}{1-zz} = -z\dot{z} - z^3\dot{z} - z^5\dot{z}$ &c. whence Fl. $\frac{M\dot{y}}{y} = -\frac{1}{2}M \times : z^2 + \frac{z^4}{2} + \frac{z^6}{3} + \frac{z^8}{4}$ &c. and log. ſine of the tables $= 10 - \frac{1}{2}M \times : z^2 + \frac{1}{2}z^4 + \frac{1}{3}z^6 + \frac{1}{4}z^8$ &c.

And thus if you have any other line or quantity in the circle given, by which you can compute the ſine,

ſine, you may, from that, find the log. ſine. And the like may be done for the tangent, or ſecant.

Cor. *Hence if* $s =$ *log. ſine of* $\frac{1}{2}$A, *half an arch given; then the log. verſed ſine of* A $= .3010300 + 2s - 10$, *for the tables.* For rad. × verſed ſine $=$ 2 ſine ſquare of the half arch, by Cor. 2. Prop. II. and $.3010300 =$ log. 2.

Or thus. Log. verſ. of arch $x = 10.3010300 + 2 \log. \frac{1}{2}x - 2M \times : \frac{x^2}{4.6} + \frac{x^4}{4^2.180} + \frac{x^6}{4^3.2835} + \frac{x^8}{4^4.37800}$ &c. for s (or the log. ſine of $\frac{1}{2}x$) $= 10 + \log. \frac{1}{2}x, - M \times \frac{x^2}{4.6} + \frac{x^4}{16.180}$ &c,

Or laſtly. If $z =$ coſine, then log. verſ. $= 10 - M \times : z + \frac{z^2}{2} + \frac{z^3}{3} + \frac{z^4}{4} + \frac{z^5}{5}$ &c. For $1 - z =$ verſed ſine, and its fluxion $\frac{-\dot{z}}{1-z} = -\dot{z} - z\dot{z} - z^2\dot{z}$ &c. whoſe Fluent is $- M \times : z + \frac{1}{2}z^2 + \frac{1}{3}z^3$ &c. to which add 10.

PROP. XVIII.

The arch x being given, to find the log. coſine.

By Cor. 1. Pr. XII. the coſine $y = 1 - \frac{xx}{2} + \frac{x^4}{24} - \frac{x^6}{720}$ &c. therefore $\frac{\dot{y}}{y} = - x\dot{x} - \frac{1}{3}x^3\dot{x} - \frac{2}{15}x^5\dot{x} - \frac{17}{315}x^7\dot{x}$ &c. and the Fl. $\frac{M\dot{y}}{y}$ or log. $y = - M \times : \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \frac{17}{2520}x^8$ &c. And the tab. log. coſ. of the arch x is $= 10 - M \times : \frac{1}{2}x^2 +$

$\frac{1}{12}x^4 + \frac{1}{45}x^6 + \frac{17}{2520}x^8 +$ &c. And that without either the nat. coſine or the table of logarithms.

Or the log. coſine may be found in the ſame manner as the ſine, in the two laſt methods in Pr. XVII.

PROP. XIX.

Given an arch z, *and the ſine and coſine of an arch* A; *to find the log. ſine and coſine of the ſum* $\overline{A+z}$, *and alſo of the difference* $\overline{A-z}$.

Let the nat. ſine and coſine of A be s, c. $y = S.\overline{A+z}$. Then by Prop. XIII. $y = s + \frac{cz}{1} - \frac{sz^2}{2} - \frac{cz^3}{6} + \frac{sz^4}{24}$ &c. and $\frac{\dot{y}}{y} = \frac{c\dot{z}}{s} - \frac{z\dot{z}}{ss} + \frac{cz^2\dot{z}}{s^3} - \frac{1+2cc}{3s^4}z^3\dot{z}$ &c. and the Fl. $M\frac{\dot{y}}{y}$ or log. $y = M \times : \frac{cz}{s} - \frac{z^2}{2ss} + \frac{cz^3}{3s^3} - \frac{1+2cc}{12s^4}z^4$ &c. but when $z = 0$, $y = s$, therefore by correction,

Log. $y -$ log. $s = M \times : \frac{cz}{s} - \frac{z^2}{2ss}$ &c. whence the

Log. $S.\overline{A+z} =$ *log.* $s + M \times : \frac{cz}{s} - \frac{z^2}{2ss} + \frac{cz^3}{3s^3} - \frac{1+2cc}{12s^4}z^4$ &c. and after the ſame manner

Log. $S.\overline{A-z} =$ *log.* $s - M \times : \frac{cz}{s} + \frac{z^2}{2ss} + \frac{cz^3}{3s^3} + \frac{1+2cc}{12s^4}z^4$ &c.

Log.

$Log.$ coſ. $\overline{A + z} = log.\, c - M \times : \frac{sz}{c} + \frac{z^2}{2cc} + \frac{sz^3}{3c^3} + \frac{1 + 2ss}{12cc} z^4$ &c.

$Log.$ coſ. $\overline{A - z} = log.\, c + M \times : \frac{sz}{c} - \frac{z^2}{2cc} + \frac{sz^3}{3c^3} - \frac{1 + 2ss}{12c^4} z^4$ &c.

PROP. XX.

The arch x being given, to find its log. tangent.

By Cor. 1. Pr. XV. the tangent $t = x + \frac{1}{3} x^3 + \frac{2}{15} x^5 + \frac{17}{315} x^7$ &c. therefore $\frac{\dot{t}}{t} = \frac{\dot{x}}{x} + \frac{2}{3} x\dot{x} + \frac{14}{45} x^3\dot{x} + \frac{124}{945} x^5\dot{x}$ &c, whence log. tan. $t =$ log. $x + M \times : \frac{1}{3} x^2 + \frac{7}{90} x^4 + \frac{62}{2835} x^6 + \frac{127}{18900} x^8$ &c. to which add 10.

Otherwiſe.

To find the log. tan. of the arch $A + x$, *the tan. of the arch* A *being given.*

Let tan. $A = a$, tan. $x = t$, $y =$ tan. $\overline{A + x}$. Then by Prop. VIII. $y = \frac{a + t}{1 - at}$, and $\dot{y} = \frac{1 - aa\, \dot{t}}{\overline{1 - at}^2}$, and $\frac{\dot{y}}{y} = \frac{\overline{1 + aa} \,.\, \dot{t}}{a + \overline{1 - aa} \,.\, t - att}$. Let $b = \frac{1 + aa}{a}$, $c = \frac{1 - aa}{a}$, then $\frac{\dot{y}}{y} = b \times : \dot{t} - ct\dot{t} + \overline{1 + cc} \,.\, tt\dot{t} - \overline{2c + c^3} \,.\, t^3\dot{t}$ &c. and Fl. $M \frac{\dot{y}}{y}$ or log. $y = b M \times :$

$t - \frac{1}{2}ctt + \frac{1+cc}{3}t^3 - \frac{2c+c^3}{4}t^4$ &c. but when x and $t = 0$, $y = a$, then by correction, log. y — log. $a = bM \times : t - \frac{1}{2}ctt$ &c. that is,

Log. tan. $\overline{A+x} =$ *log. tan.* $A + bM \times : t - \frac{1}{2}ctt + \frac{1+cc}{3}t^3 - \frac{2c+c^3}{4}t^4$ &c. and the ſame for the *log. tan.* $\overline{A-x}$, changing the ſines of the odd powers of t.

But if $A = 45°$, then $a = 1$, and $\frac{\dot{y}}{y} = \frac{2\dot{t}}{1-tt} = 2\dot{t} + 2t^2\dot{t} + 2t^4\dot{t}$ &c. And conſequently *log.* y or Fl. $\frac{M\dot{y}}{y} = 2M \times : t + \frac{1}{3}t^3 + \frac{1}{5}t^5$ &c. that is,

Log. tan. $\overline{45+x} = 2M \times : t + \frac{t^3}{3} + \frac{t^5}{5} + \frac{t^7}{7}$, &c.

But if inſtead of t you put its value $x + \frac{1}{3}x^3 + \frac{2}{15}x^5$ &c. and the fluxion for the fluxion, then will $\frac{2\dot{t}}{1-tt} = 2 \times : \dot{x} + 2x^2\dot{x} + \frac{10}{3}x^4\dot{x} + \frac{244}{45}x^6\dot{x}$ &c. whence log. y, or *log. tan.* $\overline{45+x} = 2M \times : x + \frac{2}{3}x^3 + \frac{2}{3}x^5 + \frac{244}{315}x^7$ &c. and likewiſe

Log. tan. $\overline{45-x} = -2M \times : t + \frac{t^3}{3} + \frac{t^5}{5}$ &c. $= -2M \times : x + \frac{2}{3}x^3 + \frac{2}{3}x^5$ &c. to each of which add 10 for the tabular log. tangent.

Cor.

Cor. *The log. cot. of* $x = 10 - \text{log. } x - \text{M} \times : \frac{x^2}{3} + \frac{7}{90}x^4 + \frac{62}{2835}x^6 + \frac{127}{18900}x^8$ &c.

And log. cot. $\overline{\text{A} + x} =$ is the ſame as the *log. tan.* of $\overline{\text{A} - x}$, found before.

PROP. XXI.

To find the logarithmic ſecant of a given arch.

Let x be the arch, y its ſecant; then by Cor. 2. Prop. XVI. $y = 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \frac{61}{720}x^6$ &c. whence $\frac{\dot{y}}{y} = x\dot{x} + \frac{1}{3}x^3\dot{x} + \frac{2}{15}x^5\dot{x} + \frac{17}{315}x^7\dot{x}$ &c. then log. y or Fl. $\text{M}\frac{\dot{y}}{y} = \text{M} \times : \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + \frac{31}{14175}x^{10}$ &c. *hence the tab. log. ſecant* $=$ $10 + \text{M} \times : \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520}$ &c.

Or thus.

To find the log. ſecant of $\overline{45 + z}$, let log. ſecant $45 = s$. By Prop. XIX. log. coſ. $\overline{45 + z} =$ log. coſ. $45 - \text{M} \times : \frac{sz}{c} + \frac{z^2}{2cc}$ &c. $=$ (becauſe $s = c$, $r = 1$) log. coſ. $45 - \text{M} \times : z + zz$ &c. but by Cor. 2. Pr. I. ſecant × coſine = 1. Or log. ſecant = − log. coſine. Therefore, log. ſec. $\overline{45 + z} =$ log. ſec. $45 + \text{M} \times : z + zz +$ &c. that is,

Log. ſec. $\overline{45 + z} = s + \text{M} \times : z + z^2 + \frac{2}{3}z^3 + \frac{1}{3}z^4$ &c.

Cor. *The log. coſecant of an arch* x *is* $= 10 - \text{log. } x + \text{M} \times : \frac{x^2}{6} + \frac{x^4}{180} + \frac{x^6}{2835} + \frac{x^8}{37800}$ &c. For log. coſecant = − log. ſine, when radius is 1.

Or log. coſecant of $\overline{45+z}$, *or log. ſecant of* $\overline{45-z}$ $= s - M \times : z - z^2 + \frac{2}{3}z^3 - \frac{1}{3}z^4$ &c. by this prop. changing the ſigns of the odd powers of z.

SCHOLIUM I.

Having the log. ſine, tangent or ſecant given, the arch belonging thereto may be found by the reverſion of ſeries. As ſuppoſe there is given T, the *log. tan.* $\overline{45+x}$, then $T = 10 + 2M \times : x + \frac{2}{3}x^3 + \frac{2}{5}x^5$ &c. by Pr. XX; and $x + \frac{2}{3}x^3 + \frac{2}{5}x^5$ &c. $= \frac{T-10}{2M} = l$ ſuppoſe; then by reverſion $x = l - \frac{2}{3}l^3 + \frac{14}{15}l^5$ &c. and $\frac{x}{,0174532} =$ degrees in x.

Again, let there be given K, the log. ſecant of $\overline{45+z}$, then by Prop. XXI. $K = s + M \times : z + z^2 + \frac{2}{3}z^3$ &c. then $z + z^2 + \frac{2}{3}z^3$ &c. $= \frac{K-s}{M} = l$. Then by reverſion, $z = l - ll + \frac{4}{3}l^3$ &c. which divided by ,0174532 will give the degrees in z.

SCHOLIUM II.

In any arch let log. radius $= r = 10$.

$s =$ log. ſine. $t =$ log. tangent. $ſ =$ log. ſecant.
$c =$ log. coſine. $\tau =$ log. cotangent. $\sigma =$ log. coſecant.

Then

$$s = c + t - r = t + r - ſ = c + r - \tau = 2r - \sigma.$$
$$c = s + r - t = 2r - ſ = s + \tau - r = \tau + r - \sigma.$$
$$t = s + r - c = 2r - \tau.$$
$$\tau = c + r - s = 2r - t.$$
$$ſ = t + r - s = 3r - s - \tau = \sigma + r - \tau = 2r - c.$$
$$\sigma = \tau + r - c = 3r - c - t = ſ + r - t = 2r - s.$$

And log. verſ. $=$ 2 log. ſine of $\frac{1}{2}$ arch $-$ 9.6989700.

By

By what has been delivered in the two laſt ſections, the ſines, tangents and ſecants, of all arches, whether natural or artificial, may eaſily be found; and from thence the table of ſines and tangents may be conſtructed, with great eaſe and expedition. Or any particular numbers in the tables may be computed anew, and any errors corrected therein. But the nat. ſines and tangents being firſt found, the artificial ones are moſt eaſily had by a table of logarithms; for this reaſon I have not continued the ſeveral ſeries in this laſt ſection to any great number of places; intending rather to give the reader the principles of calculation than the calculation itſelf; ſince in all probability there will be few perſons who will take the pains to calculate them anew. But if any has a mind to do it, he may himſelf continue any of theſe ſeries as far as he pleaſes.

SECT. IV.

The calculation of the ſines, coſines, cords, &c. of multiple arches.

PROP. XXII.

FIG. 19. *If a trapezoid* BCDE, *whoſe ſides* BE, CD *are parallel, be inſcribed in a circle; and the cords* BD CE *be drawn; I ſay,* $BD^2 - DE^2 = BE \times CD$.

Make angle $EBO = CBD$, then $EBD = CBO$; and ſince $BEO = CDB$, therefore the triangles BCD and BOE are ſimilar, and $BD : CD :: BE : OE$, and $BD \times OE = BE \times CD$. Alſo ſince angle $BCO = BDE$, the triangles BCO and BDE are ſimilar, whence $BD : DE :: BC : CO$, and $BD \times CO = DE \times BC = DE^2$. Therefore $BD \times \overline{OE + OC} = BE \times CD + DE^2$; that is, $BD \times CE$ or $BD^2 = BE \times CD + DE^2$, or $BE \times CD = BD^2 - DE^2 = \overline{BD + DE} \times \overline{BD - DE}$.

Cor. 1. *If* A *be any arch, and* BC *or* $DE = n$ *times* A, *and* $BD = m$ *times* A. *Then,*

As cord of $\overline{m - n}$. A :
Cord . mA + *cord* . nA ::
Cord . mA — *cord* . nA :
Cord . $\overline{m + n}$. A.

For CD is the cord of $m - n$ times A, and BE of $\overline{m + n} \times$ A.

Cor. 2. *As cord of* A :
Cord . $\overline{n + 1}$. A + *cord* . nA ::
Cord . $\overline{n + 1}$. A — *cord* . nA :
Cord . $\overline{2n + 1}$. A.

This appears by putting $m = n + 1$, or $CD = A$.

Cor.

FIG. 19.

Cor. 3. *As cord of* $\overline{m-1}\,.\,A$:
Cord . m A + *cord* . A : :
Cord . m A — *cord* . A :
Cord $\overline{m+1}\,.\,A$.
This appears by puting $n = 1$, or CB or DE = A.

Cor. 4. *If three arches* CD, BCD, BCDE *are in arithmetic progreſſion*, $BD^2 - BC^2 = CD \times BE$.

PROP. XXIII.

In the circle AEB, *whoſe diameter is* AB, *if the arches* AD, EF, FG *be taken equal to one another, and the cords* BD, BE, BF, BG *drawn. I ſay,*

As radius CB : *cord* BD : : *ſo the middle cord* BF : BE + BG, *the ſum of the extreme cords.*

For produce BE to H, and make FH = FB; then ∠BHF = HBF = FBG ; and draw the cords EF, FG. Then the angle BEF = BFG + FBG, both ſtanding on the ſame arch FGB. Alſo BEF = EFH + EHF = EFH + FBG, therefore EFH = BFG. And the triangles EFH, FBG are ſimilar and equal. Whence HE = GB, and HB = EB + BG. Alſo the triangles DCB and HFB are ſimilar, therefore BC : BD : : BF : BH or EB + BG.

Cor. *If arch* AD = A, *arch* BF = $n \times$ A. *Then*
As rad :
Cord . *ſup.* A : :
Cord . nA :
Cord . $\overline{n-1}\,.\,A$ + *cord* . $\overline{n+1}\,.\,A$.
And the ſame holds in reſpect of the cords of the ſupplements of theſe arches, if AF be ſuppoſed to be made equal to nA.

SCHOLIUM.

If the cord BG lie on the other ſide of B, then BG is negative.

PROP. XXIV.

The ſine and coſine of an arch being given; to find the ſine of n times that arch.

Let $x =$ coſine, $y =$ ſine, given. And A the given arch, and at preſent let radius $= 1$. Then by Cor. 7. Prop. III. $2x \times \text{S}.\, n\text{A} = \text{S}.\, \overline{n-1}\,.\, \text{A} + \text{S}.\, \overline{n+1}\,.\, \text{A}$, whence $\text{S}.\, \overline{n+1}\,.\, \text{A} = 2x \times \text{S}.\, n\text{A} - \text{S}.\, \overline{n-1}\,.\, \text{A}$. Or becauſe $xx + yy = 1\,.\, \text{S}.\, \overline{n+1}.\text{A} = 2x \times \text{S}.\, n\text{A} - \overline{xx+yy} \times \text{S}.\, \overline{n-1}\,.\, \text{A}$. That is, if any ſine be multiplied by $2x$, and the next preceding one multiplied by $xx + yy$, and ſubtracted from it, gives the next following ſine. Thus you will have

$$\begin{aligned}
&\text{S}.\ \text{A} = y.\\
&\text{S}.\ 2\text{A} = 2xy.\\
&\text{S}.\ 3\text{A} = 3x^2y - y^3.\\
&\text{S}.\ 4\text{A} = 4x^3y - 4xy^3.\\
&\text{S}.\ 5\text{A} = 5x^4y - 10x^2y^3 + y^5.\\
&\text{\&c.}
\end{aligned}$$

In what follows, if a denotes any variable quantity, then I denote its firſt and ſecond ſucceeding values by $\underset{1}{a}$, $\underset{2}{a}$, &c. and its firſt and ſecond preceding values by $\underset{-1}{a}$, $\underset{-2}{a}$, &c. and ſo of others. And with theſe I proceed according to the method of increments in calculating the following propoſitions.

Suppoſe then any arch, as nA, is repreſented in general thus, $\text{S}.\, n\text{A} = nx^{n-1}y - ax^{n-3}y^3 + bx^{n-3}y^5$ — &c. then according to the conſtruction of ſines, before mentioned, we ſhall have

$2nx^ny$

$$\begin{cases} 2nx^{n}y - 2ax^{n-2}y^{3} + 2bx^{n-4}y^{5} - \&c. = S.nA \times 2x, \\ \left.\begin{array}{l} -\underset{-1}{n}x^{n}y + \underset{-1}{a}x^{n-2}y^{3} - \underset{-1}{b}x^{n-4}y^{5} - \&c. \\ \qquad\quad -\underset{-1}{n} \qquad\qquad\quad +\underset{-1}{a} \end{array}\right\} = \overline{xx+yy} \times S.\overline{n-1}.A. \end{cases}$$

$$= \overline{n+1}.x^{n}y - \underset{1}{a}x^{n-2}y^{3} + \underset{1}{b}x^{n-4}y^{5} - \&c. = S.\overline{n+1}.A.$$

Hence equating the coefficients of the homologous powers, $n+1 = 2n - \underset{-1}{n}$, and $1 = n - \underset{-1}{n}$; that is, $\underset{\cdot}{n} = 1$. Likewise $\underset{1}{a} = 2a - \underset{-1}{a} + \underset{-1}{n}$, or $\underset{2}{a} = 2\underset{1}{a} - a + n$, whence $\underset{2}{a} - 2\underset{1}{a} + a = n$; that is, $\underset{\cdot\cdot}{a} = n$. And the integral is $\underset{\cdot}{a} = \frac{\underset{-1}{n}n}{2}$, and the integral of this is $a = \frac{\underset{-2}{n}\,\underset{-1}{n}\,n}{2.3}$ (for when $n = 3$, $a = \frac{\underset{-2}{n}\,\underset{-1}{n}\,n}{2.3} = 1$). Again, $\underset{1}{b} = 2b - \underset{-1}{b} + \underset{-1}{a}$, or $\underset{2}{b} = 2\underset{1}{b} - b + a$, whence $\underset{2}{b} - 2\underset{1}{b} + b = a$, or $\underset{\cdot\cdot}{b} = a = \frac{\underset{-2}{n}\,\underset{-1}{n}\,n}{2.3}$, whence $\underset{\cdot}{b} = \frac{\underset{-3}{n}\,\underset{-2}{n}\,\underset{-1}{n}\,n}{2.3.4}$, and $b = \frac{\underset{-4}{n}\,\underset{-3}{n}\,\underset{-2}{n}\,\underset{-1}{n}\,n}{2.3.4.5}$. In like manner $\underset{\cdot\cdot}{c} = b$, and $c = \frac{\underset{-6}{n}\,\underset{-5}{n}\,\underset{-4}{n}\,\underset{-3}{n}\,\underset{-2}{n}\,\underset{-1}{n}\,n}{2.3.4.5.6.7}$ &c. *whence* $S.nA = nx^{n-1}y$

$$- \frac{n.\overline{n-1}.\overline{n-2}}{1.2.3}x^{n-3}y^{3} +$$

$$\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{1.2.3.4.5}x^{n-5}y^{5},$$

&c. Or puting r = rad. $S.nA = \frac{nx^{n-1}y}{r^{n-1}} - \frac{\overline{n-1}.\overline{n-2}.yy}{2.3xx}A - \frac{\overline{n-3}.\overline{n-4}.y^{2}}{4.5xx}B - \frac{\overline{n-5}.\overline{n-6}.yy}{6.7xx}C$ &c. where A, B, C, &c. are the preceding terms with their ſigns.

Cor.

Cor. *If you ſuppoſe r to be the diameter, y the cord of an arch, x the cord of the ſupplement; then the foregoing ſeries will be the cord of the multiple arch.*

PROP. XXV.

The ſine of an arch A *being given; to find the ſine of* nA, *or n times that arch.*

Let $s =$ ſine of A, $z =$ ſine of nA. Proceeding as in Prop. XI. we ſhall have $\dot{A} = \frac{r\dot{s}}{\sqrt{rr - ss}}$, and $n\dot{A} = \frac{r\dot{z}}{\sqrt{rr - zz}} = \frac{rn\dot{s}}{\sqrt{rr - ss}}$, whence $\overline{rr - ss} \times \dot{z}^2 = nn\dot{s}^2 \times \overline{rr - zz}$, this in fluxions (puting $\dot{s}$ conſtant), will be $\overline{rr - ss} \times 2\dot{z}\ddot{z} - 2s\dot{s}\dot{z}^2 = -2nn\dot{s}^2z\dot{z}$; and $rr\ddot{z} - ss\ddot{z} - s\dot{s}\dot{z} + nnz\dot{s}^2 = 0$.

Aſſume $z = as + bs^3 + cs^5 + ds^7$ &c.

then $\dot{z} = a\dot{s} + 3bss\dot{s} + 5cs^4\dot{s} + 7ds^6\dot{s}$ &c.

and $\ddot{z} = 2.3bs\dot{s}^2 + 4.5cs^3\dot{s}^2 + 6.7ds^5\dot{s}^2$ &c.

Then by ſubſtitution,

$$\left.\begin{array}{r|lllll} rr\ddot{z} & 2.3brrs\dot{s}^2 & + 4.5crrs^3\dot{s}^2 & + 6.7drrs^5\dot{s}^2 \text{ \&c.} \\ -ss\ddot{z} & & - 2.3bs^3\dot{s}^2 & - 4.5cs^5\dot{s}^2 \text{ \&c.} \\ -s\dot{s}\dot{z} & - as\dot{s}^2 & - 3bs^3\dot{s}^2 & - 5cs^5\dot{s}^2 \text{ \&c.} \\ + nnz\dot{s}^2 & + nnas\dot{s}^2 & + nnbs^3\dot{s}^2 & + nncs^5\dot{s}^2 \text{ \&c.} \end{array}\right\} = 0.$$

Now when A is infinitely ſmall, $s =$ A, and $z = n$A $= ns = as$, therefore $a = n$. And equating the coefficients, $b = -\frac{nn - 1}{2.3rr}a$, $c = -\frac{nnb - 2.3b - 3b}{4.5rr} = -\frac{nn - 3.3}{4.5rr}b$, $d = -\frac{nn - 5.5}{6.7rr}c$, $e = -\frac{nn - 7.7}{8.9rr}d$, &c. therefore $z = ns - \frac{nn - 1}{2.3rr}as^3 - \frac{nn - 3.3}{4.5rr}bss$

$\frac{nn-3.3}{4.5rr}bs^5$ &c. $= ns - \frac{n.\overline{nn-1}}{2.3rr}s^3 + \frac{n.\overline{nn-1}.\overline{nn-9}}{2.3.4.5r^4}s^5$ &c. and ſince coſ. A $=$ $\sqrt{rr-ss} = r - \frac{ss}{2r} - \frac{s^4}{8r^3} - \frac{s^6}{16r^5}$ &c. therefore

$$\frac{z}{\sqrt{rr-ss}} = \frac{ns - \frac{n.\overline{nn-1}}{2.3rr}s^3 + \frac{n.\overline{nn-1}.\overline{nn-9}}{2.3.4.5r^4}s^5 \text{ \&c.}}{r - \frac{ss}{2r} - \frac{s^4}{8r^3} \text{ \&c.}}$$

$$= \frac{ns}{r} - \frac{nn-2.2}{2.3r^3}ns^3 + \frac{n.\overline{nn-2.2}.\overline{nn-4.4}}{2.3.4.5r^5}s^5 \text{ \&c.}$$

whence we have

z *or* $S.nA = ns - \frac{nn-1}{2.3rr}ssA - \frac{nn-3.3}{4.5rr}ssB - \frac{nn-5.5}{6.7rr}ssC - \frac{nn-7.7}{8.9rr}ssD$ &c. *where* A, B, C &c. *are the preceding terms with their ſigns; therefore if n be an odd number, the ſeries will be finite.*

Or $S.nA = \sqrt{rr-ss} \times : \frac{ns}{r} - \frac{nn-2.2}{2.3rr}ssA - \frac{nn-4.4}{4.5rr}ssB - \frac{nn-6.6}{6.7rr}ssC - \frac{nn-8.8}{8.9rr}ssD$ &c. *and therefore if n be an even number the ſeries will be finite.*

Cor. *If s be the cord of an arch, x the cord of the ſupplement, d = diameter; then*

Cord of $nA = ns - \frac{nn-1}{2.3dd}ssA - \frac{n^2-3^2}{4.5.d^2}ssB - \frac{n^2-5^2}{6.7d^2}ssC - \frac{n^2-7^2}{8.9d^2}ssD$ &c. *or*

$$\text{Cord of } n \text{ arch} = \frac{nsx}{d} - \frac{n^2 - 2^2}{2.3d^2}s^2A - \frac{n^2 - 4^2}{4.5.d^2}ssB - \frac{n^2 - 6^2}{6.7d^2}s^2C - \&c.$$

SCHOLIUM.

If z the ſine of an arch was given, and the ſine s, of the n^{th} part of the arch, was required; you'll have by reverſion of ſeries,

$$s = \frac{z}{n} + \frac{\overline{n^2 - 1}\,.\,z^2}{2.3r^2n^2}A + \frac{\overline{3n}|^2 - 1}{4.5r^2n^2}Bz^2 + \frac{\overline{5n}|^2 - 1}{6.7r^2n^2}Cz^2 \;\&c.$$

PROP. XXVI.

The ſine and coſine of an arch being given; to find the coſine of n times that arch.

Let A = arch, $y =$ ſine, $x =$ coſine. Radius = 1. Then by Cor. 7. Pr. III. $2x \times$ coſ. $nA =$ coſ. $\overline{n - 1}.A +$ coſ. $\overline{n + 1}.A$, therefore ſince $xx + yy = 1$, coſ. $\overline{n + 1}.A = 2x \times$ coſ. $nA - \overline{xx + yy} \times$ coſ. $\overline{n - 1}.A$. Therefore if you multiply any coſine by $2x$, and from it ſubtract the foregoing one multiplied into $xx + yy$, the remainder is the next ſucceeding coſine. Hence

Coſ. 0A $= r$
Coſ. 1A $= x$
Coſ. 2A $= xx - yy$
Coſ. 3A $= x^3 - 3xyy$
Coſ. 4A $= x^4 - 6x^2y^2 + y^4$
Coſ. 5A $= x^5 - 10x^3y^2 + 5xy^4$
Coſ. 6A $= x^6 - 15x^4y^2 + 15x^2y^4 - y^6$
Coſ. 7A $= x^7 - 21x^5y^2 + 35x^3y^4 - 7xy^6$,

&c. Therefore in general, let the

Coſ.

$\text{Cos.}\,\overline{n-1}.\text{A} = x^{n-1} - ax^{n-3}y^2 + bx^{n-5}y^4 - cx^{n-7}y^6$ &c.

$\text{Cos. } n\text{A} = x^n - \underset{1}{a}x^{n-2}y^2 + \underset{1}{b}x^{n-4}y^4 - \underset{1}{c}x^{n-6}y^6$ &c.

Now if the latter be multiplied by $2x$, and the former by $xx + yy$, and this product ſubtracted from the firſt, you will have

$$\left.\begin{array}{l} 2x^{n+1} - 2\underset{1}{a}x^{n-1}y^2 + 2\underset{1}{b}x^{n-3}y^4 - 2\underset{1}{c}x^{n-5}y^6 \text{ \&c.} \\ -x^{n+1} + ax^{n-1}y^2 - bx^{n-3}y^4 + cx^{n-5}y^6 \\ \qquad\quad - x^{n-1}y^2 + ax^{n-3}y^4 - bx^{n-5}y^6 \end{array}\right\}$$

$$= x^{n+1} - \underset{2}{a}x^{n-1}y^2 + \underset{2}{b}x^{n-3}y^4 - \underset{2}{c}x^{n-5}y^6 \text{ \&c.}$$

$= \text{cos.}\,\overline{n+1}\,.\,\text{A}$. Then comparing the coefficients, we have $\underset{2}{a} = 2\underset{1}{a} - a + 1$, that is, by the method of increments, $a + 2\underset{.}{a} + \underset{..}{a} = 2a + 2\underset{.}{a} - a + 1$, or $\underset{..}{a} = 1 = \underset{.}{n}$; therefore $\underset{.}{a} = n$, the integral, and $\underset{1}{a} = \frac{\overset{n\;n}{-1}}{2}$, for when $n = 2$, $\underset{1}{a}$ or $\frac{\overset{n\;n}{-1}}{2} = 1$. Likewiſe $\underset{2}{b} = 2\underset{1}{b} - b + a$, and $\underset{..}{b} = a = \frac{\overset{n\;\;n}{-2-1}}{2}$, and $\underset{.}{b} = \frac{\overset{n\;\;n\;\;n}{-3-2-1}}{2\cdot3}$, and $b = \frac{\overset{n\;\;n\;\;n\;\;n}{-4-3-2-1}}{2\cdot3\cdot4}$, (for when $n = 5$, $b = 1$.) and $\underset{1}{b} = \frac{\overset{n\;\;n\;\;nn}{-3-2-1}}{2\cdot3\cdot4}$. Again, $\underset{..}{c} = b = \frac{\overset{n\,.\,.\,n}{-4\;\;-1}}{2\cdot3\cdot4}$, $\underset{.}{c} = \frac{\overset{n\,.\,.\,n}{-5\;\;-1}}{2\cdot3\cdot4\cdot5}$, and $c = \frac{\overset{n\,.\,.\,n}{-6\;\;-1}}{2\,.\,.\,.\,.\,6}$, and therefore $\underset{1}{c} = \frac{\overset{n\ldots n}{-5}}{2\,.\,.\,.\,.\,6}$. Likewiſe $\underset{1}{d} = \frac{\overset{n\ldots n}{-7}}{2\,.\,.\,.\,7}$, &c.

then ſince $\underset{\cdot}{n} = 1$, therefore n, $\underset{-1}{n}$, $\underset{-2}{n}$, &c. are equal to n, $n-1$, $n-2$, &c. whence puting r for radius,

$$Coſ.\, nA = \frac{1}{r^{n-1}} \times : x^n - \frac{n.\overline{n-1}}{2} x^{n-2}y^2 + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{2.3.4} x^{n-4}y^4 - \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}.\overline{n-5}}{2.3.4.5.6} x^{n-6}y^6$$

&c. or

$$Coſ.\, nA = \frac{x^n}{r^{n-1}} - \frac{n.\overline{n-1}}{1.2} A\frac{yy}{xx} - \frac{\overline{n-2}.\overline{n-3}}{3.4} B\frac{yy}{xx} - \frac{\overline{n-4}.\overline{n-5}}{5.6} C\frac{yy}{xx} - \frac{\overline{n-6}.\overline{n-7}}{7.8} D\frac{yy}{xx},$$

&c. *where* A, B, C &c. *are the foregoing terms with their ſigns.*

Cor. *If you ſuppoſe* r *the diameter*, y *the cord of an arch*, x *the cord of the ſupplement; then the foregoing ſeries will be the cord of the ſupplement of the multiple arch.*

PROP. XXVII.

The ſine s *of an arch* a *being given; to find the coſine of* na, *or* n *times that arch.*

Let $z =$ coſ. $n\,a$, rad. $= 1$, then by Cor. 1. Pr. 12. $z = 1 - \frac{n^2a^2}{2} + \frac{n^4a^4}{24} - \frac{n^6a^6}{720} + \frac{n^8a^8}{40320}$ &c. but by

Prop. 11. $a = s + \frac{s^3}{6} + \frac{3s^5}{40} + \frac{5s^7}{112}$ &c.

whence $aa = ss + \frac{s^4}{3} + \frac{8s^6}{45} + \frac{4s^8}{35}$ &c.

and

and
$$\begin{array}{llll} a^4 = & s^4 + \frac{2}{3}s^6 & + \frac{7}{15}s^8 \ \&c. \\ a^6 = & s^6 & + s^8 \\ a^8 = & & s^8 \\ \&c. \end{array}$$

then all theſe values of the powers of a being ſubſtituted in the value of z, gives

$$\begin{array}{lllll} z = 1 - \frac{nnss}{2} & - \frac{n^2s^4}{6} & - \frac{4n^2s^6}{45} & - \frac{2n^2s^8}{35} \ \&c. \\ & + \frac{n^4s^4}{24} & + \frac{n^4s^6}{36} & + \frac{7n^4s^8}{360} \\ & & - \frac{n^6s^6}{720} & - \frac{n^6s^8}{720} \\ & & & + \frac{n^8s^8}{40320} \ \&c. \end{array}$$

whence

$$z = 1 - \frac{n^2}{2}s^2 + \frac{nn.\overline{n^2-2^2}}{2.3.4}s^4 - \frac{n^2.\overline{n^2-2^2}.\overline{n^2-4^2}}{2.3.4.5.6}s^6$$

&c. that is, if r = radius.

$$\text{Cof. } na = r - \frac{n^2s^2}{1.2r^2}\text{A} - \frac{n^2-2^2}{3.4r^2}ss\text{B} - \frac{n^2-4^2}{5.6r^2}ss\text{C} - \frac{n^2-6^2}{7.8r^2}ss\text{D} \ \&c.$$

where A, B, C &c. *are the foregoing terms with their ſigns. Therefore, if n be an even number, the ſeries will be finite.*

And ſince $\sqrt{rr-ss} = r - \frac{ss}{2r} - \frac{s^4}{8r^3} - \frac{s^6}{16r^5} - \frac{5s^8}{128r^7}$ &c. = coſ. A, therefore dividing the foregoing ſeries by this, and then multiply by $\sqrt{rr-ss}$, and you will have $\text{coſ. } na = \sqrt{rr-ss} \times :$

$$1 - \frac{n^2-1}{1.2r^2}ss\text{A} - \frac{n^2-3^2}{3.4r^2}ss\text{B} - \frac{n^2-5^2}{5.6r^2}ss\text{C} - \frac{n^2-7^2}{7.8r^2}ss\text{D}$$

$\frac{n^2 - 7^2}{7.8r^2}ssD$ &c. *and therefore if n be an odd number, the ſeries will be finite.*

Cor. *The foregoing ſeries will expreſs the cord of the ſupplement of the n^{th} arch; puting s for the cord of the ſingle arch, and r for the diameter.*

PROP. XXVIII.

The cord of an arch, and the cord of its ſupplement being given; to find the cord of n times that arch.

Let A be the arch, y the cord, x the cord of the ſupplement, radius $= 1$; then by cor. Pr. XXIII. $x \times \text{cord} . nA = \text{cord} . \overline{n - 1} . A + \text{cord} . \overline{n + 1} . A$, and $\text{cord} . \overline{n + 1} . A = x \times \text{cord} . nA - \text{cord} . \overline{n - 1} . A$; that is, if any cord be multiplied by x, and the foregoing one ſubtracted, will give the next following cord. Whence

$$\text{Cord} . \; A = y$$
$$\text{Cord} . \; 2A = xy$$
$$\text{Cord} . \; 3A = x^2y - y$$
$$\text{Cord} . \; 4A = x^3y - 2xy$$
$$\text{Cord} . \; 5A = x^4y - 3x^2y + y, \text{ \&c.}$$

And in general, let the cord of nA be $= yx^{n-1} - ax^{n-3}y + bx^{n-5}y - cx^{n-7}y$ &c. then will

$$\left.\begin{array}{l} yx^n - ayx^{n-2} + byx^{n-4} - cyx^{n-6} \text{ \&c.} \\ \quad\;\; - yx^{n-2} + \underset{-1}{a}yx^{n-4} - \underset{-1}{b}yx^{n-6} \text{ \&c.} \end{array}\right\} =$$

$$yx^n - \underset{1}{a}yx^{n-2} + \underset{1}{b}yx^{n-4} - \underset{1}{c}yx^{n-6} \text{ \&c.} = \text{cord} . \overline{n + 1} . A.$$

Then equating the coefficients $\underset{1}{a} = a + 1$, and $\underset{1}{a} - a$ or $\underset{\cdot}{a} = 1 = \underset{\cdot}{n}$, and the integral is $a = \underset{-2}{n}$ (for when $n = 3$, a or $\underset{-2}{n} = 1$). Again, $\underset{1}{b}$ or

$b +$

$b + \underset{1}{b} = b + \underset{-1}{a}$, or $\underset{\cdot}{b} = \underset{-1}{a} = \underset{-3}{n}$, and $b = \frac{\overset{n}{-3}\overset{n}{-4}}{2}$, alſo $\underset{1}{c} = c + \underset{-1}{b}$, or $\underset{\cdot}{c} = \underset{-1}{b} = \frac{\overset{n}{-4}\overset{n}{-5}}{2}$, and $c = \frac{\overset{n}{-4}\overset{n}{-5}\overset{n}{-6}}{2\cdot 3}$. In like manner $d = \frac{\overset{n}{-5}\overset{n}{-6}\overset{n}{-7}\overset{n}{-8}}{2\cdot 3\cdot 4}$, &c. whence the *cord* . $n\mathrm{A} = yx^{n-1} - \frac{n-2}{1}yx^{n-3} + \frac{\overline{n-3}\cdot\overline{n-4}}{1\cdot 2}yx^{n-5} - \frac{\overline{n-4}\cdot\overline{n-5}\cdot\overline{n-6}}{1\cdot 2\cdot 3}yx^{n-7} + \frac{\overline{n-5}\cdot\overline{n-6}\cdot\overline{n-7}\cdot\overline{n-8}}{1\cdot 2\cdot 3\cdot 4}yx^{n-9}$ &c. that is, puting radius $= r$, then

$$Cord\ .\ n\ arch = \frac{yx^{n-1}}{r^{n-1}} - \frac{n-2}{1}\mathrm{A}\,\frac{rr}{xx} - \frac{\overline{n-3}\cdot\overline{n-4}}{2\cdot\overline{n-2}}\mathrm{B}\,\frac{rr}{xx} - \frac{\overline{n-5}\cdot\overline{n-6}}{3\cdot\overline{n-3}}\mathrm{C}\,\frac{rr}{xx} - \frac{\overline{n-7}\cdot\overline{n-8}}{4\cdot\overline{n-4}}\mathrm{D}\,\frac{rr}{xx} - \frac{\overline{n-9}\cdot\overline{n-10}}{5\cdot\overline{n-5}}\mathrm{E}\,\frac{rr}{xx}\ \text{\&c.}$$

Cor. *If* r = *radius*, y = *ſine*, x = *coſine of an arch, then the ſine of the* n^{th} *arch* $= \frac{\overline{2x}^{\,n-1}}{r^{n-1}}y - \frac{\overline{n-2}\cdot rr}{4xx}\mathrm{A} - \frac{\overline{n-3}\cdot\overline{n-4}\cdot rr}{4\cdot 2\cdot\overline{n-2}\cdot xx}\mathrm{B} - \frac{\overline{n-5}\cdot\overline{n-6}\cdot rr}{4\cdot 3\cdot\overline{n-3}\cdot xx}\mathrm{C} - \frac{\overline{n-7}\cdot\overline{n-8}\cdot rr}{4\cdot 4\cdot\overline{n-4}\cdot xx}\mathrm{D}$ &c. this is plain by puting $\frac{1}{2}r$ for r in the foregoing ſeries.

PROP. XXIX.

The ſquare of the cord of an arch being given; to find the ſquare of the cord of n times the arch.

Let a, b, c be the cords of three arches in arithmetic progreſſion, rad. $= 1$, $x =$ cord of the common difference, and $\sqrt{4-xx} =$ cord of the ſup. common difference. Then by Prop. XXIII. rad. × ſum of the cords of the extreme arches = cord mean × cord ſup. common difference; that is, $b\sqrt{4-xx} = a + c$. And by Cor. 4. Pr. XXII. $ac = bb - xx$. Whence $4bb - bbxx = aa + cc + 2ac = aa + cc + 2bb - 2xx$, therefore $bb \times \overline{2-xx} + 2xx - aa = cc$; that is, in any ſeries of the ſquares of cords, if the ſquare of any cord be multiplied by $-xx + 2$, and the preceding one ſubtracted, and $2xx$ added, you will have the ſquare of the next following cord. Thus, ſquare of the cord

$$\begin{array}{ll}
\text{of } A = & xx \\
2A = & -x^4 + 4x^2 \\
3A = & x^6 - 6x^4 + 9x^2 \\
4A = & -x^8 + 8x^6 - 20x^4 + 16x^2 \\
5A = & x^{10} - 10x^8 + 35x^6 - 50x^4 + 25x^2 \\
\&c. &
\end{array}$$

therefore let any one in general be expreſſed thus,

$$x^{v} - ax^{v-2} + bx^{v-4} - cx^{v-6} \ \&c.$$

then the next ſucceeding one (changing the ſigns) will be $-\overset{v}{x'} + a\overset{v-2}{x'} - \underset{1}{b}\overset{v-4}{x'} + \underset{1}{c}\overset{v-6}{x'}$ &c. =

(by conſtruction to)

$$\left.\begin{array}{rrrr}
-x^{v+2} + ax^{v} & - bx^{v-2} & + cx^{v-4} \\
+ 2x^{v} & - 2ax^{v-2} & + 2bx^{v-4} \\
 & + x^{v-2} & - ax^{v-4} \\
 & & -1 \\
\end{array}\right\} \&c. + 2xx.$$

Then

Then comparing the homologous terms, we have, $\underset{1}{v} = v + 2$, or $\underset{1}{v} - v$, that is, $\underset{\cdot}{v} = 2 = 2\underset{\cdot}{n}$, and $v = 2n$. Again, $\underset{1}{a} = a + 2$, or $\underset{1}{a} - a$ or $\underset{\cdot}{a} = 2 = \underset{\cdot}{v}$, whence $a = v = 2n$. Alſo $\underset{1}{b} = b + 2a - 1$, or $\underset{\cdot}{b} = 2a - 1 = 2v - \frac{1}{2}\underset{\cdot}{v} = \underset{\cdot}{v}v - \frac{1}{2}\underset{\cdot}{v}$, therefore $b = \frac{\overset{v\,v}{-1}}{2} - \frac{v}{2}$.

Again, $\underset{1}{c} = c + 2b - \underset{-1}{a}$, or $\underset{\cdot}{c} = \underset{-1}{v}v - v - \underset{-1}{v} = \underset{-1}{v}v - 2v + 2 = \underset{-1}{v}v - 2v + \underset{\cdot}{v}$, and $c = \frac{\overset{v}{-2}\,\overset{v\,v}{-1}}{2\cdot 3} - \frac{\overset{v\,v}{-1}}{2} + v$.

In like manner $\underset{\cdot}{d} = 2c - \underset{-1}{b} = \frac{\overset{v}{-2}\,\overset{v\,v}{-1}}{3} - \frac{3\overset{v\,v}{-1}}{2} + \frac{9v}{2} - \frac{5\underset{\cdot}{v}}{2}$; and $d = \frac{\overset{v}{-3}\,\overset{v}{-2}\,\overset{v\,v}{-1}}{2\cdot 3\cdot 4} - \frac{\overset{v}{-2}\,\overset{v\,v}{-1}}{2\cdot 2} + \frac{\overset{9v\,v}{-1}}{2.2.2} - \frac{5v}{2}$.

But ſince $\underset{\cdot}{v} = 2 \,.\, \underset{-1}{v} = v - 2 \,.\, \underset{-2}{v} = \overline{v - 4}$ &c. we ſhall find $b = v \times \frac{v - 3}{2}$, $c = v \times \frac{v - 4}{2} \times \frac{v - 5}{3}$, $d = v \times \frac{\overline{v - 5} \times \overline{v - 6} \times \overline{v - 7}}{2\cdot 3\cdot 4}$ &c. *therefore if x is the cord of an arch, and b the cord of n times the arch, then will*

$$x^{2n} - 2nx^{2n-2} + 2n \times \frac{2n - 3}{2}x^{2n-4} - 2n \times \frac{\overline{2n - 4}}{2}$$

$$\frac{\overline{2n-4}\,.\,\overline{2n-5}}{2\,.\,3}x^{2n-6} + 2n \times \frac{\overline{2n-5}\,.\,\overline{2n-6}\,.\,\overline{2n-7}}{2\,.\,3\,.\,4}x^{2n-8}\ \text{\&c.} = \pm\, bb,$$

according as n is odd or even; and the ſeries continued to n terms. The laſt term being always nnxx.

$$\text{Or}\ \frac{x^{2n}}{r^{2n-2}} - \frac{2nr^2}{xx}A - \frac{\overline{2n-3}}{2xx}rrB - \frac{\overline{2n-4}\,.\,\overline{2n-5}}{3.\overline{2n-3}.xx}rrC - \frac{\overline{2n-6}\,.\,\overline{2n-7}}{4.\overline{2n-4}\,.\,xx}rrD - \frac{\overline{2n-8}\,.\,\overline{2n-9}}{5.\overline{2n-5}.xx}rrE - \frac{\overline{2n-10}\,.\,\overline{2n-11}}{6.\overline{2n-6}.xx}rrF -$$

$$\text{\&c.} = \pm\, bb.$$

Otherwiſe.

Since the law of the continuation of the ſeries of the ſquares of the cords is this, that any cord multiplied by $2 - xx$, and the foregoing one ſubtracted, and $2xx$ added, gives the next. Therefore

$$\begin{array}{rll} \square\ 1\ \text{Cord} & = & xx \\ 2. & = & 4x^2 - x^4 \\ 3. & = & 9x^2 - 6x^4 + x^6 \\ 4. & = & 16x^2 - 20x^4 + 8x^6 - x^8 \\ 5. & = & 25x^2 - 50x^4 + 35x^6 - 10x^8 + x^{10} \\ \text{\&c.} & & \end{array}$$

and in general $ax^2 - bx^4 + cx^6 - dx^8$ &c. $=$ ſquare of the cord of the n^{th} arch. Whence by the conſtruction,

$$\left.\begin{array}{l} 2a_1x^2 - 2b_1x^4 + 2c_1x^6 - 2d_1x^8 \\ + 2x^2 - a_1x^4 + b_1x^6 - c_1x^8 \\ - ax^2 + bx^4 - cx^6 + dx^8 \end{array}\right\} = \begin{array}{l} \\ \\ a_2x^2 - b_2x^4 + c_2x^6 - d_2x^8\ \text{\&c.} \end{array}$$

Here

Here then $\underset{2}{a} = 2\underset{1}{a} - a + 2$, that is, by the method of increments, $a + 2\underset{\cdot}{a} + \underset{\cdot\cdot}{a} = 2a + 2\underset{\cdot}{a} - a + 2$, that is, $\underset{\cdot\cdot}{a} = 2 = 2\underset{\cdot}{n}$, and $\underset{\cdot}{a} = 2n + \text{A}$, and $a = \frac{2\,\underset{-1}{n}\,n}{2} + \text{A}n$; and when $n = 1$, $\text{A} = 1$, therefore $a = \underset{-1}{n}\,n + n$.

Again, $\underset{2}{b} = 2\underset{1}{b} + \underset{1}{a} - b$, and their increments $b + 2\underset{\cdot}{b} + \underset{\cdot\cdot}{b} = b + 2\underset{\cdot}{b} + a + \underset{\cdot}{a} - b$, and $\underset{\cdot\cdot}{b} = a + \underset{\cdot}{a}$, likewiſe $\underset{\cdot\cdot}{c} = b + \underset{\cdot}{b}$, $\underset{\cdot\cdot}{d} = c + \underset{\cdot}{c}$ and ſo on; that is, $\underset{\cdot\cdot}{b} = \underset{1}{a}$, $\underset{\cdot\cdot}{c} = \underset{1}{b}$, $\underset{\cdot\cdot}{d} = \underset{1}{c}$ &c. therefore $\underset{\cdot\cdot}{b} = n\underset{1}{n} + \underset{1}{n}$, and $\underset{\cdot}{b} = \frac{\underset{-1}{n}\,n\,\underset{1}{n}}{3} + \frac{n\,\underset{1}{n}}{2}$, and $b = \frac{\underset{-2}{n}\,\underset{-1}{n}\,n\,\underset{1}{n}}{3 \cdot 4} + \frac{\underset{-1}{n}\,n\,\underset{1}{n}}{2 \cdot 3}$. Again, $\underset{\cdot\cdot}{c} = \frac{\underset{-1}{n}\,n\,\underset{1}{n}\,\underset{2}{n}}{3 \cdot 4} + \frac{n\,\underset{1}{n}\,\underset{2}{n}}{2 \cdot 3}$, and $\underset{\cdot}{c} = \frac{\underset{-2}{n}\,\underset{-1}{n}\,n\,\underset{1}{n}\,\underset{2}{n}}{3 \cdot 4 \cdot 5} + \frac{\underset{-1}{n}\,n\,\underset{1}{n}\,\underset{2}{n}}{2 \cdot 3 \cdot 4}$, and $c = \frac{\underset{-3}{n} \ldots \underset{2}{n}}{3 \cdots 6} + \frac{\underset{-2}{n} \ldots \underset{2}{n}}{2 \cdot\cdot 5}$. In like manner $d = \frac{\underset{-4}{n} \ldots \underset{3}{n}}{3 \cdots 8} + \frac{\underset{-3}{n} \ldots \underset{3}{n}}{2 \cdot\cdot 7}$, and $e = \frac{\underset{-5}{n} \ldots \underset{4}{n}}{3 \cdot\cdot 10} + \frac{\underset{-4}{n} \ldots \underset{3}{n}}{2 \cdot\cdot 9}$, and ſo on. Then ſince $\underset{\cdot}{n} = 1$, and n, $\underset{-1}{n}$, $\underset{1}{n}$, $\underset{-2}{n}$, $\underset{2}{n}$ &c. $= n$, $n - 1$, $n + 1$, $n - 2$, $n + 2$ &c. reſpectively. Therefore by reduction will be found $a = nn$, $b = \frac{nn \,.\, \overline{nn - 1}}{2 \,.\, 2 \,.\, 3}$, $c = \frac{nn \,.\, \overline{nn - 1} \,.\, \overline{nn - 4}}{3 \cdot 1 \,.\, 2 \cdot 3 \cdot 4 \cdot 5}$, $d = \frac{nn \,.\, \overline{nn - 1} \,.\, \overline{nn - 4} \,.\, \overline{nn - 9}}{4 \,.\, 1 \cdots 7}$ &c. *whence the*

ſquare

ſquare of the cord of the n^{th} arch $= nnx^2 -$

$$\frac{nn.\overline{nn-1}}{2.1.2.3}x^4 + \frac{nn.\overline{nn-1}.\overline{nn-4}}{3.1.2.3.4.5}x^6 - \frac{nn.\overline{nn-1}.\overline{nn-4}.\overline{nn-9}}{4.1.2.3.4.5.6.7}x^8$$

&c. *or puting* $r =$ *radius,* □ cord n^{th} arch $= nnx^2 - \frac{\overline{nn-1}.x^2}{2.2.3r^2}A - \frac{2.\overline{nn-4}.x^2}{3.4.5.rr}B - \frac{3.\overline{nn-3^2}}{4.6.7r^2}x^2C - \frac{4.\overline{n^2-4^2}.x^2}{5.8.9r^2}D - \frac{5.\overline{n^2-5^2}.x^2}{6.10.11.r^2}E$ &c. that is, □ cord of the n^{th} arch $= nnxx - \frac{\overline{nn-1}.xx}{3.4rr}A - \frac{\overline{nn-4}.xx}{5.6rr}B - \frac{\overline{nn-3^2}.xx}{7.8rr}C - \frac{\overline{n^2-4^2}.xx}{9.10}D - \frac{\overline{n^2-5^2}.xx}{11.12rr}E -$ &c.

Where x^{2n} *will always be the laſt term,* n *being an integer.*

Cor. 1. *If* x *be the cord of an arch, radius* $= r$, *and* c *the cord of* n *times the arch; then*

$$\frac{x^n}{r^{n-1}} - \frac{nx^{n-2}}{r^{n-3}} + n\times\frac{\overline{n-3}.x^{n-4}}{2r^{n-5}} - n\times\frac{\overline{n-4}.\overline{n-5}.x^{n-6}}{2.3r^{n-7}} + n\times\frac{\overline{n-5}.\overline{n-6}.\overline{n-7}}{2.3.4r^{n-9}}x^{n-8} - n\times\frac{\overline{n-6}.\overline{n-7}.\overline{n-8}.\overline{n-9}}{2.3.4.5r^{n-11}}x^{n-10}\ \text{\&c.} = \pm c,$$

according as $\frac{n+1}{2}$ *is odd or even, the ſeries continued to* $\frac{n+1}{2}$ *terms: that is,* $+c$ *when odd, and* $-c$ *when even.*

Or

Or thus; $\sqrt{4rr - xx} : x$ into $\frac{x^{n-1}}{r^{n-1}} - \frac{\overline{n-2}\,.\,x^{n-3}}{r^{n-3}} + \frac{\overline{n-3}\,.\,\overline{n-4}}{1\,.\,2r^{n-5}}x^{n-5} - \frac{\overline{n-4}\,.\,\overline{n-5}\,.\,\overline{n-6}}{1\,.\,2\,.\,3r^{n-7}}x^{n-7} + \frac{\overline{n-5}\,.\,\overline{n-6}\,.\,\overline{n-7}\,.\,\overline{n-8}}{1\,.\,2\,.\,3\,.\,4r^{n-9}}x^{n-9} - \&c. = \pm c$, *according as* $\frac{1}{2}n$ *is odd or even, and the ſeries continued to* $\frac{n}{2}$ *terms.*

For the former ſeries is the root of $x^{2n} - 2nx^{2n-2} + 2n \times \frac{2n-3}{2}x^{2n-4}$ &c. the ſquare of the cord, found above by the firſt method. And the latter ſeries is found, by dividing the ſeries $x^{2n} - 2nx^{2n-2}$ &c. by $-xx + 4$ (or rather by $xx - 4$), and extracting the root of the quotient.

Cor. 2. *If* v *be the verſed ſine of an arch,* V *the verſed ſine of* n *times the arch, then will* $\frac{\overline{2v}^{n}}{2r^{n-1}} - \frac{2nr}{2v}A - \frac{\overline{2n-3}\,.\,r}{2\,.\,2v}B - \frac{\overline{2n-4}\,.\,\overline{2n-5}\,.\,r}{3\,.\,\overline{2n-3}\,.\,2v}C - \frac{\overline{2n-6}\,.\,\overline{2n-7}\,.\,r}{4\,.\,\overline{2n-4}\,.\,2v}D - \frac{\overline{2n-8}\,.\,\overline{2n-9}\,.\,r}{5\,.\,\overline{2n-5}\,.\,2v}E$ &c. $= \pm V$, *according as* n *is odd or even, the ſeries continued to* n *terms.*

Or $V = n^2v - \frac{\overline{n^2-1}\,.\,v}{2\,.\,3r}A - \frac{\overline{n^2-2^2}\,.\,v}{3\,.\,5r}B - \frac{\overline{n^2-3^2}\,.\,v}{4\,.\,7r}C - \frac{\overline{n^2-4^2}\,.\,v}{5\,.\,9r}D - \&c.$ This appears by ſubſtituting $2rv$ for xx, and $2rV$ for bb, to which they are equal, by Cor. 8. Prop. I.

Cor. 3. *The ſeries in* Cor. 1. *will give the ſine of n times the arch; puting* $x =$ *ſine of the ſingle arch,* $\frac{1}{2}r$ *for r in the ſeries.*

PROP. XXX.

The cord of the ſupplement of an arch being given; to find the cord of the ſupplement of n times the arch.

Let radius $= r = 1$, $x =$ cord of the ſupplement, $A =$ arch; then by cor. Prop. XXIII. $1 : x ::$ cord. ſup. $nA ::$ cord. ſup. $\overline{n-1}.A +$ cord. ſup. $\overline{n+1}.A$. Whence cord. ſup. $\overline{n+1}.A = x \times$ cord. ſup. $nA -$ cord. ſup. $\overline{n-1}.A$; that is, if the cord of the ſup. of any arch, in a ſeries thereof, be multiplied by x, and the preceding cord of the ſup. ſubtracted, you will have the next ſucceeding one. Hence

$$\begin{aligned}\text{Cord ſup. } 0A &= 2\\ 1A &= x\\ 2A &= x^2 - 2\\ 3A &= x^3 - 3x\\ 4A &= x^4 - 4x^2 + 2\\ 5A &= x^5 - 5x^3 + 5x\\ 6A &= x^6 - 6x^4 + 9x^2 - 2\\ &\text{\&c.}\end{aligned}$$

and in general, $nA = x^n - ax^{n-2} + bx^{n-4} - cx^{n-6}$ &c. then

$$\overline{n+1}.A = x^{n+1} - \underset{1}{a}x^{n-1} + \underset{1}{b}x^{n-3} - \underset{1}{c}x^{n-5} \text{ \&c.}$$

$$= \left\{\begin{array}{l} x^{n+1} - ax^{n-1} + bx^{n-3} - cx^{n-5} \\ \quad\;\; - x^{n-1} + \underset{-1}{a}x^{n-3} - \underset{-1}{b}x^{n-5} \end{array}\right\} \text{\&c.}$$

by conſtruction. Hence $\underset{1}{a} = a + 1$, and $a = 1 = n$, whence $a = n$.

Alſo

Alſo $b_1 = b + a_{-1}$, and $b_{\cdot} = a_{-1} = n_{-1}$, whence $b = \frac{n_{-2}\,n_{-1}}{2} - 1$, for when $n = 4$, $b = 2$; therefore $b = \frac{n_{-3}\,n_{-2}}{2} + n_{-2} - 1 = \frac{n_{-3}\,n_{-2}}{2} + n_{-3} = \frac{n\,n_{-3}}{2}$.

Again, $c_1 = c + b_{-1}$, and $c_{\cdot} = b_{-1} = \frac{n_{-4}\,n_{-3}}{2} + n_{-4}$, and $c = \frac{n_{-3}\,n_{-4}\,n_{-5}}{2\cdot 3} + \frac{n_{-4}\,n_{-5}}{2} = \frac{n\,n_{-4}\,n_{-5}}{1\cdot 2\cdot 3}$.

Again, $d_{\cdot} = c_{-1} = \frac{n_{-4}\,n_{-5}\,n_{-6}}{2\cdot 3} + \frac{n_{-5}\,n_{-6}}{2}$, and $d = \frac{n_{-4}\,n_{-5}\,n_{-6}\,n_{-7}}{2\cdot 3\cdot 4} + \frac{n_{-5}\,n_{-6}\,n_{-7}}{2\cdot 3} = \frac{n\,n_{-5}\,n_{-6}\,n_{-7}}{1\cdot 2\cdot 3\cdot 4}$. In like manner $e = \frac{n_{-6}\,n_{-7}\,n_{-8}\,n_{-9}}{2\cdot 3\cdot 4\cdot 5}$ &c. whence

The cord of the ſupplement of the n^{th} *arch is* $= x^n - nx^{n-2} + \frac{n\cdot\overline{n-3}}{1\cdot 2}x^{n-4} - \frac{n\cdot\overline{n-4}\cdot\overline{n-5}}{1\cdot 2\cdot 3}x^{n-6} + \&c. = \frac{x^n}{r^{n-1}} - \frac{n\cdot rr}{xx}A - \frac{\overline{n-3}\cdot rr}{2xx}B - \frac{\overline{n-4}\cdot\overline{n-5}\cdot rr}{3\cdot\overline{n-3}\cdot xx}C - \frac{\overline{n-6}\cdot\overline{n-7}\cdot rr}{4\cdot\overline{n-4}\cdot xx}D - \frac{\overline{n-8}\cdot\overline{n-9}\cdot rr}{5\cdot\overline{n-5}\cdot xx}E -$ &c. *continued to* $\frac{n+1}{2}$ *terms when n is odd, or to* $\frac{n+2}{2}$ *terms when even.*

Cor.

Cor. *Hence if radius* $= r$, $x =$ *coſine of an arch, then the coſine of the* n^{th} *arch* $= \frac{\overline{2x}^{n}}{2r^{n-1}} - \frac{nrr}{4.xx}A - \frac{\overline{n-3}.rr}{4.2xx}B - \frac{\overline{n-4}.\overline{n-5}.rr}{4.3.\overline{n-3}.xx}C - \frac{\overline{n-6}.\overline{n-7}.rr}{4.4.\overline{n-4}.xx}D$ — &c. *continued to* $\frac{n+1}{2}$ *terms when* n *is odd, or to* $\frac{1}{2}n + 1$ *terms when even.*

This follows from the foregoing ſeries, puting $\frac{1}{2}r$ for r.

PROP. XXXI.

Given the cord of an arch, and the cord of the ſupplement of twice the arch; to find the cord of $2m + 1$ *times the arch.*

Let radius $= r = 1$, arch $= A$, cord of $A = y$, cord ſup. of $2A = z$. Then by cor. Pr. XXIII. $1 : z :: \text{cord of } \overline{2m+1}.A : \text{cord of } \overline{2m-1}.A + \text{cord of } \overline{2m+3}.A$. Therefore cord of $\overline{2m+3}.A = z \times \text{cord of } \overline{2m+1}.A - \text{cord of } \overline{2m-1}.A$; that is, in a ſeries of theſe cords, if any one be multiplied by z, and the foregoing one ſubtracted, you will have the next following one. Whence

Cord $A = y$
$3A = z + 1 : \times y$
$5A = z^2 + z - 1 : \times y$
$7A = z^3 + z^2 - 2z - 1 : \times y$
$9A = z^4 + z^3 - 3z^2 - 2z + 1 : \times y$
$11A = z^5 + z^4 - 4z^3 - 3z^2 + 3z + 1 : \times y$
$13A = z^6 + z^5 - 5z^4 - 4z^3 + 6z^2 + 3z - 1 : \times y$
$15A = z^7 + z^6 - 6z^5 - 5z^4 + 10z^3 + 6z^2 - 4z - 1 : \times y$;

and in general, cord $\overline{2m+1}.A = z^m + z^{m-1} -$

az^{m-2}

$az^{m-2} - bz^{m-3} + cz^{m-4} + dz^{m-5} - ez^{m-6} - fz^{m-7}$ &c. then by conſtruction,

$$\left\{\begin{array}{l} z^{m+1} + z^m - az^{m-1} - bz^{m-2} + cz^{m-3} + dz^{m-4} - ez^{m-5} \\ \qquad\qquad - z^{m-1} - z^{m-2} + \underset{-1}{a}z^{m-3} + \underset{-1}{b}z^{m-4} - \underset{-1}{c}z^{m-5} \end{array}\right.$$

$$= z^{m+1} + z^m - \underset{1}{a}z^{m-1} - \underset{1}{b}z^{m-2} + \underset{1}{c}z^{m-3} + \underset{1}{d}z^{m-4} - \underset{1}{e}z^{m-5} \text{ \&c.}$$

&c. $=$ cord of $\overline{2m+3}.A$. Whence $\underset{1}{a} = a + 1$, and $\underset{1}{a} - a$ or $\underset{\cdot}{a} = 1 = \underset{\cdot}{m}$, whence $a = \underset{-1}{m}$, for when $m = 2$, $a = 1$.

Again, $\underset{1}{b} - b$ or $\underset{\cdot}{b} = 1 = \underset{\cdot}{m}$, and $b = \underset{-2}{m}$, for when $m = 3$, $b = 1$.

Alſo $\underset{1}{c} = c + \underset{-1}{a}$, and $\underset{1}{c} - c$ or $\underset{\cdot}{c} = \underset{-2}{m}$, whence $c = \dfrac{\underset{-3}{m}\,\underset{-2}{m}}{2}$.

Likewiſe $\underset{1}{d} - d$ or $\underset{\cdot}{d} = \underset{-1}{b} = \underset{-3}{m}$, and $d = \dfrac{\underset{-4}{m}\,\underset{-3}{m}}{2}$.

Alſo $\underset{\cdot}{e} = \underset{-1}{c}, = \dfrac{\underset{-4}{m}\,\underset{-3}{m}}{2}$, and $e = \dfrac{\underset{-5}{m}\,\underset{-4}{m}\,\underset{-3}{m}}{2\,.\,3}$, likewiſe $\underset{\cdot}{f} = \underset{-1}{d}$, and $f = \dfrac{\underset{-6}{m}\,\underset{-5}{m}\,\underset{-4}{m}}{2\,.\,3}$, $g = \dfrac{\underset{-7}{m}\,\underset{-6}{m}\,\underset{-5}{m}\,\underset{-4}{m}}{2\,.\,3\,.\,4}$, $h = \dfrac{\underset{-8}{m}\,\underset{-7}{m}\,\underset{-6}{m}\,\underset{-5}{m}}{2\,.\,3\,.\,4}$, $i = \dfrac{\underset{-9}{m}\ldots\underset{-5}{m}}{2.3.4.5}$, $k = \dfrac{\underset{-10}{m}\ldots\underset{-6}{m}}{2.3.4.5}$ &c. whence the

Cord of the $\overline{2m+1}$. *arch* $= y \times$ *into* $\underline{z^m} + \underline{z^{m-1}} - \dfrac{m-1}{1}z^{m-2} - \dfrac{m-2}{1}z^{m-3} + \dfrac{\overline{m-2}\,.\,\overline{m-3}}{2}z^{m-4}$

$$+\frac{\overline{m-3}\cdot\overline{m-4}}{2}z^{m-5}-\frac{\overline{m-3}\cdot\overline{m-4}\cdot\overline{m-5}}{2\cdot 3}z^{m-6}$$

$$-\frac{\overline{m-4}\cdot\overline{m-5}\cdot\overline{m-6}}{2\cdot 3}z^{m-7}+\frac{\overline{m-4}\ldots\overline{m-7}}{2\cdot 3\cdot 4}z^{m-8}$$

$$+\frac{\overline{m-5}\ldots\ldots\overline{m-8}}{2\cdot 3\cdot 4}z^{m-9}-\frac{\overline{m-5}\ldots\ldots\overline{m-9}}{2\cdot 3\cdot 4\cdot 5}z^{m-10}$$

$$-\frac{\overline{m-6}\ldots\overline{m-10}}{2\cdot 3\cdot 4\cdot 5}z^{m-11}$$ &c. continued to $m+1$ terms.

PROP. XXXII.

Given the cord of the ſupplement of an arch, and the cord of the ſupplement of twice the arch; to find the cord of the ſupplement of $\overline{2m+1}$ times the arch.

Let radius $= 1$, arch $= A$, cord ſup. $A = x$, cord ſup. $2A = z$, then as in the laſt prop. cord. ſup. $\overline{2m+3}\,.\,A = z \times$ cord. ſup. $\overline{2m+1}\,.A$ — cord. ſup. $\overline{2m-1}\,.A$; that is, if, in a ſeries, any one be multiplied by z, and the foregoing ſubtracted, you will have the next following one. Hence cord. ſup.

of $A = x$
$3A = z - 1 : \times x$
$5A = z^2 - z - 1 \times x$
$7A = z^3 - z^2 - 2z + 1 \times x$
$9A = z^4 - z^3 - 3z^2 + 2z + 1 \times x$
$11A = z^5 - z^4 - 4z^3 + 3z^2 + 3z - 1 \times x$ &c.

And in general, cord. ſup. $\overline{2m+1}\,.\,A = z^m - z^{m-1} - az^{m-2} + bz^{m-3} + cz^{m-4} - dz^{m-5} - ez^{m-6}$ &c. therefore by conſtruction

$$\left.\begin{array}{l} z^{m+1} - z^m - az^{m-1} + bz^{m-2} + cz^{m-3} \\ \phantom{z^{m+1} - z^m} - 1z^{m-1} + 1z^{m-2} + \underset{-1}{a}z^{m-3} \end{array}\right\}\text{ &c.} =$$

$$z^{m+1} - z^m - a_1 z^{m-1} + b_1 z^{m-2} + c_1 z^{m-3} = \text{cord. ſup.}$$

$\overline{2m+3}\,.\,A$; hence $a_1 = a + 1$, $b_1 = b + 1$, $c_1 = c + \underset{-1}{a}$,

$d_1 =$

$\underset{1}{d} = d + \underset{-1}{b}$ &c. all as in the laſt prop. and therefore the coefficients a, b, c, &c. will be the very ſame, whence the

Cord. ſup. $\overline{2m+1}\,.\,A = x \times$ into $z^m - z^{m-1} - \frac{m-1}{1} z^{m-2} + \frac{m-2}{1} z^{m-3} + \frac{\overline{m-2}\,.\,\overline{m-3}}{2} z^{m-4} - \frac{\overline{m-3}\,.\,\overline{m-4}}{2} z^{m-5} - \frac{\overline{m-3}\,.\,\overline{m-4}\,.\,\overline{m-5}}{2\,.\,3} z^{m-6} + \frac{\overline{m-4}\,.\,\overline{m-5}\,.\,\overline{m-6}}{2\,.\,3} z^{m-7} + \frac{\overline{m-4} \ldots \overline{m-7}}{2\,.\,3\,.\,4} z^{m-8}$ &c.

Cor. 1. *If rad.* $= 1$, $z =$ *cord . ſup. of* $\frac{1}{2m+1}$ *part of the circumference, then* $0 = z^m - z^{m-1} - \frac{m-1}{1} z^{m-2} + \frac{m-2}{1} z^{m-3} + \frac{\overline{m-2}\,.\,\overline{m-3}}{2} z^{m-4}$ &c.

For if $\overline{2m+1} \times A =$ ſemicircumference, then the cord of the ſup. and the general ſeries is $= 0$, and may be divided by x. And in that ſeries $z =$ cord. ſup. 2A, or of the $\overline{2m+1}^{\text{th}}$ part of the circumference.

PROP. XXXIII.

The tangent of an arch A *being given; to find the ſine and coſine of n times the arch.*

Let $t =$ tan. A, then by Prop. XXIV. S $nA = \frac{nx^{n-1}y}{r^{n-1}} - \frac{\overline{n-1}\,.\,\overline{n-2}\,.\,yy}{2\,.\,3\,.\,xx}$ A &c. but $\frac{y}{x} = \frac{t}{r}$, and $x = \frac{rr}{\sqrt{rr+tt}}$, therefore $\frac{nx^{n-1}y}{r^{n-1}}$ or $\frac{nx^n ry}{r^n x} = \frac{nx^n t}{r^n} = \frac{nr^n t}{\overline{rr+tt}|^{\frac{n}{2}}}$, whence S.$nA = \frac{nr^n t}{\overline{rr+tt}|^{\frac{n}{2}}} + \frac{\overline{n-1}\,.\,\overline{n-2}\; tt}{2\,.\,3rr}$ A &c.

Likewiſe by Prop. XXVI. Coſ. $nA = \frac{x^n}{r^{n-1}} - \frac{n.\overline{n-1}.yy}{2xx}A$ &c. that is $= \frac{r^{n+1}}{\overline{rr+tt}|^{\frac{n}{2}}} - \frac{n.\overline{n-1}.tt}{2rr}A$ &c. whence

$$S.nA = \frac{nr^n t}{\overline{rr+tt}|^{\frac{n}{2}}} - \frac{\overline{n-1}.\overline{n-2}.tt}{2.3rr}A - \frac{\overline{n-3}.\overline{n-4}.tt}{4.5rr}B - \frac{\overline{n-5}.\overline{n-6}.tt}{6.7rr}C - \frac{\overline{n-7}.\overline{n-8}.tt}{8.9rr}D - \&c.$$

and

$$Coſ.\ nA = \frac{r^{n+1}}{\overline{rr+tt}|^{\frac{n}{2}}} - \frac{n.\overline{n-1}.tt}{2rr}A - \frac{\overline{n-2}.\overline{n-3}.tt}{3.4rr}B - \frac{\overline{n-4}.\overline{n-5}.tt}{5.6rr}C - \frac{\overline{n-6}.\overline{n-7}.tt}{7.8rr}D\ \&c.$$

Cor. 1. Hence ſince $t = \frac{ry}{x}$, therefore *tan.* $nA =$

$$\frac{nr^n t - \frac{n.\overline{n-1}.\overline{n-2}}{2.3}r^{n-2}t^3 + \frac{n\ldots\ldots\overline{n-4}}{2.3.4.5}r^{n-4}t^5\ \&c.}{r^n - \frac{n.\overline{n-1}}{1.2}r^{n-2}tt + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{2.3.4}r^{n-4}t^4\ \&c.}$$

Cor. 2. If ſ = ſecant of A, and t = tan. A, then

$$\textit{ſecant}\ nA = \frac{\text{ſ}^n}{r^{n-1} - \frac{n.\overline{n-1}}{2}r^{n-3}tt + \frac{n\ldots\overline{n-3}}{1\ldots.4}r^{n-5}t^4 - \&c.}$$

for $\sqrt{\overline{rr+tt}} = \text{ſ}$, and $\frac{rad.^2}{coſine}$ = ſecant.

SCHOLIUM.

Tho' the principal uſe of the foregoing propoſitions is when n is an integer number, in which caſe the ſeries will always terminate. Yet moſt of theſe ſeries will be equally true when n is any fractional number, if they do but converge. For the inveſtigation of any of theſe propoſitions does not ſuppoſe that n is always an integer, it only ſuppoſes that n is 1.

SECT. V.

Angular ſections, the inſcription of pollygons, the properties of the cords or ſubtenſes of arches.

PROP. XXXIV.

If any arch of a circle as AR *be given, and there be taken the arch* $AD = \frac{1}{n}$ *the arch* AR; *n being any integer number; and if the whole circumference of the circle be divided into n equal parts at the points* D, E, F, G, H, *&c. beginning at* D: *Then I ſay any of the arches* AD, AE, ADF, AEG, AEH, *&c. repeated n times, from* A, *will always end as* R. FIG. 7.

Call the whole circumference C, then by conſtruction, $n \times AD = AR$, alſo $n \times DE = C$, and $n \times DEF = 2C$, and $n \times DEG = 3C$, $n \times DFH = 4C$, *&c.* therefore

$n \times AE$ or $n \times AD + n \times DE = AR + C$
and $n \times AF$ or $n \times AD + n \times DF = AR + 2C$
and $n \times AEG$ or $n \times AD + n \times DEG = AR + 3C$
$n \times AFH$ or $n \times AD + n \times EFH = AR + 4C$ *&c.*

Therefore $n \times AE$, $n \times AF$, $n \times AEG$, *&c.* $= 1, 2, 3$, *&c.* entire circumferences + the arch AR. And ſince any number of circumferences will end at the point A where they began; conſequently by the addition of the arch AR, they will neceſſarily all end at R.

Cor. *If the cords* AR, BR, AD, AE, AF, *&c. are drawn; then the cord* AR= *cord of n times* AD, *or of n times* AE, *or of n times* AEF, *or of n times* AEG, &c. *and* BR *is the cord of the ſupplement of each of theſe.*

PROP. XXXV.

FIG. *To divide an arch* AR *into any number of equal parts, as n.*

7. Let the radius $=1$, $b=$ cord of the ſup. of AR, or the cord of BR, $x=$ BD the cord ſup. firſt part AD. Then by prop. XXX. $x^n - nx^{n-2} + \frac{n.\overline{n-3}}{2}x^{n-4}$ &c. $= \pm b$, according as the arch AR is leſſer or greater than a ſemicircle. Therefore x, the greateſt root of this equation, being extracted, will give BD, the cord of the ſupplement of the arch required.

EXAMPLE.

If the arch AR be leſs than a ſemicircle, and is to be divided into 7 equal parts; then $n=7$, and you will have $x^7 - 7x^5 + 14x^3 - 7x = b$, whoſe greateſt root x is BD, the cord of the ſupplement of AD required.

Otherwiſe.

Let rad. $=1$, cord AR $=b$, cord AD $=x$, then, by prop. XXIX, you will have $x^{2n} - 2nx^{2n-2} + 2n \times \frac{2n-3}{2}x^{2n-3}$, &c. to n terms $=\pm bb$, according as n is odd or even. Then extract the leaſt root xx of this equation, and then its ſquare root x will be $=$ AD, the cord of the firſt of the equal parts. *Or thus,* put $nnxx - \frac{nn.\overline{nn-1}}{2.1.2.3}x^4 + \frac{nn.\overline{nn-1}.\overline{nn-4}}{3.1.2.3.4.5}x^6$ &c. $= bb$, &c.

EXAMPLE.

If AR was to be divided into 5 equal parts, then $n=5$, and $x^{\ 0} - 10x^8 + 35x^6 - 50x^4 + 25x^2 = bb$, whoſe leaſt root xx is the ſquare of AD, and $x=$ AD, the cord of $\frac{1}{n}$ part of the arch AR.

Cor.

Cor. *If you continue to extract all the roots x, you will get the values of all the cords* BE, BF, BG, &c. *by the former method*; *or of the cords* AE, AF, AG, &c. *by the latter method*; *as appears by cor. Prop. XXXII.* FIG. 7.

SCHOLIUM.

If n is not a prime number, but compounded of two or more primes; it will be eaſier to divide the arch into a number of parts denoted by one of theſe compounding numbers, and then the firſt of theſe parts into as many parts as is denoted by another of them, and ſo on to the laſt.

PROP. XXXVI.

To inſcribe any regular polygon in a circle.

Let the number of its ſides be $= 2m + 1$, an odd number. Radius $= 1$, $z =$ the unknown cord BD, or the cord of the ſupplement of one part AD. Then by cor. Pr. XXXII. $z^m - z^{m-1} - \frac{m-1}{1} z^{m-2} + \frac{m-2}{1} z^{m-3}$ &c. $= 0$, *therefore the greateſt root z will give* BD, *and* $\sqrt{4 - zz} =$ *ſide of the polygon* AD, *required.* 8.

EXAMPLE.

In a heptagon, $2m + 1 = 7$, or $m = 3$, then by the former equation $z^3 - z^2 - 2z + 1 = 0$, and the greateſt root z gives the value of BE, and $\sqrt{4 - zz} =$ AE, the ſide of the heptagon. 9.

Otherwiſe.

Let $n =$ number of ſides, rad. $= 1$, $x =$ cord of the ſup. of the n^{th} part of the circle. Then ſince

FIG. 9. the cord ſubtending the whole circle is 0; therefore by Prop. XXVIII. $yx^{n-1} - \frac{\overline{n-2}}{1}yx^{n-3}$, &c. $= 0$, therefore, dividing by y, you will have $x^{n-1} - \frac{\overline{n-2}}{1}x^{n-3} + \frac{\overline{n-3}\cdot\overline{n-4}}{1\cdot 2}x^{n-5}$ &c. $= 0$, *extract the greateſt root x of this equation, then $\sqrt{4-xx} =$ ſide of the polygon.*

EXAMPLE.

In a pentagon $n = 5$, then $x^4 - 3x^2 + 1 = 0$, and $x^2 = \frac{3+\sqrt{5}}{2}$, the greateſt root, and $\sqrt{4-xx} = \sqrt{\frac{5-\sqrt{5}}{2}}$ the ſide of the pentagon.

SCHOLIUM.

When the number of ſides is not prime; it will be eaſier to divide the circumference into a number of parts denoted by one of the primes that compoſe it; and then one of theſe parts into as many parts as is denoted by another of the compounding prime numbers, and ſo on till all the primes which compoſe the given number be exhauſted.

PROP. XXXVII.

10. *If the arch* DAF *of a circle, whoſe diameter is* AE, *be divided into an even number of equal parts at* C, B, A, H, G, *&c. and there be drawn the lines* BH, CG, DF, &c. *perpendicular to the diameter* AE; *and likewiſe the cords* AB, BE: *I ſay, the ſum of the lines,* BH, CG, &c. + *half the laſt, that is,* $BH + CG + DO = \frac{AO \times EB}{AB}$.

For draw the lines CH, DG, &c. then all the triangles BAK, KHL, LCM, MGN, NDO, &c. are

FIG. 10.

are ſimilar. Therefore BK : KA : : HK : KL : : CM : ML : : GM : MN : : DO : ON, &c. therefore BK : AK : : BK + KH + CM + GM + DO : AK + KL + LM + MN + NO : : BH + CG + DO : AO. But EB : AB : : BK : AK : : BH + CG + DO : AO, therefore $AB \times \overline{BH + CG + DO} = AO \times EB$.

Cor. 1. *If the circle* ADG *be divided into an even number of equal parts at* A, B, C, D, E, F, &c. *and all the lines* BH, CG, DF, &c. *be drawn, and alſo* AB, BE. *Then the ſum of all the lines* BH + CG + DF, &c. $= \frac{AE \times EB}{AB}$.

For here the baſe of the ſegment is 0, and the points D, O, F, E coincide.

Cor. 2. *If a ſemicircle* ABE *be divided into any number of equal parts, as n, at the points* B, C, D, &c. *and the ſines* BK, CM, DO, &c. *be drawn; then I ſay, the ſum of all the ſines,* BK + CM + DO, &c. $= \frac{AE \times EK}{2BK}$.

For by ſimilar triangles $\frac{EB}{AB} = \frac{EK}{BK}$; and taking half of the quantities in Cor. 1. BK + CM + DO $= \frac{AE \times EB}{2AB} = \frac{AE \times EK}{2BK}$. Hence

Cor. 3. *Let radius* $= r$, $\frac{180}{n} = A$, *n being any integer.* $s = S.\ A.$ $c = coſ.\ A$, *then the ſum of all the ſines of* A, 2A, 3A, 4A, &c. *in the ſemicircle, is* $= \frac{r + c}{s} r$.

FIG.

10. Cor. 4. *Hence the area of the curve, made by erecting all the ſines perpendicular to a line equal to the ſemicircumference, or as it is commonly called, the area of the figure of ſines, is* $= 2rr$.

For if n be infinite and A be $= 1 = s$, then $c = r$, and the ſum of all the ſines or area of the curve $= \frac{2rr}{s} = 2rr$.

PROP. XXXVIII.

10. *If the circumference of the circle* ADG *be divided into an even number of equal parts, at the points* A, B, C, D, &c. *and the parallel chords* AB, CH, DG, &c. *be drawn; I ſay, the ſum of all the cords* AB + CH + DG + EF, &c. $= \frac{AE^2}{AB}$.

For by ſimilar triangles, AE : AB : : AB : AK : : HL : KL : : CL : ML : : GN : MN : : DN : NO : : FE : EO; and by compoſition AE : AB : : AB + CH + DG + FE : AE.

Cor. 1. *If a circle is divided into an even number of equal parts, and* A *be one of the parts. Then the ſum of all the cords of* 2A, 4A, 6A, 8A, &c. *is to the ſum of all the cords of* A, 3A, 5A, 7A, &c. : : *as* EB : *to* EA.

For BH, CG, DF, &c. are the cords of 2A, 4A, 6A. And BA, CH, DG, &c. are the cords of 1A, 3A, 5A, &c. And the firſt ſum, is to the laſt, as $\frac{AE \times EB}{AB}$ to $\frac{AE^2}{AB}$, or as EB : to AE.

Cor. 2. *If a ſemicircle is divided into any even number of equal parts, as* n; *and if* A *be one of theſe parts, and* $s = S.$ A, *radius* $= r$; *then the ſum of the ſines of* A, 3A, 5A, 7A, &c. $= \frac{rr}{s} =$ *coſec.* A.

This

This follows from this prop. taking half the cord for the ſine of half the arch. FIG. 10.

Cor. 3. *If a ſemicircle is divided into an even number of equal parts, and* A *be one of theſe parts, and rad.* $= r$, *S.* A $= s$, *coſ.* A $= c$; *then the ſum of all the even ſines in the ſemicircle, that is, the ſum of the ſines of* 2A, 4A, 6A, 8A, &c. $= \frac{cr}{s} =$ *cotan.* A.

For by Cor. 3. Pr. 37. ſum of the ſines of } A, 2A, 3A, 4A &c. $= \frac{r+c}{s}r$.

And by the laſt cor. ſum of the ſines of } A, 3A, 5A &c. $= \frac{rr}{s}$

Therefore, dif. = 2A 4A, 6A &c. $= \frac{cr}{s}$.

P R O P. XXXIX.

If the circumference of a circle, whoſe diameter is AB *and center* C, *be divided into any odd number of equal parts, as* n, *at the points* D, E, F, G, H, I, K; *and if, from any point* A, *there be drawn the cords* AD, AE, AF, AG, AH, &c. *and if there be taken the arch* AR $= n \times$ *arch* AD, *and the cord* AR *drawn; I ſay, the product of all the cords* $AD \times AE \times AF \times AG \times AH \times AI \times AK = AR \times CB^{n-1}$. 7.

1. Let the line QAP be ſuppoſed to revolve about the point A, the common origin of all the arches; then will the part AP paſs thro' the circumference of the circle AEBHA. And if the line ſtill continues to revolve about A, then the other end AQ will alſo paſs thro' all the points of the circumference AEBHA. And if the line ſtill revolves forward, then the part AP paſſes again thro' the circle AEBHA, &c. and ſo on alternately. Now if from A to any point of interſection as R, the cord AR be drawn,

FIG. 7. drawn, this cord, AR will be affirmative in going the firſt time about the circle; but negative, the ſecond time about; becauſe then AR will lie from A towards Q, in the revolving line, which is the contrary way to what it laid before; likewiſe in going the third time about, AR will be affirmative, and the fourth time negative, and ſo on alternately. Therefore in general, let C = circumference, A = any arch leſs than C; then the cords of A, 2C + A, 4C + A, &c. or of any even number of C's + A, will be affirmative. And the cords of C + A, 3C + A, 5C + A, &c. or of any odd number of C's + A, will be negative.

2. Let AR = c, x = cord of the ſingle arch. Now ſince n times the arch AD, AE, AF &c. all end at R (by Prop. XXXIV.) and therefore have the ſame cord AR; conſequently, by Cor. 1. Pr. XXIX. we ſhall have

Caſe 1. $x^n - nx^{n-2}$ &c. $= + c$, the cords of A, 2C + A, 4C + A &c.
or the cords of $n \times$ AD, $n \times$ AF, $n \times$ AH &c.

Caſe 2. $x^n - nx^{n-2}$ &c. $= - c$, the cords of C + A, 3C + A, 5C + A &c.
or the cords of $n \times$ AE, $n \times$ AG, $n \times$ AEH &c.

And in caſe 1ſt, the root AD, being affirmative, the others AF, AH &c. are affirmative for the ſame reaſon.

And in caſe 2d, the root AE is affirmative, and likewiſe the roots AG, AI. But it is ſhewn in algebra, that if the ſigns of all the even powers in an equation be changed, the affirmative roots are changed into negatives, and the contrary. And in caſe 2d, all the even powers of x are wanting, except c, therefore in caſe 2d, changing $- c$ into $+ c$, we have $x^n - nx^{n-2}$ &c. $= + c$, in which equation, AE, AG, AI &c. are negative roots. But this equation is now the ſame as Caſe 1. therefore, in general, FIG. 7. general, in the equation $x^n - nx^{n-2}$ &c. $= c$, the affirmative roots are AD, AF, AH &c. and the negative one AE, AG, AI &c.

3. It is ſhewn in algebra, that in any equation, as $x^n - nx^{n-2}$ &c. $= c$, or rather $x^n - nr^2x^{n-2}$ &c. $= cr^{n-1}$, the abſolute number is equal to the product of all the roots; therefore $AD \times AE \times AF \times AG$ &c. $= cr^{n-1}$.

Cor. 1. *The ſum of all the odd cords is equal to the ſum of all the even ones,* $AD + AF + AH + AK = AE + AG + AI$.

For it is ſhewn in algebra, that the coefficient of the ſecond term is = ſum of all the roots; and here the ſecond term is wanting, and therefore $AD - AE + AF - AG$ &c. $= 0$.

11. Cor. 2. *If the ſemicircumference be divided into any odd number of equal parts, as n; whereof the two firſt are* BF, *the two next* EF, *the two next* ED *&c. and if the cords* AF, AE, AD *&c. be drawn: then the product of all theſe cords,* $AF \times AE \times AD = CB^{\frac{n-1}{2}}$.

7. For if $AD = AK$, then R and G fall in B, and $AE = AI$, $AF = AH$ &c. therefore by this prop. $AD^2 \times AE^2 \times AF^2 \times AB = AB \times r^{n-1}$.

Cor. 3. *The ſame things ſuppoſed as in the laſt cor. I ſay,* $AF - AE + AD$ *&c.* $= CB$.

For by Cor. 1. $2AD + 2AF = 2AE + AB$.

Cor. 4. *If arch* $BR = n \times BG$, *and the cord* AR *drawn, and the reſt be done as is ſuppoſed in the prop. then the product of all the cords* $AD \times AE \times AF$ *&c.* $= AR \times CB^{n-1}$, *and* $AD + AF + AH + AK = AE + AG + AI$.

FIG. 7. For it will appear with a little conſideration, that $AD + BG =$ half of $DE = \frac{C}{2n}$, or $n \times AD + n \times BG = \frac{1}{2}C$. But $n \times BG = BR$, therefore $n \times AD = \frac{1}{2}C - BR = AR$, therefore this cor. comes to the caſe of this propoſition.

Cor. 5. *If a quadrant be divided into any odd number of equal parts, as* n; *and* A *be one of theſe parts, radius* $= r$. *Then the product of all the ſines, of* 1A, 3A, 5A, 7A *&c. in the quadrant* $=$

$$2 \times \left.\overline{\frac{r}{2}}\right|^{\frac{n+1}{2}}.$$

This follows from Cor. 2. taking half the cord for the ſine of half the arch, and half radius for CB, and multiplying both ſides by r.

PROP. XL.

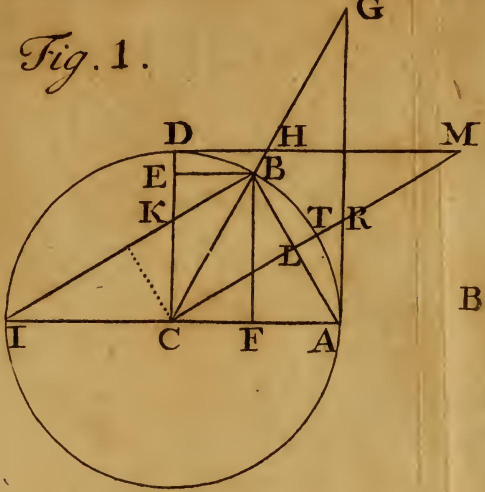
9. *If the circumference of a circle, whoſe center is* C, *be divided into any odd nnmber of equal parts, as* n, *at the points* A, E, F, G, H, I, K, *and from any one, as* A, *to all the reſt, there be drawn the cords* AE, AF, AG *&c. I ſay the product of all the cords,* $AE \times AF \times AG \times AH \times AI \times AK = n \times AC^{n-1}$.

For let the arches AD, AR be infinitely ſmall, and they will coincide with their cords; then the cord AD, is to the cord AR, as 1 to n, and with this ratio the cords AD, AR vaniſh when A and D coincide, and therefore by Prop. XXXVII. $1 \times AE \times AF \times AG$ &c. $= n \times CB^{n-1}$.

Otherwiſe.

If AD be $= 0$, then n times the arch AE, AF, AG &c. always ends at A, and therefore c, the cord of

Fig. 1.



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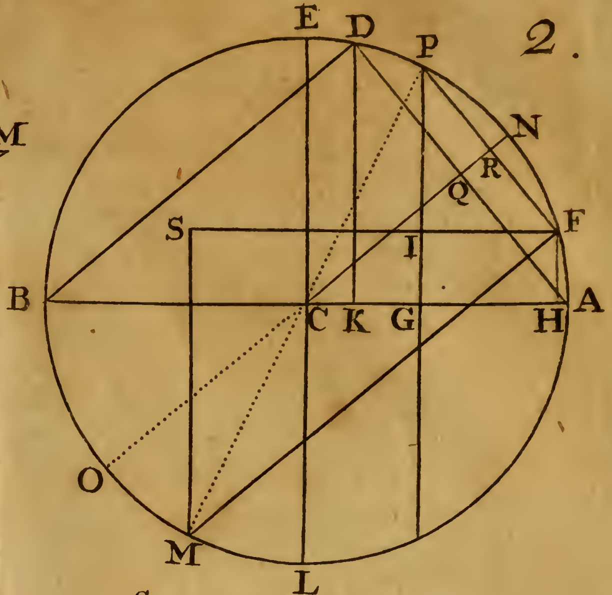

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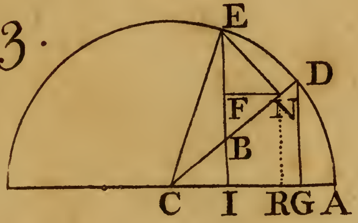

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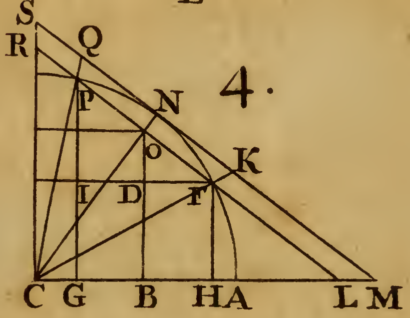

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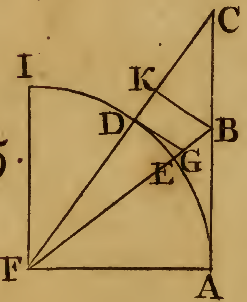

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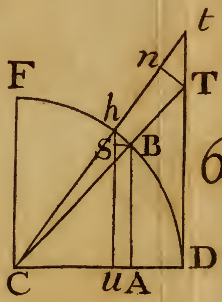

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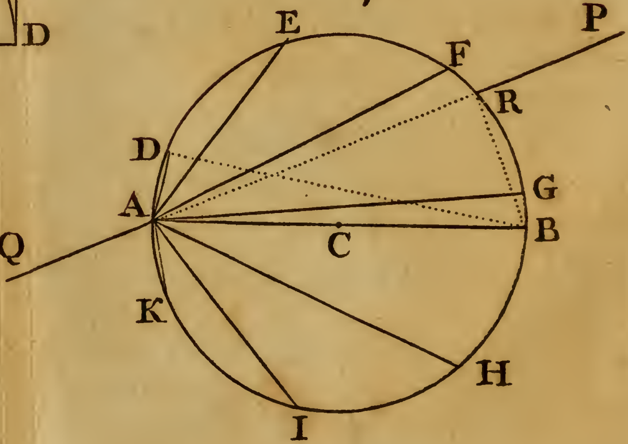

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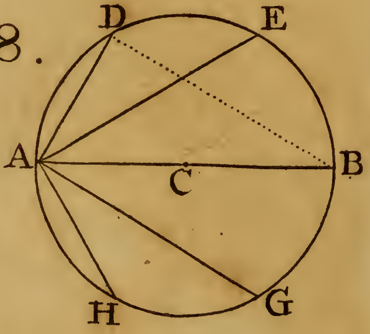


Trigonometry Pl. I. pa. 78.

of theſe multiple arches is = 0, therefore if s = cord FIG. 7.
of the ſingle arch, we ſhall have (by Cor. 2. Prop. XXV.) $ns - \frac{n . \overline{nn-1}}{24r^2}s^3$ &c. = 0, wherein one root s is = 0, and the equation being divided, we have $n - \frac{n^3 - n}{24rr}s^2$ &c. = 0, wherein the abſolute number is n or rather nr^{n-1}, which therefore is equal to the product of all the roots, whoſe number is now $n - 1$.

Cor. 1. *If the ſemicircumference* AEB *be divided into any odd number of equal parts, as n; whereof the two firſt is* AE, *the two next* EF, *the two next* FG *&c. And the cords* AE, AF, AG *&c. be drawn: I ſay, the product of all the cords* AE × AF × AG *&c.* $= \overline{CB}^{\frac{n-1}{2}} \times \sqrt{n}$. 9.

For AE = AK, AF = AI &c. therefore $AE^2 \times AF^2 \times AG^2 = n \times AC^{n-1}$.

Cor. 2. *If the quadrant of a circle be divided into any odd number of equal parts, as n, and* A *be one of them; then the product of all the coſines of* A, 3A, 5A, 7A *&c.* $= \frac{r}{2}\Big|^{\frac{n-1}{2}} \times \sqrt{n}$.

This follows from the laſt cor. becauſe ½AG, ½AF &c. are the coſines of ½BG, ½BF &c. or of A, 3A &c.

Cor. 3. *Hence alſo, the ſame things ſuppoſed as in this prop. the ſum of the ſquares of the cords* $AE^2 + AF^2 + AG^2 + AH^2 + AI^2 + AK^2 = 2n \times CB^2$.

For in the equation $ns - \frac{n^3 - n}{24rr}s^3$ &c. = 0, or rather by Cor. 1. Prop. XXIX. $x^n - nx^{n-2}$ &c. = 0, one root $x = 0$, which being divided, $x^{n-1} - nx^{n-3}$ &c. = 0, ſince here the index decreaſes by 2, therefore

FIG. 9. fore xx is the root of the equation, and n or nr^2 the coefficient of the ſecond term is the ſum of the roots, that is, of the ſquares of AE, AF, AG; which doubled gives the ſum of all the ſquares.

PROP. XLI.

12. *If the circumference of a circle, whoſe center is* C, *and diameter* AB, *be divided into any even number of equal parts, at the points* D, E, F, G, H, I; *and from any point* A, *there be drawn the cords* AD, AE, AF &c. *And the arch* AR *be taken* $= n \times$ *arch* AD, *and the cord* AR *drawn*; *then the product of all the cords* $AD \times AE \times AF \times AG \times AH \times AI = AR \times CB^{n-1}$.

Let cord $AR = c$, and $x =$ cord of the ſingle arch, then by Prop. XXIX. we have $- x^{2n} + 2nx^{2n-2}$ &c. $= cc$. But xx or the roots of this equation are AD^2, AE^2, AF^2 &c. and therefore their product $AD^2 \times AE^2$ &c. $= cc$ or rather ccr^{2n-2}, and extracting the root, $AD \times AE \times$ &c. $= cr^{n-1}$.

Cor. 1. *The ſum of the ſquares of all the cords* $AD^2 + AE^2 + AF^2 + AG^2 + AH^2 + AI^2 = 2n \times CB^2$.
For the coefficient of the ſecond term is the ſum of all the roots $= 2n$ or $2nr^2$.

13. Cor. 2. *If the ſemicircle* AEB *be divided into an even number of equal parts, as n*; *and the odd cords* AD, AE, AF *drawn*; *I ſay their product* $AD \times AE \times AF = CA^{\frac{n}{2}} \times \sqrt{2}$.

12. For in this caſe, $AD = AI$, $AE = AH$, and $AF = AG$, and $AR = 2CB$, therefore $AD^2 \times AE^2 \times AF^2 = AR \times CB^{n-1} = 2CB^n$.

PROP.

PROP. XLII.

If the circumference of a circle, whoſe center is C, *be divided into any even number of equal parts, at* A, D, E *&c. and from one of them as* A, *there be drawn to all the reſt the cords* AD, AE, AB, AG, AH; *I ſay the product of all the cords*, AD × AE × AB × AG × AH $= n \times CB^{n-1}$. FIG. 8.

For $n \times$ AD, $n \times$ AE &c. all end at A, and their cords $= 0$, therefore by Cor. 1. Prop. XXIX. $x^{n-1} - \overline{n-2} . x^{n-3}$ &c. ... $+ \frac{1}{2} nx : \times \sqrt{4 - xx}, = 0$, and one root $x = 0$, and dividing the equation by it and the correſpondent cord of the ſupplement $\sqrt{4 - xx}$ (which here is AB), and we ſhall have $\pm x^{n-2} \mp \overline{n-2} . x^{n-4} \ldots + \frac{1}{2}n = 0$, where $\frac{1}{2}n$ or $\frac{1}{2}nr^{n-2}$ is the rectangle of the remaining roots AD, AE, AG, AH, which multiply by $2r$, and you have AD × AE × AB &c. $= nr^{n-1}$.

This prop. is alſo evident from the laſt prop. by ſuppoſing AD, AR infinitely ſmall, and to vaniſh in the ratio of 1 to n.

PROP. XLIII.

If the circumference of a circle AFHA, *whoſe diameter is* AB *and center* C, *be divided into any even number of equal parts, as* n, *at the points* D, E, F, G, H, I; *and from any point* B *in the circumference, there be drawn the cords* BD, BE, BF *&c. and if the arch* AR *be taken* $= n \times$ *arch* AD, *and the cord* BR *drawn*; *I ſay*, 14. 15.

The product of the ſquares of the odd cords $BD^2 \times BF^2 \times BH^2 = \overline{BA + BR} \times CB^{n-1}$, *when* $\frac{1}{2}n$ *is an odd number*; *and* $= \overline{BA - BR} \times CB^{n-1}$, *when* $\frac{1}{2}n$ *is an even number.*

FIG. *And the product of the ſquares of the even cords*
14. $BE^2 \times BG^2 \times BI^2 = \overline{BA - BR} \times CB^{n-1}$, *when*
15. $\frac{1}{2}n$ *is odd, and* $= \overline{BA + BR} \times CB^{n-1}$, *when* $\frac{1}{2}n$ *is even.*

For let A be the common origin of the arches; and let the line PQ revolve about B, beginning at A, and going through the ſemicircumference AEB, in which any cord as BR being drawn, will be affirmative. But after the point of interſection paſſes through B, the other part BQ paſſes through the whole circumference, in which any cord BR will be negative; then the part PB paſſes through the next circumference from B, and any cord drawn in it will be affirmative; and ſo on alternately. Therefore if C = circumference, A = arch AR leſs than $\frac{1}{2}$C; then will the cords of A, 2C + A, 4C + A &c. be affirmative, and the cords of C + A, 3C + A, 5C + A &c. negative. Put a = cord of BR, then

CASE I.

In the arches AD, AF, AH, whoſe ſup. cords are BD, BF, BH; we have $n \times AD = A$, $n \times AF = 2C + A$, $n \times AFH = 4C + A$ &c. wherefore the cords of $n \times AD$, $n \times AF$, $n \times AFH$ are affirmative. Whence by Prop. XXX,

$x^n - nx^{n-2}$ &c. $- 2 = + a$, when $\frac{1}{2}n$ is odd,
and $x^n - nx^{n-2}$ &c. $+ 2 = + a$, when $\frac{1}{2}$ is even,
that is $x^n - nx^{n-2}$ &c. $= a \pm 2$, according as $\frac{1}{2}n$ is odd or even. In which equation the ſquares of BD, BF, BH, the odd cords, are the roots; therefore their product is $2 \pm a$, or rather $\overline{2r \pm a} \times r^{n-1}$, according as $\frac{1}{2}n$ is odd or even.

CASE II.

In the arches AE, AEG, AEI, whoſe ſup. cords. FIG.
are BE, BG, BI; we have $n \times AE = C + A$, 14.
$n \times AEG = 3C + A$, $n \times AEI = 5C + A$. 15.
Therefore the cords of $n \times AE$, $n \times AEG$, $n \times AEI$, are negative, whence by Prop. XXX.

$x^n - nx^{n-2}$ &c. $- 2 = - a$, when $\frac{1}{2}n$ is odd, and $x^n - nx^{n-2}$ &c. $+ 2 = - a$, when $\frac{1}{2}n$ is even; that is, $x^n - nx^{n-2}$ &c. $+ a \mp 2 = 0$, as $\frac{1}{2}n$ is odd or even. Wherein the ſquares of BE, BG, BI, the even cords are the roots, and therefore their product is $a \mp 2$ or rather $\overline{2r \mp a} \times r^{n-1}$, according as $\frac{1}{2}n$ is odd or even.

Cor. 1. *The ſame things ſuppoſed; the ſum of the ſquares of the odd cords, as alſo the ſum of the ſquares of the even cords*, $BD^2 + BF^2 + BH^2$ or $BE^2 + BG^2 + BI^2 = n \times CB^2$. 14.

For the coefficient of the ſecond term in both equations, is n or nr^2, the ſum of the roots.

Cor. 2. *The product of the ſquares of the odd cords + the product of the ſquares of the even cords, that is*, $BD^2 \times BF^2 \times BH^2 + BE^2 \times BG^2 \times BI^2 = 4CB^n$.

Cor. 3. *Hence alſo if the circumference be divided into an even number of equal parts as n; and cords be drawn from any point* B *to all the reſt, then the ſum of the ſquares of the alternate cords will be equal to each other, and to n* $\times CB^2$; $BA^2 + BF^2 + BH^2 + BL^2 + BN^2 = BE^2 + BG^2 + BM^2 + BO^2 = n \times CB^2$. 16.

This follows from Cor. 1. ſuppoſing the point G to coincide with B, and D with A in Fig. 14, 15.

FIG. 16. Cor. 4. *If the arch* AR, *the multiple of* AD, *be greater than half the circumference; then the cord of the ſupplement will be* — *a. In which caſe you muſt write* — BR *for* + BR, *and* + BR *for* — BR, *in the preſent propoſition.*

SCHOLIUM.

Since the ſquare of the cord = verſed ſine × diameter, therefore, in the foregoing propoſitions, and their corollaries, you may, by ſubſtitution, find the ſums or products of the verſed ſines of arches, inſtead of the ſums or products of the ſquares of the cords correſponding to them.

PROP. XLIV.

17. 18. *If the circumference of a circle, whoſe diameter is* AR, *and center* O, *be divided into any number of equal parts as* n, *at the points* A, C, E, G *&c. beginning at* A; *and from any point* P *in the diameter* AR, *be drawn the lines* PC, PE, PG *&c. I ſay, the product of all the lines* $AP \times PC \times PE \times PG$ *&c. thro' the whole circle is* $= AO^n - OP^n$.

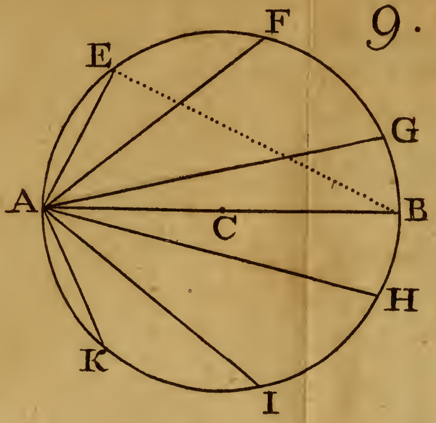
For draw the cords AC, AE, AG; and from C, E *&c.* let fall perpendiculars upon the diameter AR, as CI *&c.* and let AO or $OR = r$, $PO = x$, $AP = y = r - x$. Then $PC^2 = AP^2 + AC^2 - 2PA \times AI = yy + AC^2 - \frac{y}{r}AC^2$,

$$\left.\begin{array}{ll} \text{that is, } & PC^2 = y^2 + \frac{x}{r}\,AC^2 \\ \text{likewiſe } & PE^2 = y^2 + \frac{x}{r}\,AE^2 \\ \text{alſo } & PG^2 = y^2 + \frac{x}{r}\,AG^2 \text{ \&c.} \end{array}\right\}\text{ theſe multiplied}$$

together, produce $PC^2 \times PE^2 \times PG^2 = y^6 + \frac{x}{r}\,y^4 \times$

AC

9.



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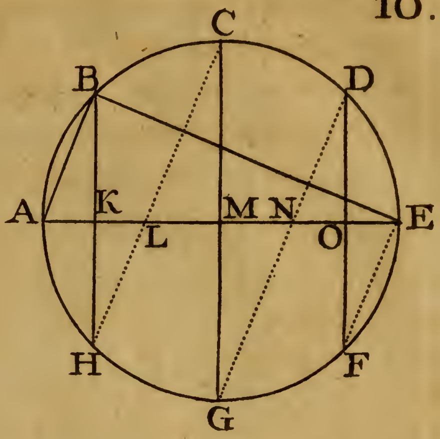

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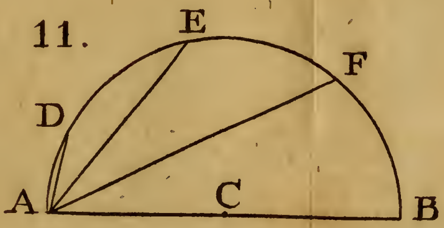

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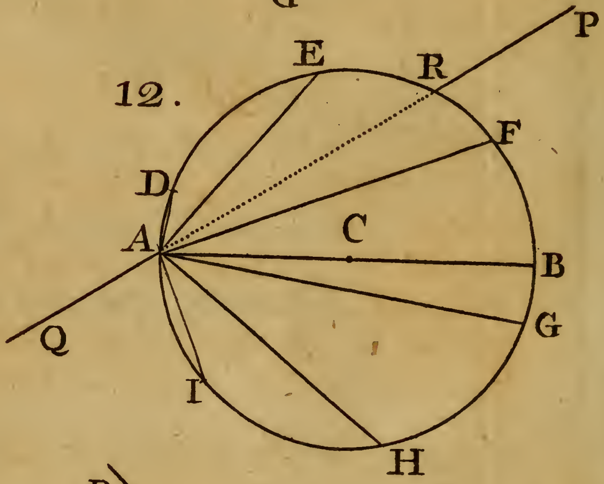

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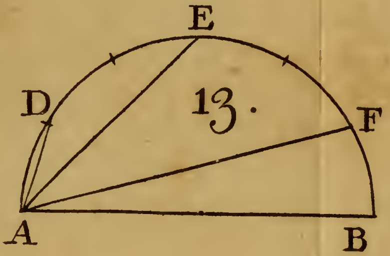

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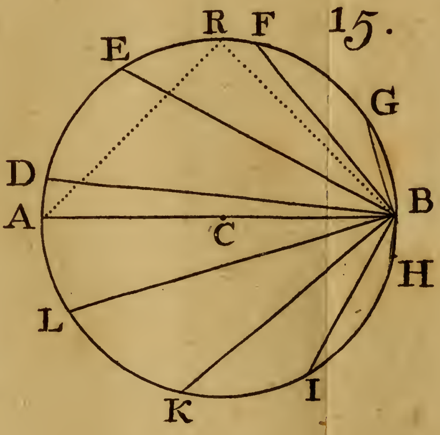

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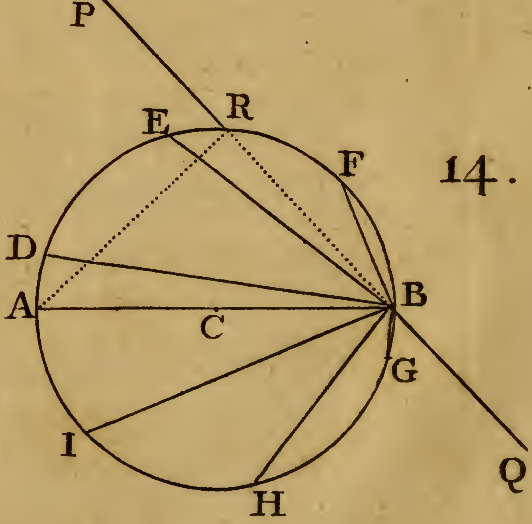


Pl.11. *pa 84.*

$\overline{AC^2+AE^2+AG^2}+\frac{xx}{rr}y^2\times\overline{AC^2\times AE^2+AC^2\times AG^2}$ $\overline{+AE^2\times AG^2}+\frac{x^3}{r^3}\times AC^2\times AE^2\times AG^2=PC\times Pc\times$ $PE\times Pe\times PG\times Pg$. But in the ſecond term of this equation, $AC^2+AE^2+AG^2$ is the ſum of the ſquares of all the cords (AC, AE, AG) which call A. And in the third term is the ſum of all their rectangles $=B$. In the fourth term, the ſum of all the ſolids $=C$, and ſo on. Likewiſe the index of y in the firſt term is $n-1$, when n is odd, or $n-2$ when even. This being premiſed; the equation will be expreſſed in general thus, $PC\times Pc\times PE$ Pe *&c.* $=y^{n-1}+\frac{x}{r}y^{n-3}A+\frac{xx}{rr}y^{n-5}B+\frac{x^3}{r^3}y^{n-7}C+$ $\frac{x^4}{r^4}y^{n-9}D$ *&c.* when n is odd. And $PC\times Pc\times PE\times$ Pe &c. $=y^{n-2}+\frac{x}{r}y^{n-4}A+\frac{xx}{rr}y^{n-6}B+\frac{x^3}{r^3}y^{n-8}C$ &c. when n is even. Let $x=$ cord of AC, AE, &c. then,

CASE I.

If n be an odd number; then by Cor. 1. Prop. 17.
XXIX. the cord of n times that arch $=\frac{x^n}{r^{n-1}}-\frac{nx^{n-2}}{r^{n-3}}$ &c. $=0$ in this caſe. But here one root is $x=0$, by which dividing the equation, and multiplying by r^{n-1}, you will have $x^{n-1}-nr^2x^{n-3}+\frac{n.\overline{n-3}}{2}r^4x^{n-5}-\frac{n.\overline{n-4}.\overline{n-5}}{6}r^6x^{n-7}$ &c. $=0$, wherein the roots are AC, AE, AG, *Ag*, *Ae*, *Ac*. But ſince $AC=Ac$, $AE=Ae$, $AG=Ag$, and ſince in this equation there are only the even powers of x; therefore xx

is

FIG. 17. is the algebraic root of it, and all the particular roots are AC^2, AE^2, AG^2.

But in every equation, the coefficient of the ſecond term is = the ſum of all the roots, therefore $AC^2 + AE^2 + AG^2 = nr^2 = A$. The coefficient of the third term = ſum of all the rectangles = $\frac{n.\overline{n-3}}{2}r^4$ = B, the coefficient of the fourth term = ſum of all the ſolids = C = $\frac{n.\overline{n-4}.\overline{n-5}}{6}r^6$, and ſo on. Therefore puting for A, B, C &c. theſe their values, and we ſhall have $PC \times PE \times PG \times Pg$ &c. = $y^{n-1}+\frac{x}{r}y^{n-3}\times nr^2+\frac{xx}{rr}y^{n-5}\times\frac{n.\overline{n-3}}{2}r^4$ &c. = $y^{n-1}+nrxy^{n-3}+\frac{n.\overline{n-3}}{2}r^2x^2y^{n-5}+\frac{n.\overline{n-4}.\overline{n-5}}{6}r^3x^3y^{n-7}$ &c. that is (puting $r-x$ for y and involving)

$$=r^{n-1}-\overline{n-1}.r^{n-2}x+\frac{\overline{n-1}.\overline{n-2}}{2}r^{n-3}x^2-\frac{\overline{n-1}.\overline{n-2}.\overline{n-3}}{6}r^{n-4}x^3\text{ \&c.}$$

$$+nr^{n-2}x-\frac{n.\overline{n-3}}{1}r^{n-3}x^2+\frac{n.\overline{n-3}.\overline{n-4}}{2}r^{n-4}x^3\text{ \&c.}$$

$$+\frac{n.\overline{n-3}}{2}r^{n-3}x^2+\frac{n.\overline{n-3}.\overline{n-5}}{2}r^{n-4}x^3\text{ \&c.}$$

$$+\frac{n.\overline{n-4}.\overline{n-5}}{6}r^{n-4}x^3\text{ \&c.}$$

$$+\dots\dots x^{n-1}$$

$$=r^{n-1}+r^{n-2}x+r^{n-3}x^2+r^{n-4}x^3\text{ \&c.}$$

$$\dots+x^{n-1}$$

Then multiply by PA or $r-x$, and you will have $PA \times FC \times PE \times PG \times Pg \times Pe \times Pc = r^n-x^n$.

CASE

CASE II.

FIG. 18.

If *n* be an even number; then by Cor. 1. Prop. XXIX. the cord of *n* times the arch $=\sqrt{4rr-xx}\times\frac{x^{n-1}}{r^{n-1}}-\frac{\overline{n-2}.x^{n-3}}{r^{n-3}}$ &c. $=0$, in which is one root $x=0$, and dividing the equation by *x* and the correſpondent cord of its ſupplement $\sqrt{4rr-xx}$ (which here is AR), and multiplying by r^{n-1}, you will have $x^{n-2}-\frac{\overline{n-2}}{1}r^2x^{n-4}+\frac{\overline{n-3}.\overline{n-4}}{2}r^4x^{n-6}-\frac{\overline{n-4}.\overline{n-5}.\overline{n-6}}{6}r^6x^{n-8}$ &c. $=0$, in which there being only the even powers of *x*, the roots are AC², AE², AG².

But in this equation, the coefficient of the ſecond term is = ſum of all the roots $AC^2+AE^2+AG^2=\overline{n-2}\times r^2=A$. The coefficient of the 3ᵈ term is the ſum of all their rectangles $=\frac{\overline{n-3}.\overline{n-4}}{2}r^4=B$. The coefficient of the fourth term is the ſum of all the ſolids $=\frac{\overline{n-4}.\overline{n-5}.\overline{n-6}}{6}r^6=C$, and ſo on. Therefore puting for A, B, C, &c. their values in the general equation, we have $PC\times PE\times PG\times Pg$ &c. $=y^{n-2}+\frac{x}{r}y^{n-4}\times\overline{n-2}.r^2+\frac{x^2}{r^2}y^{n-6}\times\frac{\overline{n-3}.\overline{n-4}}{2}r^4$ &c. $=y^{n-2}+\overline{n-2}.rxy^{n-4}+\frac{\overline{n-3}.\overline{n-4}}{2}r^2x^2y^{n-6}+\frac{\overline{n-4}.\overline{n-5}.\overline{n-6}}{6}r^3x^3y^{n-8}$ &c. that is (puting $r-x$ for *y* and involving)

$$\begin{array}{llll}
= r^{n-2} & -\overline{n-2}.r^{n-3}x & +\dfrac{\overline{n-2}.\overline{n-3}}{2}r^{n-4}x^2 & -\dfrac{\overline{n-2}.\overline{n-3}.\overline{n-4}}{6}r^{n-5}x^3 \ \&c. \\
 & +\overline{n-2}. & -\dfrac{\overline{n-2}.\overline{n-4}}{1} & +\dfrac{\overline{n-2}.\overline{n-4}.\overline{n-5}}{2}\&c. \\
 & & +\dfrac{\overline{n-3}.\overline{n-4}}{2} & -\dfrac{\overline{n-3}.\overline{n-4}.\overline{n-6}}{2}\&c. \\
 & & & +\dfrac{\overline{n-4}.\overline{n-5}.\overline{n-6}}{6}\&c. \\
 & & & \&c.
\end{array}$$

$$\ldots\ldots \&c. + x^{n-2}$$

$$= r^{n-2} \quad * \quad + r^{n-4}x^2 \quad * \quad + \quad r^{n-6}x^4 \ \&c. \ \ldots \ldots + x^{n-2}$$

Multiply by PA × PR or $rr - xx$, and you will have, PA × PC × PE × PG × PR × P*g* × P*e* × P*c* $= r^n - x^n$. *Q. E. D.*

FIG. 18. Cor. 1. *If the ſemicircumference* AER *be divided into any number of equal parts as n; at the points* A, B, C, D *&c. and from any point* P *in the diameter* AR *be drawn lines to all the even points of diviſion, as* PC, PE, PG; *I ſay, the product of* PA *into that of all the ſquares of theſe lines,* $PA \times PC^2 \times PE^2 \times PG^2$, *if n is an odd number, or of* PA *and* PR *into theſe ſquares,* $PA \times PR \times PC^2 \times PE^2 \times PG^2$ *if n is even, is* $= AO^n - OP^n$.

For if $n \times AC$ = whole circumference, then $n \times AB$ = half the circumference, and P*c* = PC, P*e* = PE &c.

Cor. 2. *If a ſemicircle be divided into n equal parts, and one of them be* K, *and s, t, u, &c. be put for the coſines of* 2K, 4K, 6K *&c. to* $\overline{n-1}.K$, *if n is odd, or to* $\overline{n-2}.K$ *if even; then I ſay,* $\overline{r-x} \times \overline{rr-2sx+xx} \times \overline{rr-2tx+xx} \times \overline{rr-2ux+xx}$, *&c. if n be an odd number, or* $\overline{r-x} \times \overline{r+x} \times \overline{rr-2sx+xx} \times \overline{rr-2tx+xx} \times \overline{rr-2ux+xx}$ *&c. (if n be even)* $= r^n - x^n$.

For let *a*, *b*, *c* &c. be the ſines of 2K, 4K, 6K &c. or of AC, AE, AG &c. then CI $= a$, OI $= s$, PI =

FIG. 18.

$=s-x$, $AP=r-x$, $PR=r+x$, and $PC^2=aa+\overline{s-x}^2=aa+ss-2sx+xx=rr-2sx+xx$, after the ſame manner $PE^2=rr-2tx+xx$, and $PG^2=rr-2ux+xx$, &c. and by Cor. 1. the product of theſe into $r-x$, or elſe into $\overline{r-x}\times\overline{r+x}$, is $=r^n-x^n$. But obſerve that the coſines of arches greater than 90°, will be negative.

Cor. 3. *Hence alſo if d, e, f &c. be the ſines of K, 2K, 3K, &c. then* $y\times\overline{y^2+\frac{4ddx}{r}}\times\overline{y^2+\frac{4eex}{r}}\times\overline{y^2+\frac{4ffx}{r}}$ *&c. if n be odd,*

Or $y\times\overline{r+x}\times\overline{y^2+\frac{4ddx}{r}}\times\overline{y^2+\frac{4eex}{r}}\times\overline{y^2+\frac{4ffx}{r}}$ *&c. if n be even,* $=r^n-x^n$.

This follows from the demonſtration of this Prop. for $2d=AC$, $2e=AE$, $2f=AG$, &c.

Cor. 4. *Alſo, if a, b, c be the verſed ſines of* 2K, 4K, 6K *&c.*

$$\left.\begin{array}{l}\text{then if } n \text{ be odd, } y\\ \text{if } n \text{ be even, } y\times\overline{r+x}\end{array}\right\}\times\overline{y^2+2ax}\times\overline{y^2+2bx}\times\overline{y^2+2cx},$$

&c. $=r^n-x^n$. *For* $a=\frac{2dd}{r}$, $b=\frac{2ee}{r}$, $c=\frac{2ff}{r}$.

Cor. 5. *And if* P *be taken without the circle at* π, *the product of all the lines* $\pi A\times\pi C\times\pi E\times\pi G$ *&c.* $=\pi O^n-AO^n$.

For let $O\pi:OA::OA:OP$. And let the lines PC, PE &c. be drawn; then ſince $\pi O:CO::CO:OP$, and angle O common, therefore the triangles πOC and COP are ſimilar, and for the ſame reaſon, πEO, and EOP, &c. Whence $\pi O:CO::\pi C:PC::\pi E:PE$ &c. $::\pi R:PR::\pi A:PA$. Therefore $PA=\frac{r\times\pi A}{\pi O}$, $PR=\frac{r\times\pi R}{\pi O}$, $PC=\frac{r\times\pi C}{\pi O}$, $PE=\frac{r\times\pi E}{\pi O}$ &c. $PO=\frac{rr}{\pi O}$. Whence $AP\times PC$

×

FIG. $\times PE \times PG\ \&c. = \dfrac{\pi A \times \pi C \times \pi E \times \pi G\ \&c.}{\pi O^n}$, $r^n = r^n - \dfrac{r^{2n}}{\pi O^n}$, or $\pi A \times \pi C \times \pi E\ \&c. = \pi O^n - r^n$.

P R O P. XLV.

17. 18. *If the circumference of a circle* AEG, *whoſe diameter is* AR, *and center* O, *be divided into any number of equal parts as n, at the points* B, D, F &c. *and the part* B*b* *be biſected by the diameter* AR; *and from any point* P *therein there be drawn the lines* PB, PD, PF *&c. I ſay, the product of all theſe lines* PB×PD×PF *&c. thro' the whole circle is* $=AO^n + OP^n$.

For let all the parts BD, DF &c. be biſected in C, E, &c. and PC, PE &c. drawn, and then the circle will be divided into 2*n* parts, therefore

by the laſt prop. } $PA \times PB \times PC \times PD \times PE\,\&c. = AO^{2n} - OP^{2n}$

and by the ſame } $PA \times \quad PC \times \quad PE\,\&c. = AO^n - OP^n$

therefore by diviſion } $PB \times \quad PD \times PF \times PR \times Pf \times Pd \times Pb$ when *n* is odd,

or $PB \times \quad PD \times PF \times PH \times Ph \times Pf \times Pd \times Pb$,

when *n* is even, is $= \dfrac{AO^{2n} - OP^{2n}}{AO^n - OP^n} = AO^n + OP^n$.

Cor. 1. *If the ſemicircumference* AER *be divided into any number of equal parts as n, in the points* A, B, C, D *&c. and from any point* P *in the diameter* AR, *the lines* PB, PD, PF, *&c. be drawn to all the odd points of diviſion: then, the product of all their ſquares, if n is even,* $PB^2 \times PD^2 \times PF^2 \times PH^2$; *or that product* ×*d by* PR, *that is,* $PB^2 \times PD^2 \times PF^2 \times PR$, *if n is odd, is* $= AO^n + OP^n$.

For if $n \times Bb =$ whole circumference, then $n \times AB = \frac{1}{2}$ circumference; and $Pb = PB$, $Pd = PD$ &c.

Cor. 2. *If a ſemicircle be divided into any number of equal parts, as n, and one of them be* K, *and* *s, t, u, &c. be the coſines of* 1K, 3K, 5K *&c. to* $\overline{n-1}.$K *when n is even, or to* $\overline{n-2}.$K *when n is odd. Then I ſay* FIG. 17. 18.

$\overline{rr-2sx+xx}\times\overline{rr-2tx+xx}\times\overline{rr-2ux+xx}$ &c. $=r^n+x^n$, when n is even, or

$\overline{r+x}\times\overline{rr-2sx+xx}\times\overline{rr-2tx+xx}\times\overline{rr-2ux+xx}$ &c. $=r^n+x^n$, when n is odd.

For if a, b, c &c. be the ſines of 1K, 3K, 5K &c. then the ſame way as in Cor. 2. Pr. XLIV. will be found $PB^2=rr-2sx+xx$, $PD^2=rr-2tx+xx$, $PF^2=rr-2ux+xx$ &c. whoſe product alone, or elſe multiplied by $r+x$, is, by Cor. 1. $=r^n+x^n$.

If P *be without the circle as at* π, *it will ſtill be* $\pi B\times\pi D\times\pi F$ &c. $=AO^n+O\pi^n$.

For by this Prop. and Cor. 5. of the laſt; that product will be $\dfrac{O\pi^{2n}-AO^{2n}}{O\pi^n-AO^n}=O\pi^n+AO^n$.

PROP. XLVI.

If the arch Ae *of a circle be divided into any number of equal parts as n, of which* AB *is one; and if the whole circumference be divided into the ſame number of equal parts, at the points* B, C, D, E *&c. beginning at* B; *and if from any point* P, *taken in the diameter* AR, *the lines* PB, PC, PD &c. *be drawn, and* eF *be let fall perpendicular to* AR *to interſect it in* F, *between the center* O *and* R. *And if* $AO=r$, $OP=x$, $OF=b$; *then I ſay the product of all the ſquares* $PB^2\times PC^2\times PD^2\times PE^2$ &c. $=r^{2n}+2br^{n-1}x^n+x^{2n}$. 20.

For let $AO=r=1$. $AP=y=1-x$, then you will find as in Prop. XLIV. $PB^2=y^2+x\times AB^2$, $PC^2=y^2+x\times AC^2$, $PD^2=y^2+x\times AD^2$ &c. which multiplied together, produce $PB^2\times PC^2\times PD^2$ &c. $=y^\nu+Axy^{\nu-2}+Bx^2y^{\nu-4}+Cx^3y^{\nu-6}$ &c. $+Fx^n$, putting $\nu=2n$, A $=$ ſum of the ſquares of the cords

AB²

FIG. 20. $AB^2 + AC^2 + AD^2$ &c. $B =$ ſum of their rectangles, $C =$ ſum of their ſolids &c. and F product of them all.

Now if z be the cord of an arch, c the cord of n times the arch, then by Prop. XXIX. $z^v - vz^{v-2} + v . \frac{\overline{v-3}}{2} z^{v-4} - v . \frac{\overline{v-4} . \overline{v-5}}{2 . 3} z^{v-6}$ &c. $= \pm cc$.

In which equation, the root zz repreſents indifferently the ſquare of any cord AB, AC, AD &c. Therefore by common algebra, the ſum of the ſquares of the cords, $AB^2 + AC^2 + AD^2$ &c. $= v$, the ſum of their products $= v . \frac{\overline{v-3}}{2}$; ſum of their ſolids $= v \times \frac{\overline{v-4} . \overline{v-5}}{2 . 3}$, &c. and the product of all $= cc$. Whence we get $A = v$, $B = \frac{v . \overline{v-3}}{2}$, $C = \frac{v . \overline{v-4} . \overline{v-5}}{2 . 3}$ &c. and $F = cc$. Conſequently $PB^2 \times PC^2 \times PD^2$ &c. $= y^v + vxy^{v-2} + v . \frac{\overline{v-3}}{2} x^2 y^{v-4} + \frac{v . \overline{v-4} . \overline{v-5}}{2 . 3} \times x^3 y^{v-6}$ &c. $+ ccx^n$; and puting $1 - x$ for y, and involving, we have $PB^2 \times PC^2 \times PD^2$ &c. $=$

$$\begin{array}{l}
1 - vx + v . \frac{\overline{v-1}}{2} x^2 - v . \frac{\overline{v-1}}{2} . \frac{\overline{v-2}}{3} x^3 \text{ \&c.} \ldots \text{\&c.} - vx^{v-1} + x^v \\
\quad + vx - v . \frac{\overline{v-2}}{1} x^2 + v . \frac{\overline{v-2}}{1} . \frac{\overline{v-3}}{2} x^3 \text{ \&c.} \qquad \text{\&c.} + vx^{v-1} \\
\qquad + v . \frac{\overline{v-3}}{2} x^2 - v . \frac{\overline{v-3}}{2} . \frac{\overline{v-4}}{1} x^3 \text{ \&c.} \\
\qquad\qquad\qquad\qquad \text{\&c.} \\
\qquad\qquad\qquad\qquad + ccx^n
\end{array}$$

Hence it will appear, by ſuming them up, that the coefficients of the ſecond, third &c. terms are 0, till you come to the middle term, where you have

ccx^n (the index n being $\frac{1}{2}v$); likewiſe beyond the FIG. middle term they are 0, becauſe the coefficients are 20. the ſame at equal diſtances on each ſide from the middle; whence they all vaniſh till the laſt x^v.

Now to find the coefficient of the middle term, it muſt be obſerved, that ccx^n ſupplies the place of the firſt term of the next horizontal ſeries; which term, if it were there, then the ſum of the coefficients (in that perpendicular column) would there alſo be $= 0$, as before. Hence it follows, that if $v = 2, 4, 6$ &c. that the term $vx, v.\frac{v-3}{2}x^2, v.\frac{v-4}{2}, \frac{v-5}{3}x^3$ &c. will be wanting, reſpectively, which term is $2x$, $2x^2$, $2x^3$, &c. reſpectively. And in general, the term wanting is always $\frac{1.2.3\ldots\overline{n-1}\times v}{1.2.3\ldots\overline{n-1}\,.\,n}x^n = \frac{v}{n}x^n = 2x^n$. Conſequently the middle term will be found to be $-2x^n + ccx^n$. But $cc = \text{AR} \times \text{AF} = 2 + 2b$, therefore $-2x^n + \overline{2+2b}\,.\,x^n = 2bx^n$ is the middle term. And therefore $\text{PB}^2 \times \text{PC}^2 \times \text{PD}^2$ &c. thro' the whole circle, $= 1 + 2bx^n + x^{2n}$, or rather $r^{2n} + 2r^{n-1}bx^n + x^{2n}$.

Cor. 1. *If the arch* Ae *be leſs than a quadrant,* 21. *and* F *lie from* O *towards* A; *then will* $\text{PB}^2 \times \text{PC}^2 \times \text{PD}^2 \times \text{PE}^2$ &c $= r^{2n} - 2br^{n-1}x_n + x^{2n}$.

This follows from this Prop. putting $-b$ inſtead of $+b$, as lying on the contrary ſide of O.

Cor. 2. *If* P *be without the circle as at* π, *then* 20. *will* $\pi\text{B}^2 \times \pi\text{C}^2 \times \pi\text{D}^2$ &c. $= r^{2n} \pm 2br^{n-1}x^n + x^{2n}$, 21. *according as* F *falls beyond, or on this ſide of* O, *in reſpect to* P.

The demonſtration of this Prop. equally proves this, ſince the even powers of $x - 1$, are the ſame with thoſe of $1 - x$, and v or $2n$ is an even number.

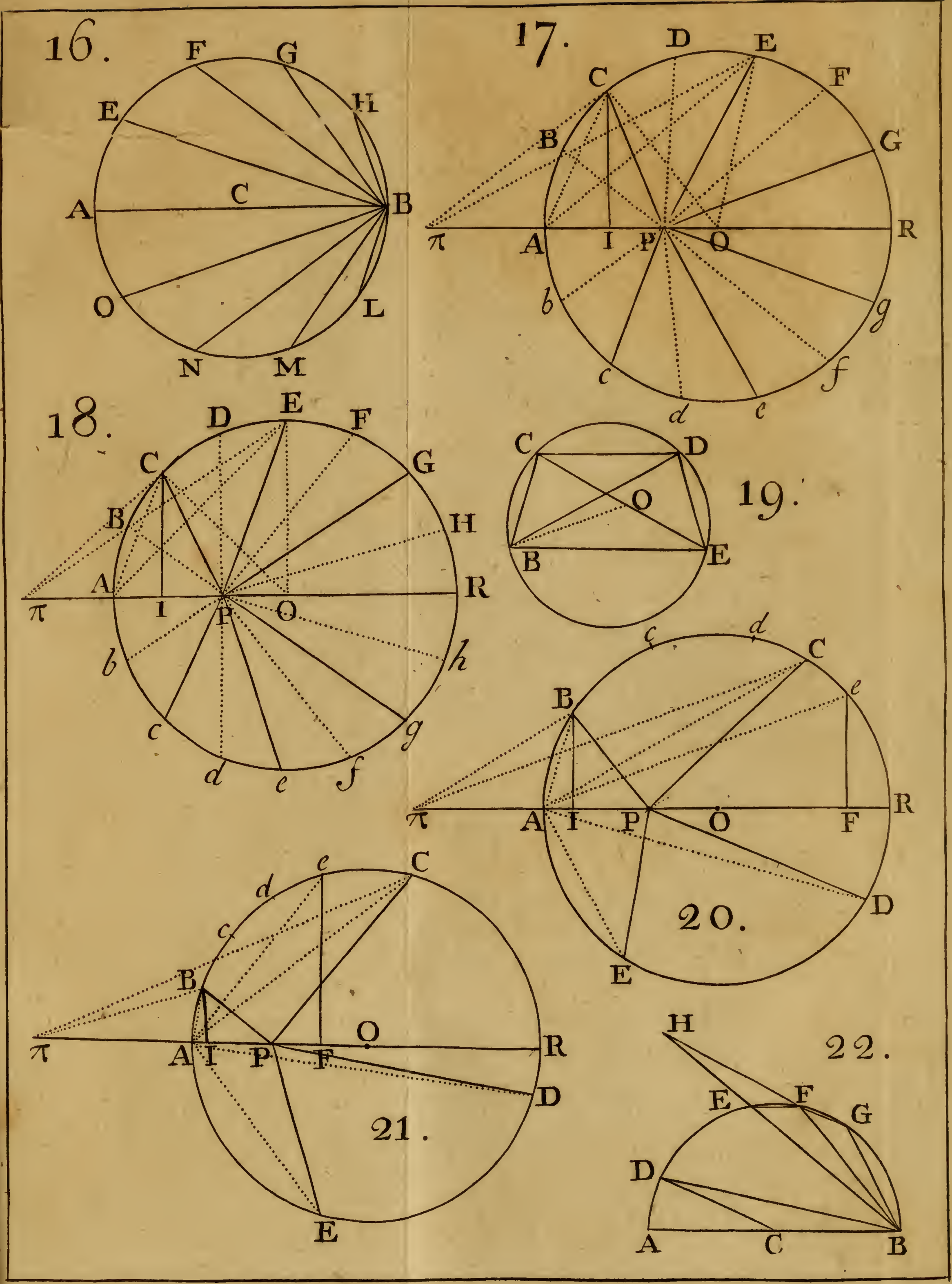
Cor.

FIG. 21.

Cor. 3. *Let* $b =$ *coſine of an arch* A *or* Ae, *to the radius* r; *and* $\frac{A}{n} = B$, $\frac{360}{n} = C$, *and* s, t, u &c. *the coſines of* B, B+C, B+2C, B+3C, &c. *to* $B+\overline{n-1}.C$, *to the radius* r, $x =$ *any quantity*; *then I ſay*, $\overline{rr-2sx+xx} \times \overline{rr-2tx+xx} \times \overline{rr-2ux+xx}$ &c. *to* n *terms, is* $= r^{2n} - 2br^{n-1}x^{n} + x^{2n}$.

For let $a = BI$ the ſine of B, $s = OI$ the coſine, then $PB^2 = BI^2 + PI^2 = aa + \overline{s-x}^2 = aa + ss - 2sx + xx = rr - 2sx + xx$, for the ſame reaſon, $PC^2 = rr - 2tx + xx$, $PD^2 = rr - 2ux + xx$, &c. but by Cor. 1. the product of theſe is $= r^{2n} - 2br^{n-1}x^{n} + x^{2n}$.

If A be greater than a quadrant, write $+b$ for $-b$. Likewiſe any of the coſines, s, t, u, &c. of arches between 90, and 270, is negative, and therefore in that caſe write $+s$, $+t$, $+u$ inſtead of $-s$, $-t$, $-u$, &c.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Pl. III. *pa.* 94

THE ELEMENTS OF TRIGONOMETRY.

BOOK II.

Plain trigonometry, or the doctrine of plain triangles.

PLAIN *trigonometry* teaches the relation of the ſides and angles of plain triangles, and how to calculate their ſides and angles.

A plain triangle is made by three right lines, which are called it's ſides, and if one ſide be perpendicular to another, or makes a right angle with it; then it is called *a right-angled triangle*, otherwiſe it is an oblique triangle.

A right angle is meaſured by an arch of 90°, an *acute angle* is leſs, and an *obtuſe* one greater than a right angle.

In a right-angled triangle, the ſide oppoſite to the right angle is called the *hypothenuſe*; and the other two, the *legs* or *ſides*, or ſometimes the *perpendicular and baſe*.

In all plain triangles, the ſum of the three angles together is 180 degrees: and in a right angle-triangle, the ſum of the two oblique angles is 90°.

When three letters denote an angle, the middle letter is always at the angle.

SECT. I.

The relations, and ſeveral proportions among the ſides, and angles of plain triangles.

PROP. I.

FIG. 1. *In any right-angled triangle* CAB, *let the hypothenuſe* CA $= h$, *and* CA — CB *or* BD $= v$; *then,* $57.29578\sqrt{\frac{2v}{h}} + \frac{v}{3.4h}A + \frac{3^2v}{5.8h}B + \frac{5^2v}{7.12h}C$, &c. $=$ *angle* ACB *in degrees; where* A, B, C *are the preceding terms.*

For with the center C deſcribe the arch AD, then BD or v is the verſed ſine of the arch AD, and radius h; therefore (by Prop. XIV. B. I.) arch AD $= \sqrt{2hv} + \frac{v}{3.4h}A + \frac{3^2v}{5.8h}B + \frac{5^2v}{7.12h}C$ &c. but as half the circumference $3.14159h$: 180 degrees : : AD : $\frac{180\,AD}{3.14159h} =$ degrees in the arch AD $= \frac{57.295779}{h} \times$ AD, that is, $\angle$ ACD $= \frac{57.29}{h} \times \sqrt{2hv} + \frac{v}{3.4h}A$: &c. $= 57.2957\sqrt{\frac{2v}{h}} + \frac{v}{3.4h}A + \frac{3^2v}{5.8h}B$ &c.

Cor. 1. *As the ſum of the hyp. and* $\frac{1}{2}$ *longer leg*
AC $+ \frac{1}{2}$ CB :
To 86 : :
So the ſhorter leg AB :
To its oppoſite angle C, *nearly.*

Let CB $= b$, and AB $= c$, $a = \angle$C in degrees, then by the nature of the circle, $c = \sqrt{2hv - vv} =$

$\sqrt{2hv}$

$\sqrt{2bv}-\frac{v}{4b}\sqrt{2bv}$ &c. by evolution. Then by this prop. FIG. 1. $a=\frac{57.3}{b}\times:\sqrt{2bv}+\frac{v}{12b}\sqrt{2bv}$: nearly, multiply this equation by the former, and you have $a\times:\sqrt{2bv}-\frac{v}{4b}\sqrt{2bv}=\frac{57.3c}{b}\times:\sqrt{2bv}+\frac{v}{12b}\sqrt{2bv}$, and dividing by $\sqrt{2bv}$, $a\times\overline{1-\frac{v}{4b}}=\frac{57.3c}{b}\times\overline{1+\frac{v}{12b}}$.

And $\frac{57.3c}{b}=a\times\frac{1-\frac{v}{4b}}{1+\frac{v}{12b}}=$ (by division) $a\times\overline{1-\frac{v}{3b}}$ &c. $=a\times\frac{3b-v}{3b}=a\times\frac{3b-b+b}{3b}=a\times\frac{2b+b}{3b}$, whence $a\times\overline{2b+b}=171.9c=172c$ nearly, or $\overline{b+\frac{1}{2}b}\times a=86c$.

SCHOLIUM.

If $c=1$, $n=57.3$, then $\frac{3}{2}n=86$, and by Cor. 1. $b+\frac{1}{2}b:1::\frac{3}{2}n:a$, and $2b+b=\frac{3n}{a}$, and $b=\frac{3n}{2a}-\frac{b}{2}=\frac{3n}{2a}-\frac{\sqrt{bb-1}}{2}$, whence $\frac{3n}{a}-2b=\sqrt{bb-1}$. This equation reduced gives, $b=\frac{2n}{a}-\sqrt{\frac{nn}{aa}-\frac{1}{3}}$. Whence $b=\frac{3n}{a}-2b$; which is *Forster*'s rule.

PROP. II.

FIG. 1. *In any right-angled triangle* CAB, *let* $a =$ *degrees in the leſſer* ∠ C, $m = \dfrac{a}{57.29578}$. *Then*

As 1 :

To $1 - \dfrac{m^2}{1.2} - \dfrac{m^2}{3.4}A - \dfrac{m^2}{5.6}C - \dfrac{m^2}{7.8}D$ &c. : :

So the hypothenuſe CA :

To the adjoining ſide CB.

where A, B, C &c. are the foregoing terms, with their ſigns.

For let radius CA or CD $= b$. Arch AD $= z$, then $z = \dfrac{ab}{57.295 \text{ \&c.}} = mb$, and by Cor. 1. Pr. XII. B. I. coſine CB $= b - \dfrac{z^2}{1.2b} + \dfrac{z^4}{1.2.3.4b^3}$ &c. $= b - \dfrac{m^2b^2}{1.2b} + \dfrac{m^4b^4}{1.2.3.4b^3} -$ &c. $=$ CA $\times \; 1 - \dfrac{m^2}{1.2} + \dfrac{m^4}{1.2.3.4} - \dfrac{m^6}{1\ldots6} + \dfrac{m^8}{1\ldots.8}$ &c.

Cor. 1. *If a be the leſſer angle, in degrees, then it will be,*

As $\dfrac{57.3}{a} + \dfrac{3a}{1000}$: 1 : : *hyp.* CA : *op. ſide* AB, *nearly.*

For let $n = 57.3$, then CB$^2 = bb - z^2 + \dfrac{z^4}{3b^2}$ &c. and AB$^2 =$ CA$^2 -$ CB$^2 = z^2 - \dfrac{z^4}{3b^2}$, and ex-

FIG. 1.

tracting the root, $AB = z - \frac{z^3}{6bb} = mb - \frac{m^3b}{6} = b \times \overline{\frac{a}{n} - \frac{a^3}{6n^3}}$; and dividing both ſides by $\overline{\frac{a}{n} - \frac{a^3}{6n^3}}$, $b = AB \times \overline{\frac{n}{a} + \frac{a}{6n}} = : \frac{57.3}{a} + \frac{3a}{1031} \times : AB.$

Cor. 2. *If a be the leſſer angle in degrees, then As* 1 : $1 - \frac{1\frac{1}{2}aa}{10000}$: : *hyp.* CA : *adjoining ſide* BC, *nearly.*

For $CB = CA \times \overline{1 - \frac{m^2}{2}} = CA \times \overline{1 - \frac{aa}{2nn}} = CA \times \overline{1 - \frac{1.52aa}{10000}}$.

SCHOLIUM.

Let $q = 90°$, $y = q - a$, then ſince $CB = CA \times \overline{1 - \frac{aa}{2nn}}$, by diviſion $CA = CB \times \overline{1 + \frac{aa}{2nn}}$, and $CA \times y = CB \times \overline{y + \frac{4aa}{\frac{8nn}{y}}}$. But $\frac{8nn}{y} = \frac{8nn}{q - a} = \frac{8nn}{q} + \frac{8nna}{qq}$ &c. $= 292 + 3a$. Therefore $CA \times y = CB \times \overline{y + \frac{4aa}{300 + 3a}}$, nearly, which is *Wilſon*'s rule.

Since in a right-angled triangle, the ſquare of the hypothenuſe is equal to the ſum of the ſquares of the ſides. And in any triangle, the baſe is to the ſum of the ſides, as their difference, to the difference of the

FIG. the ſegments made by a perpendicular. Therefore by theſe and the two foregoing propoſitions and their corollaries, all the caſes of plain triangles may be reſolved, without any tables whatever: dividing oblique triangles into two right-angled ones, by a perpendicular let fall from the end of a given ſide, and oppoſite to a given angle.

PROP. III.

1. *In any right-angled triangle* CAB, *any one of the three ſides being made a radius*; *and a circle deſcribed from the end of it as a center*; *then the other ſides will repreſent ſines, tangents, ſecants, &c. of the reſpective angles: then it will be, as any ſide : to what it repreſents in the table : : ſo is any other ſide : to what it repreſents in the table.*

Deſcribe the arches AD and BE, from the center C; then if CA be made radius, AB will be the ſine, and CB the coſine, of the angle C. And if CB be made radius, AB will be the tangent, and CA the ſecant of the angle C. And becauſe the ſines, tangents &c. of any angles may be found in the tables; therefore, when any triangle is propoſed, there may always be found a ſimilar one in the tables; and conſequently the ſides about the equal angles are proportional. Therefore it will be,

As radius : AC : : S.C : AB : : coſ. C : CB.
and rad. : CB : : tan. C : AB : : ſec. C : CA.

PROP. IV.

2. *In any right-angled triangle*, CDE,
As the ſum of the hyp. and one ſide, CE + ED :
Radius : :
So the other ſide, CD :
To tan. half the op. angle, CED.

Produce

Produce DE to B, ſo that $EB = EC$, then $\angle B$ FIG. $= \angle BCE$, and angle $DEC = \angle B + BCE = 2 \angle B$. But in the triangle CDB, by Prop. III. it is, as DB : radius : : CD : tan. B; that is, $DE + EC$: rad. : : CD : tan. $\frac{1}{2}$ E.

PROP. V.

The ſides of any triangle, are proportional to the 3. *ſines of their oppoſite angles,* AB : S. F : : AF : S.B : : FB : S.A.

On AB, AF let fall the perpendiculars FH, B*b*;

$$\begin{array}{rcccc} \text{then by Prop. III.} & AF : FH & :: & R : S.A \\ & FH : FB & :: & S.B : R \\ \cline{2-5} \text{therefore ex equo,} & AF : FB & :: & S.B : S.A. \end{array}$$

Again,

$$\begin{array}{rcccc} \text{As} & AB : Bb & :: & R : S.A \\ \text{and} & Bb : BF & :: & S.F : R \\ \cline{2-5} \text{and ex equo,} & AB : BF & :: & S.F : S.A. \end{array}$$

Cor. 1. *If a line* AD *be drawn from the vertex of* 4. *any triangle cuting the baſe; the rectangle of the ſides and ſines of the vertical angles, are directly as the ſegments of the baſe.*

For S.CDA : CA : : S.CAD : CD $= \frac{CA \times S.CAD}{S.CDA}$. And S.CDA or BDA : AB : : S.DAB : DB $= \frac{AB \times S.DAB}{S.CDA}$, therefore, CD : DB : : $CA \times S.CAD$: $AB \times S.DAB$.

Cor. 2. *If in a triangle two ſides* AC, CB, *and* 5. *the angle* A *oppoſite to the leſſer ſide* CB, *be given; the angle oppoſite to the greater may be either leſs or*

FIG. 5. *greater than a right angle; alſo the baſe and it's oppoſite angle, will have each of them, two ſeparate values. Therefore the three things required are ambiguous.*

For make CD = CB, then $\angle$ CDB = $\angle$ B, and there are the two triangles ACB, ACD, wherein two ſides and an oppoſite angle are given the ſame in both; therefore the other three things may be found in either triangle; for the angle oppoſite to AC may be either B or it's ſup. ADC, and the baſe may be either AB or AD, and it's op. angle may be either ACB or ACD.

PROP. VI.

6. *In any triangle* ABC,
As the ſum of any two ſides BC + AB :
To their difference BC − AB : :
Tan. half the ſum of their op. angles, tan. $\frac{A+C}{2}$:
Tan. half their difference, tan. $\frac{A-C}{2}$.

Make BD = BA, and draw AD, and BFG $\perp$ to it; likewiſe FE $\parallel$ AC, then will AF = FD, DE = EC. The $\angle$ BAF $= \frac{BAD + BDA}{2} = \frac{BAC + C}{2}$, and $\angle$ GAF = BAC − BAF $= \frac{BAC - C}{2}$.

The triangles BFE, BGC are ſimilar, whence BE : EC : : BF : FG. But AF : R : : BF : tan. BAF : : FG : tan. FAG, or BF : FG : : tan. BAF : tan. FAG. Conſequently BE or $\frac{BD + BC}{2}$: EC or $\frac{BC - BD}{2}$: : tan. BAF : tan. FAG.

Other-

Otherwiſe.

BC : BA : : S.A : S.C, and BC + BA : BC — BA : : S.A + S.C : S.A — S.C : : (by Cor. 1. Pr. VII.) tan. $\frac{A+C}{2}$: tan. $\frac{A-C}{2}$. FIG. 6.

Note, Inſtead of the tan. half the ſum of the op. angles, you may take the cotangent of half the included angle, which is equal to it.

Cor. 1. *As the baſe* CA :
Sum of the ſides, CB + BA : :
So diff. ſides CD :
Diff. ſegments of the baſe made by a perpendicular (or the alternate baſe) : :
So coſ. $\frac{1}{2}$ *ſum op. angles, or S.* $\frac{1}{2}$ *the vertical angle* :
Coſ. $\frac{1}{2}$ *diff. op. angles, or coſ.* $\frac{1}{2}$ *diff. vertical angles, made by a perp.*

For ſince DF = $\frac{1}{2}$ DA, FE = $\frac{1}{2}$ AC; and in the triangle FBE, FE : BE : : S.FBD or coſ. BDF : S.BFE or GFE or coſ. DFE. But DFE or DAC = BDA — C = BAD — C, and 2 DAC = BAC — C, alſo $\frac{1}{2}$ B = comp. $\frac{A+C}{2}$, and A — C = diff. vert. ∠s made by a perpendicular.

Likewiſe it is evident from geometry, that CA : CB + BA : : CD : diff. ſegments of the baſe. Suppoſing a circle deſcribed with center B and radius BA.

Cor. 2. *As the baſe* CA :
Difference of the ſides CD : :
So ſum of the ſides CB + BA :
Diff. ſegments by a perpendicular (or the alternate baſe) :

FIG. 6.

So ſine half ſum op. angles, $\frac{A+C}{2}$:
Sine of half their difference, $\frac{A-C}{2}$.

For in the triangle ADC, AC : DC : : S.ADB : S.CAD.

Cor. 3. *As ſquare of the baſe* AC^2 :
Difference of the ſquares of the ſides, $BC^2 - BA^2$: :
S. ſum op. angles, $A+C$:
S. their difference, $A-C$.

For by multiplying the two proportions in Cor. 1. and 2. $AC^2 : \overline{CB+BA} \times \overline{CB-BA} : : \text{coſ.}\,\frac{A+C}{2} \times S.\,\frac{A+C}{2} : \text{coſ.}\,\frac{A-C}{2} \times S.\,\frac{A-C}{2} : :$ (by Cor. 3. Pr. II. B. I.) $S.\overline{A+C} : S.\overline{A-C}$.

Cor. 4. *As half the ſum of the ſides :*
Tan. half the ſum of the op. angles : :
Greater ſides :
Tan. $\frac{1}{2}$ *ſum + tan.* $\frac{1}{2}$ *diff. op. angles : :*
Leſſer ſide :
Tan. $\frac{1}{2}$ *ſum — tan.* $\frac{1}{2}$ *diff. op. angles.*

For ſince $BC+BA : BC-BA : : T.\,\frac{A+C}{2} : T.\,\frac{A-C}{2}$. Therefore, by compoſition and diviſion,

$$BC+BA : \left\{\begin{array}{l} 2BC : : \\ 2BA : : \end{array}\right\} T.\,\frac{A+C}{2} : \left\{\begin{array}{l} T.\,\frac{A+C}{2} + T.\,\frac{A-C}{2} \\ T.\,\frac{A+C}{2} - T.\,\frac{A-C}{2}. \end{array}\right.$$

Cor. 5. In any triangle ACB, if CD be drawn biſecting the vertical angle C; it will be, FIG. 70.

As twice the rectangle of the ſides, 2ACB :
To the rectangle of the biſecting line and ſum of the ſides, $CD \times \overline{AC + CB}$: :
So radius :
Coſ. half the vertical angle C.

For produce AC, and make CF = CB, and draw FB and CP ⊥ to it. Then AF = AC + CB, and ∠ F = ∠ CBF, and ∠ ACB = F + CBF = 2F, or ACD = F. And the triangles ACD, AFB are ſimilar, therefore, AC : CD : : AF : FB = $\frac{CD \times AF}{AC}$. And CF (CB) : rad. : : FP ($\frac{1}{2}$FB) : S.FCP, or coſ. F; that is,

$$CB : \frac{CD \times AF}{2AC} :: \text{rad.} : \text{coſ.}\ \tfrac{1}{2}ACB.$$

Cor. 6. *As the leſſer ſide* : *to the greater* : : 6.
So radius : *to tan. of an arch* A.
From A take 45°, then
As radius : *tan.* $\overline{A - 45}$, *the remaining arch* : :
So tan. $\frac{1}{2}$ *ſum of the op. angles* : *to tan.* $\frac{1}{2}$ *their difference.*

For let L be the leſſer, G the greater ſide, then ſince $\frac{rG}{L}$ = tan. A, we ſhall have (by Cor. 2. Pr. IX. L. I.) the tan. $\overline{A - 45} = \dfrac{\frac{rG}{L} - r}{\frac{rG}{L} + r} \times r = \dfrac{G - L}{G + L} r$, and therefore r : tan. $\overline{A - 45}$: : G + L : G — L : : tan. $\frac{1}{2}$ ſum of the oppoſite angles : tan. half their difference (by this prop.)

FIG. *The ſame will hold good, if you make, as the greater : to the leſſer ſide : : rad. : tan.* A, *and then take rad. : tan.* $\overline{45 - \mathrm{A}}$: : *tan.* $\frac{1}{2}$ *ſum : tan.* $\frac{1}{2}$ *diff. op. angles.*

PROP. VII.

7. *In any triangle* AFB,
As twice the rectangle of the legs, $2\mathrm{AF} \times \mathrm{FB}$:
Sum of ſquares of the legs — ſquare baſe : :
Radius :
Coſ. vertical angle, F.

On FB let fall the perpendicular AC, then (by Geom. II. 23) $\mathrm{AB}^2 + 2\mathrm{BF} \times \mathrm{CF} = \mathrm{AF}^2 + \mathrm{FB}^2$, and $2\mathrm{BF} \times \mathrm{FC} = \mathrm{AF}^2 + \mathrm{FB}^2 - \mathrm{AB}^2$. But by Prop. III. AF : FC : : rad. : coſ. F. And multiplying the two firſt terms by 2FB, then will $2\mathrm{AF} \times \mathrm{FB} : 2\mathrm{BF} \times \mathrm{FC}$ or $\mathrm{AF}^2 + \mathrm{FB}^2 - \mathrm{AB}^2$: : rad. : coſ. F.

Cor. 1. *Hence theſe are reſpectively proportional,*
As twice the rectangle of the legs :
Rectangle of the baſe + diff. legs, and baſe — diff. legs :
Rectangle of the ſum of the three ſides, and ſum legs — baſe : :
So radius :
Verſed ſine of the included angle :
Verſed ſine of the ſum of the oppoſite angles.

For let $m = \mathrm{AF}$, $n = \mathrm{BF}$, $b = \mathrm{AB}$, then by this prop.

r : coſ. F : : $2mn : mm + nn - bb$; and by compoſition,

$r : r \pm$ coſ. F : : $2mn : 2mn \pm mm \pm nn \mp bb$,

that is, by ſchol. Prop. I.

$$r : \left\{ \begin{array}{l} \text{verſ. ſup. F.} \\ \text{verſ. F.} \end{array} \right\} : : 2mn : \left\{ \begin{array}{l} \overline{m+n}^2 - bb = \overline{m+n+b} \times \overline{m+n-b} \\ bb - \overline{m-n}^2 = \overline{b+\overline{m-n}} \times \overline{b-\overline{m-n}}. \end{array} \right.$$

Cor.

FIG. 7.

Cor. 2. *From half the ſum of the three ſides, ſubtract each ſide ſeverally, and note the three remainders. Then,*

As half the rectangle of the legs :
To ſquare root of the product of this half ſum and the three remainders : :
So radius :
Sine of the vertical angle.

For let $s = b + m + n$, $v =$ verſ. F. $V =$ verſ. ſup. F. Then by Cor. 1.

$$v = \frac{\overline{b + m - n} \times \overline{b + n - m}}{2mn} r = \frac{\overline{s - 2n} \times \overline{s - 2m}}{2mn} r,$$

$$\text{and } V = \frac{\overline{b + m + n} \times \overline{m + n - b}}{2mn} r = \frac{s \times \overline{s - 2b}}{2mn} r.$$

And by Schol. I. Prop. I.

$$\sqrt{Vv} = \text{S.F} = \frac{r}{2mn} \sqrt{s \times \overline{s - 2b} \times \overline{s - 2m} \times \overline{s - 2n}}$$

$$= \frac{2r}{mn} \times \sqrt{\frac{s}{2} \times \frac{s - 2b}{2} \times \frac{s - 2m}{2} \times \frac{s - 2n}{2}}.$$

Cor. 3. *As the rectangle of the legs :*
Radius ſquare : :
So the rectangle $\frac{1}{2}$ *ſum 3 ſides — one leg, and that* $\frac{1}{2}$ *ſum — other leg :*
Sine ſquare of half the vertical angle : :
And ſo the rectangle $\frac{1}{2}$ *ſum 3 ſides, and that* $\frac{1}{2}$ *ſum — baſe :*
Coſine ſquare of half the vertical angle.

For multiplying the three laſt terms in Cor. 1. by $\frac{1}{2}r$; then $2mn : \overline{b + m - n} \times \overline{b + n - m} : s \times \overline{s - b} : : \frac{1}{2}rr : \frac{1}{2}rv : \frac{1}{2}rV$. But by Prop. II. $\frac{1}{2}rv =$ ſine ſquare of half F, and $\frac{1}{2}rV =$ coſ. ſquare of $\frac{1}{2}$ F. Therefore $2mn : \overline{s - 2m} \times \overline{s - 2n} : s \times \overline{s - b} : : \frac{1}{2}rr :$ ſine^2 :

FIG. 7. ſine^2 : coſine^2. or $mn : \frac{s-2m}{2} \times \frac{s-2n}{2} : \frac{s}{2} \times \frac{s-b}{2}$: : rr : ſine^2 : coſine^2, of $\frac{1}{2}$F. And hence alſo follows,

Cor. 4. *As four times the rectangle of the legs :*
To radius ſquare : :
So rectangle of the baſe + diff. legs, and baſe — diff. legs :
Sine ſquare of half the vertical angle : :
And ſo rectangle of ſum legs + baſe, and ſum legs — baſe :
Coſine ſquare of half the vertical angle.

Cor. 5. *Rectangle of ſum of the three ſides, and ſum legs — baſe :*
Rectangle baſe + diff. legs, and baſe — diff. legs : :
Radius ſquare :
Tangent ſquare of half the angle at the vertex.

This follows from Cor. 4. becauſe coſine : ſine : : rad. : tangent, by Prop. I. B. I.

SCHOLIUM.

Inſtead of the *rectangle* of the legs, you may take a *quarter of the rectangle*, of the *ſum* + the *difference* of *the legs*, into the *ſum* — *diff. legs*; which is equal thereto.

Likewiſe inſtead of *half the vertical angle*, you may take the *complement of half the ſum* of *the oppoſite angles*, which is alſo equal to it.

PROP.

PROP. VIII.

In any triangle AFB, *I ſay*, FIG. 8.

$$AB - \frac{AF \times cof.\ A}{rad.} : S.A :: AF : tan.\ oppoſite \angle B.$$

For let fall the perpendicular FR on the baſe; then in the triangle AFR, rad. : AF :: S.A : FR $= \frac{AF \times S.A}{r}$ :: coſ. A : AR $= \frac{AF \times cof.\ A}{r}$. And BR $= AB - \frac{AF \times cof.\ A}{r}$, then BR : rad :: FR or $\frac{AF \times S.A}{r}$: tan. B. Or BR : S.A :: AF : tan. B.

Cor. $\sqrt{: AB^2 + AF^2 - \frac{2AB \times AF}{rad.} \times cof.\ A} := FB.$ For $FB^2 = AB^2 + AF^2 - 2AB \times AR.$

PROP. IX.

In the triangle AFB, *draw* FR *perpendicular to the baſe, and make* BR = RD; *then,* 8.

As the baſe, AB :
To the difference of the ſegments AD ::
So ſine of the vertical angle F :
To ſine of the diff. vert. angles, or S. diff. angles at the baſe B, A.

For AB : S.AFB :: FB or FD : S.A :: AD : S.AFD.
And $\angle B - \angle A = \angle FDB - \angle A = \angle AFD.$

PROP.

PROP. X.

FIG. 9. *As coſ. diff. angles at the baſe — coſ. ſum :*
Sine of the vertical angle : :
So perpendicular :
Half the baſe.

For let ADB be the triangle, DP the perpendicular, C the center of the circumſcribing circle, draw ECG perpendicular, and DF parallel to AB, &c. the ∠ ECB (= 2EBA) is the ſum of the angles A, B, whoſe verſed ſine is EG, and ECD or 2EBD is the difference of the angles A, B, whoſe verſed ſine is EF; therefore the difference of theſe verſed ſines, or the difference of the coſines, is FG or DP. Alſo GCB or ADB is the vertical angle, whoſe ſine is GB. And therefore DP : GB : : diff. coſines : S. vertical angle.

Cor. *As ſine of the diff. angles at the baſe :*
Sine of the vertical angle : :
Diſtance of the perp. from the middle of the baſe :
Half the baſe.

For FD is the ſine of the difference of the angles A, B; and FD or GP : GB : : S. diff. : S. vert. angle.

PROP. XI.

8. *As radius :*
To ſum of the cotan. half the angles at the baſe : :
So the perpendicular :
To ſum of the three ſides.

Put

Put FR $= p$, radius $= r$, then by Prop. IV. FIG. 8.

$$\tan.\tfrac{1}{2}A : r :: r : \cot.\tfrac{1}{2}A :: p : AF + AR,$$

$$\text{and} \quad r : \cot.\tfrac{1}{2}B :: p : BF + BR;$$

$$\text{whence } AF + BF + AB = \frac{\cot.\tfrac{1}{2}A + \cot.\tfrac{1}{2}B}{r}p.$$

PROP. XII.

In a right-angled triangle ACB, 10.
As radius :
Sine of double one acute angle, A *or* B : :
Square of the hypothenuſe :
Four times the area.

Draw CD $\perp$ AB, and biſect the hyp. AB in E, and draw EC; then AE $=$ EC $=$ EB, becauſe ACB may be inſcribed in a ſemicircle; then by ſimilar triangles, $AB : BC :: AC : CD = \dfrac{AC \times BC}{AB}$. Alſo $\angle CED = \angle B + BCE = 2B$, and $CEB = 2A$. Therefore by Prop. III. as rad : CE or $\tfrac{1}{2}AB$: : S.E : CD, or rad : S.E : : $\tfrac{1}{2}AB : \dfrac{AC \times CB}{AB} :: \tfrac{1}{4}AB^2 :$ $\dfrac{AC \times CB}{2}$ or the area of the triangle.

PROP. XIII.

As radius :
To ſine of any angle of a triangle, A : : 3.
So half the rect. of the including ſides, $\dfrac{AF \times AB}{2}$:
To the area of the triangle.

For

FIG. For the area of the triangle AFB is $\frac{1}{2}$ AB×FH;
3. but rad : AF :: S.A : FH $= \frac{AF \times S.A}{rad.}$, therefore the area $= \frac{AF \times AB \times S.A}{2\ rad.}$.

PROP. XIV.

9. *As coſ. diff. — coſ. ſum of the angles at the baſe :*
Sine of the vertical angle ::
So the ſquare of the perpendicular :
To the area of the triangle ::
And ſo the area of the triangle :
To the ſquare of half the baſe.

For by Prop. X. coſ. $\overline{B-A}$ — coſ. $\overline{B+A}$: S.D :: DP : GB :: DP^2 : GB × DP, the area :: GB × DP : GB^2.

PROP. XV.

3. *In any triangle,*
As radius :
Tan. half the vertical angle ::
Rect. of half the perimeter, and that half — baſe :
To the area of the triangle.

For, let $s =$ S.AFB; c, t, τ the coſine, tangent, and cotangent of half AFB, $m =$ AF, $n =$ FB, AB $= b$, $z = \frac{m+n+b}{2}$. Then by Prop. XIII. $\frac{mns}{2r} =$ area, but by ſch. Prop. II. B. I. $s = \frac{2cc}{\tau} = \frac{2cct}{rr}$, therefore $\frac{mncct}{r^3} =$ area. And by Cor. 3. Prop. VII. $\frac{mncc}{rr} = z \times \overline{z-b}$, therefore $\frac{z \times \overline{z-b}}{r} t =$ area.

Cor.

FIG. 3.

Cor. *As radius :*
Cotan. half the vertical angle : :
So rect. baſe + diff. ſides, and baſe — diff. ſides :
Four times the area of the triangle.

For if $a = $ S. $\frac{1}{2}$ angle F, then by ſch. Pr. II. B. I. $s = \frac{2aa}{t} = \frac{2aa\tau}{rr}$, and the area $= \frac{mns}{2r} = \frac{mna^2\tau}{r^3}$. But by Cor. 3. Pr. VII. $mna^2 = \overline{z-m} \times \overline{z-n} \times rr = \frac{b+n-m}{2} \times \frac{b+m-n}{2} r^2$, therefore the area $= \frac{b+n-m}{2} \times \frac{b+m-n}{2r} \tau$.

PROP. XVI.

11. *If three lines be drawn from the three angles of a triangle to any point* O, *the product of the ſines of all the alternate angles will be equal.*

Let the ſines of the angles be as in the figure, then in the triangle

$$\left.\begin{array}{llllllll} \text{POI,} & a & : & f & :: & IO & : & PO \\ \text{IOH,} & e & : & d & :: & HO & : & IO \\ \text{HOP,} & c & : & b & :: & PO & : & HO \end{array}\right\} \text{theſe multiplied,}$$

$$aec : fdb :: IO \times HO \times PO : PO \times IO \times HO.$$

But the two laſt terms are equal, and therefore the two firſt, $aec = fdb$.

Cor. 1. *If two angles,* P, H, *are biſected, by the lines* PO, HO; *then the third angle* I *will alſo be biſected.* For ſince $ace = bdf$, and $a = b$, and $c = d$, or $ac = bd$, therefore $e = f$. And the contrary.

Cor. 2. *Hence alſo three perpendiculars erected on the middle of the three ſides of a triangle, interſect in one point. And the contrary.*

FIG. 11. For if two perpendiculars be erected on two ſides PH, IH, then will $b=c$, and $d=e$, whence $a=f$; therefore the perpendicular ſtands on the middle of PI.

SCHOLIUM.

If the point O be taken out of the triangle, this Prop. will hold equally true.

PROP. XVII.

12. *If four right lines be drawn from the four angles of a trapezium, to any point in it, as* O; *the products of the ſines of the alternate angles, will be equal to each other.*

In the triangle.

SPO, $a : h :: SO : PO$
POH, $c : b :: PO : HO$
HOI, $e : d :: HO : IO$
IOS, $g : f :: IO : SO$

their product

$$aceg : hbdf :: SO.PO.HO.IO : PO.HO.IO.SO,$$

therefore

$$aceg = hbdf.$$

Cor. 1. *After the ſame manner, if lines be drawn from all the angles of any polygon whatever to a point; the products of the ſines of the alternate angles will be equal.*

13. Cor. 2. *In any trapezium, let* a, b, c, d &c. *be the ſines of the angles, as in the figure; then*

As $hbd + ace :$
$hbd - ace ::$

FIG. 3.

Tan. half the ſum of the angles $\frac{ISH+SIP}{2}$:

Tan. half the difference $\frac{ISH-SIP}{2}$.

For ſince $aceg = hbdf$, therefore $ace : hbd :: f : g$ :: SO : IO, and by compoſition and diviſion, $hbd + ace : hbd - ace$ :: IO + SO : IO — SO :: (by Prop. VI.) tan. $\frac{ISO+SIO}{2}$: tan. $\frac{ISO-SIO}{2}$.

SECT. II.

The ſolution of all the caſes of plain triangles.

From the foregoing propoſitions, all the caſes of plain triangles may be reſolved. Every triangle has 6 parts, 3 ſides and 3 angles; and any three things being given (except the 3 angles) the other 3 may be found; but if only 3 angles are given, there may be found an infinite number of triangles, that will have theſe three angles. I ſhall here give the ſolution of right-angled triangles 3 different ways, *viz.* arithmetically, logarithmically, and algebraically: and that of oblique triangles, logarithmically and algebraically. For it is needleſs to ſpend time in ſolving theſe arithmetically; ſince any oblique-angled triangle is divided into two right ones by a perpendicular, and then they are reſolved by the caſes of right-angled triangles; this method is true to 3 or 4 places of figures, and is ſufficient in common caſes. Alſo all the caſes in obliques (except the laſt) may be reſolved by the rules of right-angled triangles; but with a deal more labour; by letting fall a perpendicular from the end of a given ſide, or oppoſite to a given angle.

I have alſo omitted their ſolution by projection and inſtrumentally, as being only approximations to the truth; and they are beſides ſo eaſy in themſelves, to any body that can but handle a pair of compaſſes, as to need no particular explication. I ſhall only obſerve in general, with regard to Gunter's ſcale, &c. that any ſimple proportion may be wrought on it by this *general rule*,

Extend the compaſſes from the firſt term to one of the means, on its proper line; that extent, ſet the

the ſame way, upon its proper line, will reach from the other mean to the fourth term required; where radius is the ſine of 90°, or tan. 45°. If, in extending upon the tangents, the compaſſes reach beyond the end, ſet it ſo far back as it reaches over.

Now according to the different variety of things given and ſought, the ſolution of triangles is divided into ſeveral caſes as follows; in which I have only put the eaſieſt ſolutions. They that require more may conſult the propoſitions, from whence they are derived; the numerical operations are performed by *this rule*,

Add the logarithms of the ſecond and the third terms together, from which ſubtract the logarithm of the firſt term, and the remainder is the logarithm of the fourth term ſought, where radius is 10.

But in algebraic ſolutions you muſt uſe natural ſines, tangents, &c. and they muſt be actually multiplied; and divided, and here radius is 1.

Note, In right angled-triangles, if one acute angle is given, the other is found by ſubtracting this from 90.

And in oblique triangles, if 2 angles be given, the third is found by ſubtracting their ſum from 180. Or ſubtracting one from 180 gives the ſum of the other two.

In ſolving any of the caſes, it will be proper for diſtinction's ſake to mark what is given with a daſh (/), and what is ſought with a cypher (o).

Right-angled plain triangles.

CASE I.

FIG. 14. An angle C and the hypothenuſe BC, being given; to find a ſide, BA.

I. *Arithmetically without tables.*

Let D = degrees in the leſſer angle; then

$$\frac{57\cdot 3}{D} + \frac{3D}{1000} : 1 :: \text{hyp. BC} : \text{leſſer ſide CA.}$$

$$\text{or} \quad 1 : 1 - \frac{1\frac{1}{2}DD}{10000} :: \text{hyp. BC} : \text{greater ſide BA.}$$

II. *Logarithmically, by the table of artificial ſines*, &c.

Rad : hyp. BC : : S. an angle C : op. ſide BA.

III. *Algebraically, by the table of nat. ſines* &c.

Let hyp. BC $= h$, nat. ſine of C $= s$, rad. $= 1$. then

$1 : h :: s : sh =$ BA.

CASE II.

14. Given an angle B or C, and a leg; to find the hypothenuſe, BC.

I. *Arithmetically.*

Let D = degrees in the leſſer angle, then

$$1 : \frac{57\cdot 3}{D} + \frac{3D}{1000} :: \text{leſſer ſide AC} : \text{hyp. BC.}$$

$$\text{or } 1 - \frac{1\frac{1}{2}DD}{10000} : 1 :: \text{greater leg BA} : \text{hyp. BC.}$$

II. *Logarithmically.*

S. angle B : oppoſite ſide CA : : rad : hyp. BC. FIG. 14.

III. *Algebraically.*

Let $s = $ S.B, $p = $ AC, then $\frac{p}{s} = $ hyp. BC.

CASE III.

Given an angle, and a leg CA; to find the other leg, BA. 14.

I. *Arithmetically.*

Find the hyp. BC, by Caſe II. then $\sqrt{BC^2 - CA^2} = $ BA.

II. *Logarithmically.*

Rad : one leg CA : : tan. angle C : op. ſide BA.

III. *Algebraically.*

Let S.C $= s$, S.B $= c$, tan. C $= t$, CA $= p$, then $pt = $ BA, or $\frac{ps}{c} = $ BA.

CASE IV.

Given the hyp. BC, and a leg CA; to find an angle B, or C. 14.

I. *Arithmetically.*

Find the other leg, BA, by Caſe V, then

As hyp. BC : + ½ the longer leg BA : ſhorter leg CA : : 86 : op. angle B; and B taken from 90 gives C.

II. *Logarithmically.*

FIG. 14. As hyp. BC : rad : : given leg CA : : S. op. angle B, and B taken from 90 gives C.

III. *Algebraically.*

Let hyp. BC $= h$, leg CA $= p$, then $\frac{p}{h} =$ S.B, and $90 - B = C$.

CASE V.

14. Given the hypothenuſe BC, and a leg CA, to find the other leg BA.

I. *Arithmetically.*

$$\sqrt{BC^2 - CA^2} = BA.$$

II. *Logarithmically.*

Take the *ſum* and *difference* of the hyp. BC, and given leg AC, add their logarithms together; *half the ſum* is the logarithm of the ſide required, AB.

III. *Algebraically.*

Let $BC = h$, $CA = p$, then $\sqrt{hh - pp} = BA$.

Or let $BC + AC = z$, $\overline{BC - AC} = d$, then $\sqrt{dz} = BA$.

CASE VI.

14. The legs AB, AC, being given, to find an angle, B.

I. *Arithmetically.*

Find the hyp. BC by Caſe VII, then, as hyp. BC + half the longer leg, BA : leſſer leg CB : : 86 : angle op. B.

II. *Lo-*

II. *Logarithmically.*

As one leg AB : rad : : other leg AC : tan. B, the op. angle. FIG. 4.

III. *Algebraically.*

Let AB $= b$, AC $= p$, then $\frac{p}{b} =$ tan. B, or $\frac{p}{\sqrt{bb+pp}}$ $=$ S.B.

CASE VII.

The legs AB, AC being given; to find the hypothenuſe BC. 14.

I. *Arithmetically.*

$$\sqrt{AB^2 + AC^2} = BC.$$

II. *Logarithmically.*

Find the angle B by Caſe VI. and then the hyp. BC by Caſe II.

III. *Algebraically.*

Let AB $= b$, AC $= p$, then $\sqrt{bb+pp} =$ BC.

Oblique

Oblique plain triangles.

CASE I.

FIG. 15. 16. 17. Given two angles A, B, and an oppoſite ſide, FB; to find the other oppoſite ſide AF.

I. *Logarithmically.*

S. one angle A : *op. ſide* FB : : *S. the other angle* B : *to its oppoſite ſide* AF.

II. *Algebraically.*

Let $a = S.\ A$, $s = S.\ B$, $FB = d$, then $\frac{sd}{a} = AF$.

Alſo put $n = cos.\ A$, $m = cos.\ B$. *Then* $am + ns = S.\ F$, *and* $\frac{am + ns}{a} d = AB$. *Alſo* $\frac{ab}{am + ns} = BF$.

CASE II.

15. 16. 17. Two ſides AF, FB, and an oppoſite angle B being given; to find the other oppoſite angle A.

I. *Logarithmically.*

One ſide AF : *S. its op. angle* B : : *other ſide* FB : *S. of its op. angle* A.

If the given angle B is obtuſe, or if its oppoſite ſide AF be greater than the other ſide BF, then angle A ſought is acute; otherwiſe it is doubtful: and the ſum of A and B taken from 180, gives F.

II. *Algebraically.*

Let $s = S.\ B$, $AF = c$, $FB = d$, then $\frac{sd}{c} = S.\ A$.

Alſo

Alſo put m = *coſ.* B, *and then* $\frac{msd + s\sqrt{cc - ssdd}}{c}$ = S. F. FIG.

CASE III.

Two ſides AF, FB, and an oppoſite angle B, given; to find the third ſide AB. 15. 16. 17.

I. *Logarithmically.*

Find the angle A by caſe the ſecond, and then the angle F will be had. Then find AB by Caſe I.

II. *Algebraically.*

Let AF $= c$, BF $= d$, *S.* B $= s$, *coſ.* B $= m$. *Then* $dm \pm \sqrt{cc - ssdd}$ = AB; + if ∠ A is acute, and − if obtuſe. Where note, if B be obtuſe, m will be negative.

CASE IV.

Two ſides AF, FB, and the included angle F, being given; to find the other angles, A, B. 15. 16. 17.

I. *Logarithmically.*

As ſum of the ſides AF + FB : *their diff.* AF − FB : : *tan. half the ſum of the op. angles* $\frac{B + A}{2}$: *tan. of half their diff.* $\frac{B - A}{2}$; *then* $\frac{B + A}{2} + \frac{B - A}{2}$ = B *the greater angle, and* $\frac{B + A}{2} - \frac{B - A}{2}$ = A *the leſſer angle.*

II. *Al-*

II. *Algebraically.*

FIG. *Let* AF $= c$, BF $= d$, $x =$ *S.*F, $y =$ *coſ.* F. *Then* $\frac{dx}{c - dy} =$ *tan.* A, *and* $\frac{cx}{d - cy} =$ *tan.* B.

Or $\frac{c - dy}{\sqrt{dd + cc - 2dcy}} =$ *coſ.* A, *aud* $\frac{d - cy}{\sqrt{dd + cc - 2dcy}}$ $=$ *coſ.* B.

CASE V.

15. Given two ſides AF, FB, and the contained angle
16. F; to find the third ſide AB.
17.

I. *Logarithmically.*

Find A *or* B *by Caſe* IV. *then* AB *by Caſe* I.

II. *Algebraically.*

Let AF $= c$, BF $= d$, $y =$ coſ. F, $v =$ verſ. F. Then $\sqrt{cc + dd - 2cdy} =$ AB. Or $\sqrt{\overline{c - d}^2 + 2cdv}$ $=$ AB.

See alſo the corollaries to Prop. VII.

CASE VI.

18. Three ſides given, to find an angle F.

I. *Logarithmically.*

Let fall a perpendicular BD, on a ſide adjoining to the angle ſought, then as baſe AF : ſum, ſides AB + BF : : their diff. AB — BF : AD — DF the diff. ſegments, or the alternate baſe; then $\frac{1}{2}$ baſe AF + $\frac{1}{2}$ diff. = greater ſegment, and $\frac{1}{2}$ baſe AF — $\frac{1}{2}$ diff. = leſſer ſegment. Then there is given BF, DF to find F, by Caſe IV. of right-angled triangles. Alſo ſee Prop. VII. and cor.

II. *Algebraically.*

FIG. 18.

$$\left.\begin{array}{r}\textit{Let}\ \mathrm{AF}=c,\ \mathrm{BF}=d,\ \mathrm{AB}=b,\\ \mathrm{AF}\backsim\mathrm{BF}=n,\end{array}\right\}\left\{\begin{array}{l}\textit{then}\ \frac{cc+dd-bb}{2cd}=\textit{coſ.}\,\mathrm{F}.\\ \textit{or}\ \frac{b^2-n^2}{2dc}=\textit{verſ.}\,\mathrm{F}.\end{array}\right.$$

See alſo the corollaries to Prop. VII.

SCHOLIUM.

In the ſolutions of plane triangles there are three caſes, where the thing ſought requires two operations; 1. when two legs of a right-angled triangle are given to find the hypothenuſe; 2. when two ſides and the included angle, in an oblique triangle, are given, to find the third ſide; 3. when two ſides and an angle oppoſite are given, in an oblique triangle, to find the third ſide.

⁂

The actual ſolution of all the caſes of plain triangles in numbers.

Right-angled triangles.

CASE I.

FIG. 57. Given the ∠ C = 52 13, hyp. BC = 316, to find BA.

I. *Arithmetically.*

Let D = ∠ B = $37\frac{47}{60}$ = 37.78 in decimals, and $\frac{57.3}{D} = 1.516$, and $\frac{3D}{1000} = .113$, and $\frac{1\frac{1}{2}DD}{1000} =$ .214, and 1.516 + .113 = 1.629, and 1 − .214 = .786; therefore 1.629 : 1 : 316 : 194 = CA.
and 1 : .786 :: 316 : 248 = BA.

II. *Logarithmically.*

Rad. . . .	10.0000000
BC (316) ——	2.4996871
S.C (52 13) —	9.8978103
AB = 249.7 . . .	2.3974974

CASE II.

58. Given the ∠ B = 37 47, CA = 194. To find BC.

I. *Arithmetically.*

$\frac{57.3}{D} + \frac{3D}{1000} = 1.629$ as in the laſt, then
1 : 1.629 : : 194 : 316 = BC.

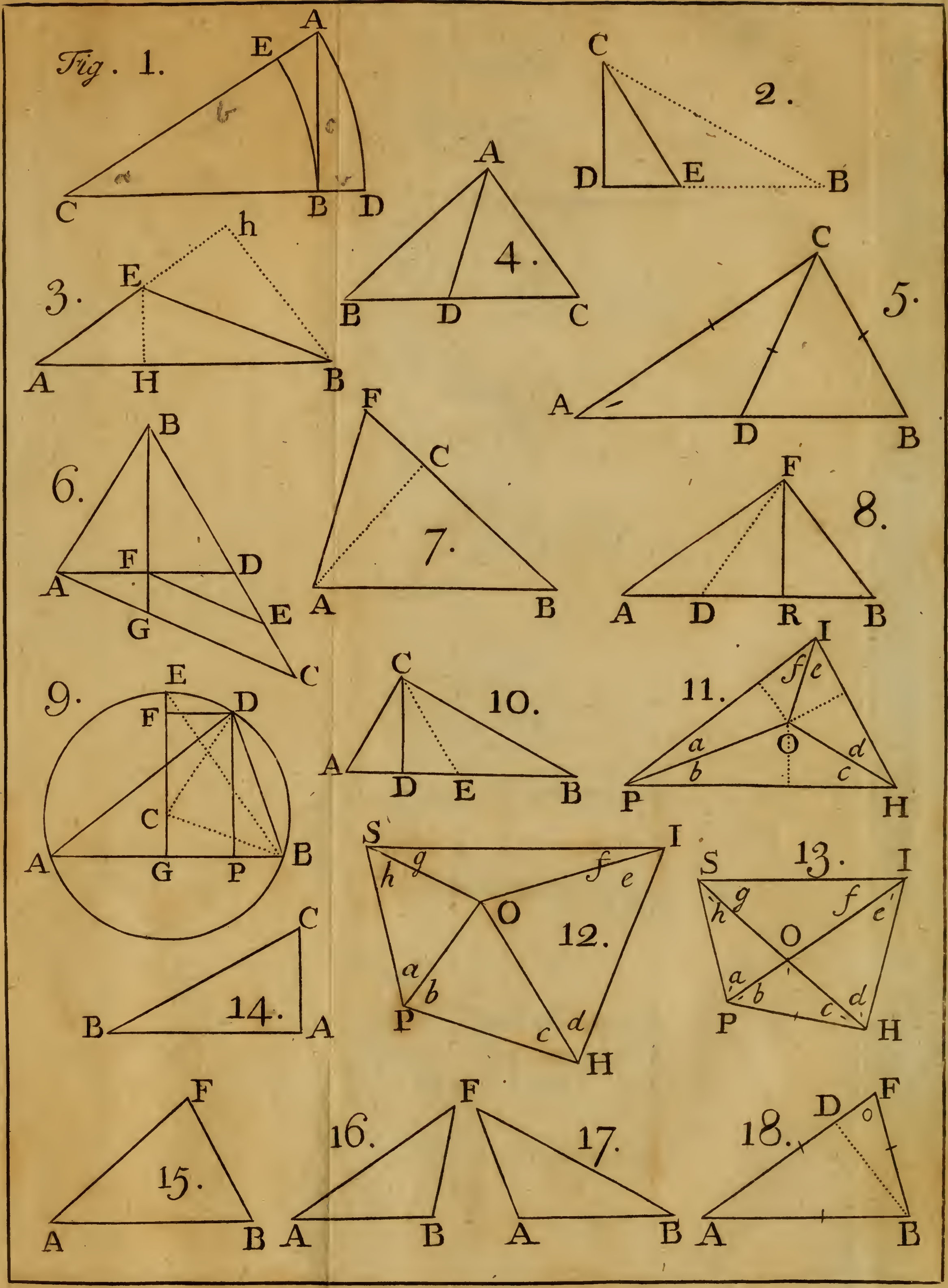


Fig. 1.
A
E
C
B
D
b
c
a
2.
C
D
E
B
3.
h
E
A
H
B
4.
A
B
D
C
5.
C
A
D
B
6.
B
F
D
A
G
E
C
7.
F
C
A
B
8.
F
A
D
R
B
9.
E
D
F
C
A
G
P
B
10.
C
A
D
E
B
11.
I
f
e
O
a
b
c
d
P
H
12.
S
I
g
h
f
e
O
a
b
c
d
P
H
13.
S
I
g
h
f
e
O
a
b
c
d
P
H
14.
C
B
A
15.
F
A
B
16.
F
A
B
17.
A
B
18.
D
F
o
A
B

Pl. IV. *pa.* 126.

II. *Logarithmically.* FIG. 58.

S.B (37 47) —— 9.7872317
CA (194) —— 2.2878017
Rad. —— —— 10.
Hyp. BC, 316.6 . . . 2.5005700

CASE III.

Given the ∠C = 52 13, CA = 194; to find BA. 59.

I. *Arithmetically.*

Find BC = 316, by Caſe II. Then
from the ſquare of BC = 99856
ſubtract the ſquare of CA = 37636
extract the ſquare root of 62220
which will be —— 249 = BA.

II. *Logarithmically.*

Rad. —— —— 10.
CA (194) —— 2.2878017
Tan. C (52 17) — 10.1116225
BA, 250.8 . . . 2.3994242

CASE IV.

There is given the hyp. BC = 316, and CA = 194; to find the angle B. 60.

I. *Arithmetically.*

From the ſquare of BC 99856, take the ſquare of CA 37636, the ſquare root of the remainder (62220) is 249 = BA; and 316 + ($\frac{1}{2}$ BA) $124\frac{1}{2}$ = $440\frac{1}{2}$, then

As $440\frac{1}{2}$: 194 : : 86 : 37.87 in decimals = 37 52.

II. *Lo-*

FIG. 60.

II. *Logarithmically.*

BC (316) ——	2.4996871
Rad. —— —	10.
CA (194) ——	2.2878017
S. B, 37 52 . . .	9.7881146

CASE V.

61. Given the hypothenuſe BC = 316, and CA = 194; to find BA.

I. *Arithmetically.*

From the ſquare of BC = 99856
take the ſquare of CA = 37636
the ſq. root of the remaind. 62220
extracted will be . . . . 249 = BA.

II. *Logarithmically.*

BC	316		
CA	194		
Sum	510	log.	2.7075702
Diff.	122	log.	2.0863598
			4.7939300
		half	2.3969650 = 249.4 = BA.

CASE VI.

The legs AB = 249, and AC = 194 being given; to find the angle B.

I. *Arith-*

FIG. 62.

I. *Arithmetically.*

To the ſquare of AB = 62001
add the ſquare of AC = 37636
The ſum —— 99637
Its ſquare root 315.6 = BC,
add ½ AB 124.5
440.1

then 440.1 : 194 : : 86 : 37.9 in decimals, or
= 37 54 = ∠ B.

II. *Logarithmically.*

AB (249 —— — 2.3961993
Rad. —— — — 10.
AC (194) —— — 2.2878017
Tan. B, 37 55 . . . 9.8916024

CASE VII.

The legs AB = 249, AC = 194, being given; to find the hyp. BC. 63.

I. *Arithmetically.*

To the ſquare of AB = 62001
add the ſquare of AC = 37636
Sum 99637
Its ſquare root 124.5 = BC.

II. *Logarithmically.*

Find the angle B = 37 55, by Caſe VI. then find the hyp. BC = 315.7, by Caſe II.

Oblique triangles.

CASE I.

FIG. 64. Given two angles A = 31 07, and B = 115 24; BF = 97, to find AF.

S. A (31 07) —— 9.7133077
FB (97) —— 1.9867717
S. B (115 24) — 9.9558490
11.9426207
AF = 169.6 . . . 2.2293130

CASE II.

65. Given AF = 320, FB = 468, and B = 32 15, to find the ∠ A.

AF (320) —— 2.5051500
S. B (32 15) —— 9.7272276
FB (468) —— 2.6702458
12.3974734
$\text{S. A}\left\{\begin{matrix}51\ 18 \text{ or}\\ 128\ 42\end{matrix}\right\}$ 9.8923234

Since AF is leſs than B, the angle A may be either acute or obtuſe.

CASE III.

66. Given $\left\{\begin{matrix}\text{AF} = 320,\ \text{B} = 32\ 15;\ \text{to find AB.}\\ \text{FB} = 468.\end{matrix}\right.$

Find the angle A = 51 18 or 128 42, by Caſe II; then we have F = 96 27 or 19 03.

Then

Then by Caſe I. find AB = 595.9 or 195.7; the ∠ A and AB being ambiguous, becauſe AF (op. to the given angle) is leſs than FB. FIG. 66.

Case IV.

$$\text{Given}\left\{\begin{array}{l} AF = 376 \\ FB = 311 \\ \quad F = 123\ 10 \end{array}\right\}\text{to find the angles A, B.}$$ 67.

AF = 376			180
FB = 311			123 10
Sum	687	Sum op. ∠s	56 50
Diff.	65	½ ſum	28 25

687 ——	2.8369567	then
65 ——	1.8129133	½ ſum 28 25
Tan. 28 25 ——	9.7332566	½ diff. 2 56
	11.5461699	31 21 = B
Tan. ½ diff. (2 56)	8.7092132	25 29 = A.

Case V.

$$\text{Given}\left\{\begin{array}{l} AF = 376 \\ FB = 311 \\ \quad F = 123\ 10 \end{array}\right\}\text{required AB.}$$ 68.

By Caſe IV. find A = 25 29.
Then AB will be found by Caſe I. = 605.1.

Case VI.

$$\text{Given}\left\{\begin{array}{l} AF = 376 \\ FB = 311 \\ AB = 605 \end{array}\right\}\text{, required } \angle \text{ F.}$$ 69.

FIG. 69. On AF let fall the perpen. BD, then

AB = 605
FB = 311
Sum 916
Diff. 294

AF (376)	——	2.5751878	then
Sum (916)	——	2.9618955	$\frac{1}{2}$AF = 188
Diff. (294)	——	2.4683473	$\frac{1}{2}$alt. = 358.1
		5.4302428	Diff. = 170.1
Alternate baſe (716.2)		2.8550550	= FD.

Then in the right ∠ triangle FDB, BF, FD are given, to find ∠ F, by Caſe IV. of right ∠ triangles; then F = 56 50, and ∠ AFB = 123 10, becauſe, the alternate baſe being greater than the true baſe; the perp. BD falls without the triangle.

THE

ELEMENTS

OF

TRIGONOMETRY.

BOOK III.

The doctrine of the ſphere and ſpherical trigonometry.

DEFINITIONS.

1. *Spherical trigonometry*, teaches the properties of ſpherical triangles, and how to calculate their ſides and angles.

2. *A circle of the ſphere* is that which is made by a plane cutting the ſurface of the ſphere. If the plane paſs thro' the center, it is a *great* circle: if not, it is a *leſſer* circle.

3. *The pole of a circle* is a point on the ſurface of the ſphere equidiſtant from every point of the circle. Every circle has two poles, diametrically oppoſite to each other.

4. *A ſpherical angle* is the inclination of two circles of the ſphere to one another, interſecting in a point called the angular point.

5. *A right angle* is 90 degrees; an *acute* angle is leſs, and an *obtuſe* angle greater than a right one.

6. *A ſpherical triangle* is made upon the ſurface of the ſphere, by the interſection of three great circles.

cles. If one angle be right, it is called a *right-angled* ſpherical triangle. If one ſide be a quadrant, it is called a *quadrantal* triangle. If no angle or ſide be 90°, it is an oblique triangle.

7. In a right-angled triangle, the ſide oppoſite to the right angle is called the *hypothenuſe*; the other two are called *legs* or ſides.

8. In a right-angled or quadrantal triangle, the five circular parts (ſetting aſide the right angle or quadrantal ſide) are, the two parts adjoining on each ſide the right angle or quadrantal ſide, and the complements of the three parts which are furtheſt diſtant from it. The *middle part* is that which is equidiſtant from other two; which two if they adjoin to the middle part, are called *extremes conjunct*; and if they be remote from it, are called *extremes disjunct*.

9. Sides and angles are ſaid to be of the *ſame affection*, when both are greater, or both leſſer, or both equal to 90°. And they are of *different affection*, if one is greater and another leſs than 90°.

10. *Like or ſimilar arches* of different circles are thoſe that contain the ſame number of degrees.

SECT. I.

The properties of ſpherical angles and arches.

PROP. I.

The ſection of a ſphere and a plane is a circle.

Let BIC be the ſection, and let the plane of the great circle ABCD be drawn through the center E of the ſphere, perpendicular to the plane BIC, and let BHC be their common interſection. Draw EH perpendicular to BC, then $BH = HC$. Take any point I in it, and draw HI, IC; and alſo BE, CE. Since EH is perpendicular to BC, and in the perpendicular plane ABCD, therefore it is perpendicular to the plane BIC, and therefore to HI. But BE, CE, IE are equal radii of the ſphere; therefore in the right-angled triangles BEH, IEH; $BH^2 + HE^2 = BE^2 = IE^2 = HI^2 + HE^2$, therefore $BH^2 = IH^2$, and $BH = HI$, therefore BIC is a circle whoſe center is H. FIG. 19.

Cor. 1. *If a great circle* ABCD *be perpendicular to any other circle* BIC, *it paſſes thro' its poles.*

For ſince it is proved that H is the center of the circle BIC, and EH perpendicular to it, produce EH to P which will be in the plane of the circle ABD, becauſe EH is in it. Draw the great circle PI, then ſince the ſines BH, HI, HC are equal, therefore the arches belonging to them BP, IP, CP are equal, therefore P is the pole. And if PE was produced to the other pole, that pole will alſo be in the plane of the circle ABC.

FIG. 19. Cor. 2. *If a great circle* ABC *paſſes thro' the pole* P *of another circle, it cuts it at right angles and into two equal parts.*

For draw PE to cut the circle in H, then is H the center, and PH perpendicular to its plane, and conſequently EH and the plane ABCD are perpendicular to it. And ſince the center H is in BC, the common interſection of the planes; therefore BIC is a ſemicircle.

PROP. II.

20. *All great circles of the ſphere cut one another into two equal parts.*

For the common ſection of their planes is the diameter of the ſphere, and conſequently their ſegments are ſemicircles.

Cor. 1. *Any ſide of a ſpherical triangle is leſs than a ſemicircle.*

For ſince ADC or ABC is a ſemicircle, therefore in the triangle ADB, AD or AB is leſs than a ſemicircle.

Cor. 2. *If two ſides of a ſpherical triangle* ABD *be produced, till they interſect in* C, *each will become a ſemicircle.*

PROP. III.

21. *If one or more circles interſect another circle in one and the ſame point* C; *the ſum of the angles* ACE, ECD *is equal to two right angles.*

Let the arch EC interſect AD in C, and draw CB perpendicular to AD; then ACB + BCD = two right angles; but ACE + ECB = ACB, and ECD — ECB = BCD, therefore ACE + ECD = ACB + BCD = two right angles.

Cor.

Cor. 1. *All the angles made about one point are* = *four right angles.* FIG. 21.

Cor. 2. *When one circle croſſes another, the vertical angles* a, c *are equal.*

For $a+b=$ two right angles, and $b+c=$ two right angles, therefore $a+b=b+c$ and $a=c$.

PROP. IV.

A ſpherical angle made by two great circles is meaſured by the arch of a great circle intercepted between the ſides, and deſcribed at a quadrant's diſtance from the angular point. 22.

Let AB = AD = 90°, and draw the arch BD, and from the center E draw BE, DE; alſo draw the tangents AT, AS, to the arches AB, AD; then the ∠ TAS = ſpherical angle BAD, and AEB, AED will be two right angles; but TAE, SAE are right angles, therefore TA, SA are $\parallel$ to BE, DE, and ∠ TAS = BED. But the arch BD is the meaſure of the angle BED or TAS, and therefore of the ſpherical angle BAD.

Cor. 1. *The angles* BAD, BCD, *made at oppoſite points of the ſemicircle, are equal. For the* ∠ BED *or arch* BD *is the meaſure of both.*

Cor. 2. *The diſtance of the poles of two circles is equal to the angle of their inclination.*

For ſince AE is $\perp$ to the plane BDE, this plane is $\perp$ to both the planes ABC, ADC; therefore the poles are in the circle BD, as ſuppoſe at P, Q. Then BP = 90° = DQ, and ſubtracting DP from both, then BD = PQ.

Cor. 3. *Two great circles* ABC, ADC *paſſing thro' the poles of another great circle* BD, *will cut all the parallels to* BD, *as* bd, *into ſimilar arches.*

For

FIG. 22. For AC is $\perp$ to *bed*, and *e* the center of *bd*, therefore $\angle$ BED = *bed*, or the arch *bd* ſimilar to BD.

Cor. 4. *Hence an angle made by two great circles of the ſphere, is equal to the angle of inclination of the planes of theſe great circles.* For $\angle$ BED or *bed* = $\angle$ TAS or BAD.

Cor. 5. *Hence alſo if bd be a parallel to the great circle* BDP; *it is as radius : coſine of the parallel's diſtance from its great circle : : ſo any arch of the great circle : to a ſimilar arch of the parallel.*
For BE : BD : : *be* : *bd*.

PROP. V.

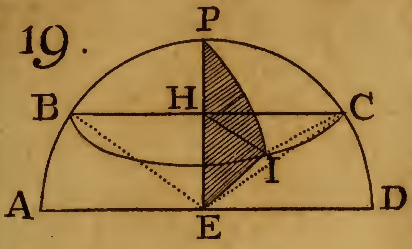
23. *If any two circles* AEB, AFB *interſect one another on the ſphere, they make the oppoſite angles* A *and* B *equal, and the oppoſite parts* AFE, BFE *ſimilar.*

Let AB be the common ſection of their planes; and in the planes of their reſpective circles, draw the tangents AC, BC to the circle AFB; and AD, BD to the circle AEB, and join CD. Then in the triangles ACD, BCD, AC = BC, being tangents to the ſame circle, and for the ſame reaſon AD = BD, and CD is common; therefore the triangles are equal and ſimilar, and the $\angle$ CAD = CBD, and therefore the ſpherical angles A and B equal to them, are alſo equal to one another.

Likewiſe the oppoſite parts of the figure AFBEA are ſimilar, being bounded by parts of the ſame circles, having the ſame poſition.

Cor. *Hence any two like arches* OI, SV, *drawn after a like manner, will be equal. That is, if* AO = BS, *and* AI = BV, *or* $\angle$ AOI = $\angle$ BSV, *then* OI = SV.

19.



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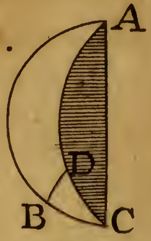

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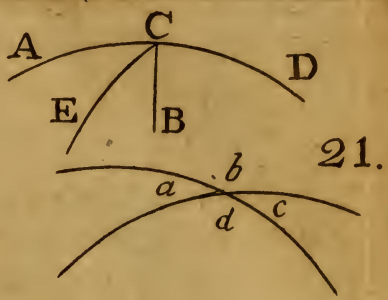

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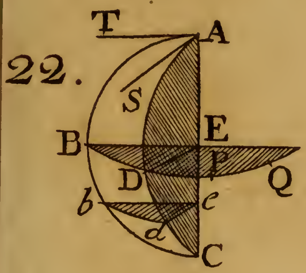

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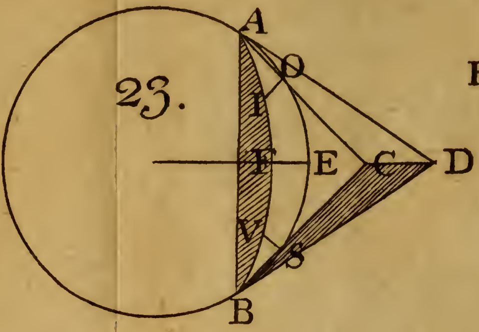

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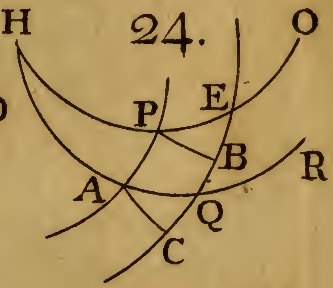

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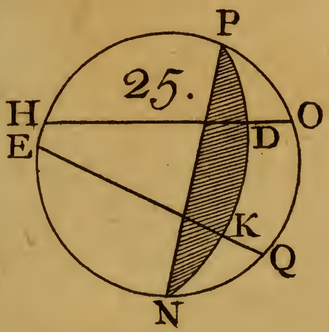

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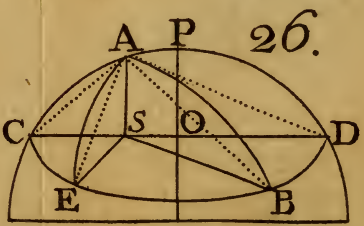

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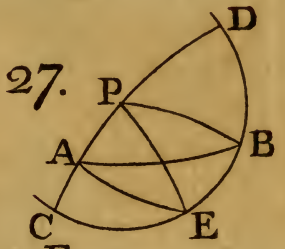

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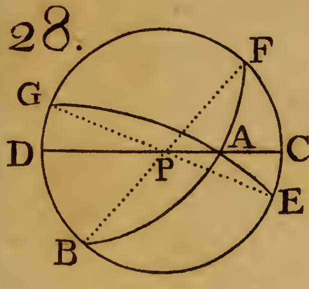

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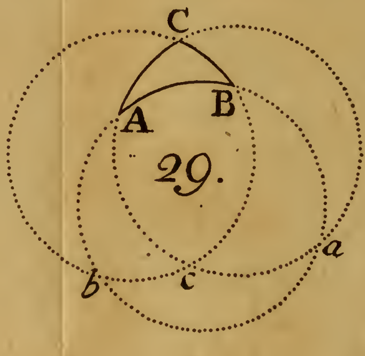

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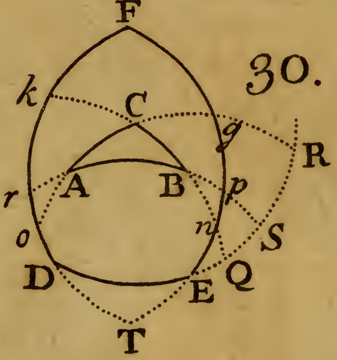

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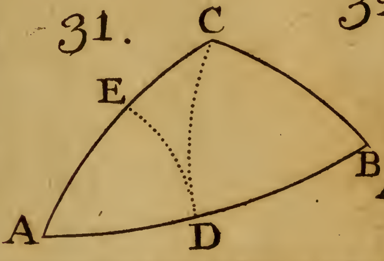

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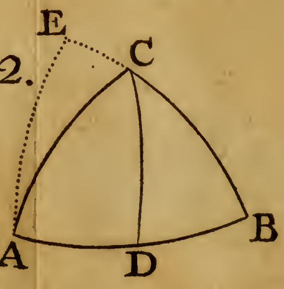

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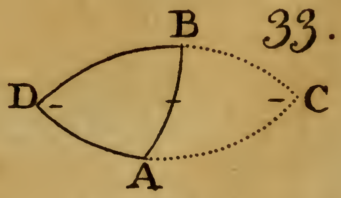

Pl. V. *pa.* 138.

SV. For then the triangles AOI, and BSV are ſimilar and equal. FIG. 23.

SCHOLIUM.

Though the oppoſite parts of the figure AFE and BFE are ſaid to be ſimilar; yet if the angle A be applied to B, the triangles AFE, BFE will not coincide. Or if the baſe FE of one be laid on the baſe EF of the other, the triangular figures AFE, BFE will lie contrary ways, one to the right, the other to the left hand.

PROP. VI.

If two great circles HO, HR *have the ſame inclination to a third* EQ; *the arches of all the parallels* (AP) *to the third, intercepted by the other two, will be ſimilar arches.* 24.

Let the great circle EC cut the circles HO, HR at equal angles in E and Q. Thro' the poles of EC draw the great circles PB, AC. Then in the triangles PEB, AQC, ∠ E = ∠ Q, and the angles at B and C are right, and PB = AC; therefore if the angle E be laid upon the angle Q, then will P fall ſomewhere in the arch PA (becauſe PB=AC); and ſomewhere in the arch QH (becauſe ∠ E = ∠ Q), therefore P falls upon A, their point of interſection; and conſequently B upon C (otherwiſe A would be the pole of CQ); therefore BE = CQ, and PE = AQ, therefore EQ = BC; but BC is ſimilar to AB (by Cor. 3. Prop. IV), therefore EQ and AP are ſimilar.

Cor. *The ſame things ſuppoſed, the arches of the two inclined great circles intercepted between any two parallels to the third, are equal,* PE = AQ.

PROP. VII.

FIG. 25. *If a plane* EPQ *be ſuppoſed to be drawn perpendicular to the planes of two equal circles* HO, EQ; *and any other plane* PDKN *be drawn thro' the two poles* P, N, *which are equidiſtant from their reſpective circles, and to revolve about* PN : *then theſe planes will intercept equal arches of theſe circles,* DO, KQ.

For let N be the pole of HO, and P of EQ, and NO = PQ, then by cor. Prop. V. ∠ NOH or NOD = PQE or PQK; and ∠ POD = NQK, and NQ = PO, and by Prop. V. ∠ N = P. Therefore the triangles POD and NQK are ſimilar and equal, and therefore arch DO = KQ. And likewiſe arch HD = EK.

PROP. VIII.

26. *Of ſeveral arches of great circles drawn from the ſame point* A *to any other circle* CED; *the greateſt* AD *is that which paſſes thro' the pole* P, *and the nearer to this as* AB *is greater than that which is further of as* AE. *And the leaſt is* AC, *the remainder of that which paſſes thro' the pole.*

Let AS be perpendicular to the plane of the circle CED, O its center, CSOD its diameter. Draw the right lines SB, SE, AD, AB, AE, AC. Then you have the right-angled plane triangles ASD, ASB, ASE, ASC, wherein the perpendicular AS is common. But (Geom. IV. 4.) the baſe SD paſſing thro' the center is the greateſt, and SB greater than SE, and SE greater than SC, which is the leaſt.

Therefore the hypothenuſe AD is the greateſt, AC the leaſt, and AB greater than AE. But theſe are the cords of arches, and to greater cords belong greater arches, conſequently the arch AD is the greateſt, arch AB (nearer to AD) is greater than arch

arch AE (further from it); and arch AC the leaſt FIG.
of all. 26.

Cor. 1. *If two ſides in two ſpherical triangles* APE, 27.
APB *be reſpectively equal; that which hath the greater included angle, hath the greater baſe: and the contrary.*

For, about the pole P, at the diſtance PE or PB (the greater *ſide* in the two triangles), deſcribe the circle EBD, and from A thro' P draw the great circle APD. Then by this Prop. if BD be leſs than ED, or ∠ APB greater than APE; then is AB greater than AE.

And if AB be greater than AE, then AB is nearer to D than AE, ∠ APB greater than APE.

Cor. 2. *The perpendicular let fall on the baſe of a ſpherical triangle, is either greater than either ſide, or leſs than either ſide.*

For it paſſes through the pole of the baſe, and is therefore either the longeſt or ſhorteſt line drawn from the vertex of the triangle.

Cor. 3. *And therefore when the perpendicular falls without the triangle, either the greater or leſſer perpendicular may be eſteemed the perpendicular upon the baſe.*

Cor. 4. *Hence if the perpendicular fall without, as* 28.
in the triangle BAE, *the greater perpendicular* AD *lies next the greateſt ſide* AB; *and the leaſt perpendicular* AC *next the leaſt ſide* AE.

But when the perpendicular falls within; if it is leſs than a quadrant, as AC (in the triangle AEF), then it lies nearer the leſſer ſide AE, and the ſegment CE is leſs than CF, and ∠ CAE leſs than CAF. But if it is greater than a quadrant, as AD (in the triangle ABG), then it lies neareſt the greater ſide AG, and then it makes the leſſer ſegment of the

FIG. 28. the baſe (DG), and leſſer vertical angle (DAG), next the greater ſide AG. And the greater ones next the leſſer ſide AB.

Cor. 5. *The perpendicular falling within, or the neareſt perpendicular falling without, is of the ſame affection, as half the ſum of the ſides.*

For in the triangle EAF, AC is leſs than a quadrant, and ſince $AF < AG$, therefore $AE + AF < EG$ or 180, and $\frac{AE + AF}{2} < 90$. But in the triangle GAB, AD is $>$ a quadrant; and ſince $AG > AF$, therefore $BA + AG > BF$ or 180, and $\frac{BA + AG}{2} > 90$.

Again, in the triangle BAE, where the perpendicular AC is leſs than a quadrant, and $CE < CF$; then by this Prop. $AE < AF$, and $AE + AB < BF$ or 180°, and $\frac{AE + AB}{2} < 90$. But in the triangle GAF, where the perpendicular AD (greater than a quadrant) is neareſt, or $DB > DG$, or $CF > CE$, then $AF > AE$, and $AF + AG > GE$ or 180, and $\frac{AF + AG}{2} > 90$.

Cor. 6. *Hence, when the perpendicular falls within, or you take the neareſt perpendicular falling without; then if half the ſum of the ſides be leſs than a quadrant, the leſſer ſegment, and leſſer vertical angle, adjoins to the leſſer ſide; but if half the ſum of the ſides be greater than a quadrant, the leſſer ſegment and leſſer vertical angle adjoins to the greater ſide.*

SECT. II.

The affections of ſpherical triangles.

PROP. IX.

Every triangle ABC *hath (by producing its ſides) another triangle abc, on the oppoſite ſide of the globe, ſimilar and equal to it.* FIG. 29.

For the point a is oppoſite to A, or a ſemicircle diſtant from it; and b is oppoſite to B, and c to C. Therefore $ab = AB$, $bc = BC$, and $ac = AC$. Likewiſe, by Cor. 1. Prop. IV. $\angle a = \angle A$, $b = B$, and $c = C$; therefore the triangle abc is equal and ſimilar to ABC.

Cor. 1. *Hence any triangle* ABC, *by producing its ſides, hath oppoſite to every angle thereof, another triangle of the ſame baſe and oppoſite angle with the former; and the other parts the ſupplements thereof.*

Thus in the triangle Bca, $\angle B = \angle ABC$, $ac = AC$, $aB =$ ſup. AB, $cB =$ ſup. CB, $\angle caB =$ ſup. CaB or A, and $\angle acB =$ ſup. C.

Cor. 2. *Any triangle* ABC, *has adjoining to every ſide thereof another triangle, having the ſame ſide and oppoſite angle, and the other parts the ſupplements thereof.*

Thus in the triangle ABc, AB and $\angle c$ is the ſame with AB and $\angle C$; Bc, Ac are the ſupplements of BC, AC; $\angle cBA =$ ſup. CBA, and $\angle cAB =$ ſup. CAB. Likewiſe there is the triangle aBC adjoining to CB; and the triangle AbC adjoining to AC.

SCHO-

SCHOLIUM.

FIG. 29. Tho' the triangles ABC, *abc* are ſaid in this Prop. to be ſimilar; yet they will not coincide when applied to one another. For laying any angle *a* upon its equal A, ſo that the concavities of the triangles lie the ſame way; the ſide *ac* will fall on AB, and *ab* on AC.

PROP. X.

30. *If from the three angles of any ſpherical triangle* ABC, *as poles, you deſcribe three great circles, they will form another ſpherical triangle* DEF *by their interſections; each ſide and angle whereof will be the ſupplements of the angle and ſide oppoſite, in the given triangle.*

Produce the ſides as in the fig. then ſince Bk or BF is a quadrant, and Ag or AF is a quadrant, therefore F is the pole of AB; and likewiſe E the pole of AC, and D the pole of CB. Then $DQ =$ a quadrant $= ER$, and $DE = QR =$ ſup. $\angle C$.

Alſo $Eg = 90^\circ = Fp = Tp$, and $TE = pg$, therefore $FE =$ ſup. ET or pg, that is, of the $\angle CAB$. Likewiſe $DT = kr$ or $\angle B$, and $DF =$ ſup. $\angle B$.

Alſo $rB = 90^\circ = Ap$, and $rB + Ap$, or $rp + AB = 180^\circ$, and rp or the $\angle F = 180 - AB =$ ſup. AB.

And thus $Ag = CR$, and $AC = gR$, and ſup. $gR =$ ſup. AC, that is, $\angle E =$ ſup. AC.

Laſtly, $CQ = 90^\circ = Bk$, and $CQ + Bk$ or $kQ + CB = 180$, and $kQ = 180 - CB$, that is, $\angle D =$ ſup. CB.

Cor. 1. *Some three poles of the ſides of any triangle, form another triangle, wherein the angles and ſides are reſpectively equal to the ſupplements of the ſides and angles*

angles of the other; and theſe two triangles are mutually ſupplemental to each other. FIG. 30.

For D, E, F are the poles of the ſides of the triangle ABC, as well as A, B, C are the poles of the ſides of DEF.

Cor. 2. *The neareſt poles* D, E, T *of the ſides of any ſpherical triangle* ABC, *form another triangle, wherein the angles and ſides are reſpectively equal to the ſides and angles of the given one: excepting only the ſupplements of one ſide and its oppoſite angle in the former, will be the correſpondent angle and ſide in the latter.*

For in the triangles DFE, DTE, DE is common, and ∠ F = ∠ T; and all the reſt are the ſupplements. There are other five, or in all, ſix ſuch triangles, and all adjoining to the angles and ſides of the triangle DEF, by Cor. 1, 2. Prop. IX. but there is only one ſupplemental triangle DEF to the triangle ABC.

SCHOLIUM.

By this prop. quadrantal triangles may be reduced to right-angled ones.

PROP. XI.

In two triangles, if three ſides in one be reſpectively equal to three ſides in the other; or the three angles in the one to the three angles in the other; theſe triangles will be equal in all reſpects.

Caſe 1. If three ſides be equal; and if the baſe of one triangle be laid on the baſe of the other, the two other ſides muſt coincide; becauſe there can be but one point from whence two given arches can be drawn to the two angles at the baſe.

Caſe 2. If three angles are equal, then by Cor. 1. Prop. X. the ſides of two triangles formed by the

poles

FIG. poles of both, will be the ſupplements of the angles of the former, and therefore reſpectively equal. Therefore, by Caſe 1. the angles of theſe ſupplemental triangles are equal, and theſe angles are the ſupplements of the ſides of the former triangles by Cor. 1. Prop. X. therefore the ſides of the former triangles are reſpectively equal.

SCHOLIUM.

This demonſtration ſuppoſes the equal ſides to lie the ſame way; but if they lie contrary ways, they cannot coincide. Yet if you will ſuppoſe the convexity of one of them to lie the contrary way, then they will coincide when laid upon one another. The ſame may be applied to the following prop.

PROP. XII.

If in two triangles, there be two ſides and the included angle; or two angles and the included ſide, reſpectively equal; the two triangles will be equal in all reſpects. For,

1. If one ſide in one triangle be laid upon its equal in the other triangle; then, by reaſon of the included angle being equal in both, the other two ſides will coincide, and ſo the whole triangle.

2. If the equal ſide in one be laid upon that in the other, and the equal angles upon one another, the whole triangles will coincide, and will therefore be equal.

PROP. XIII.

31. *The ſum of any two ſides of a ſpherical triangle is greater than the third ſide.*

Let AB be the greateſt ſide; about the pole B, thro' C deſcribe the arch CD, which will be ⊥ to BD, BD, alſo about the pole A thro' D deſcribe the arch DE, which will alſo be ⊥ to AD. And ſince the two circles CD, ED, can but touch in one point D, therefore at E, C there is a ſpace EC between them. Whence, ſince AE + CB = AD + DB or AB, therefore AE + EC + CB or AC + CB is greater than AB. FIG.

Cor. 1. *The ſum of the three ſides of any ſpherical triangle* DEF *is leſs than a circle.* 30.

For DE is leſs than DT + TE, and therefore DE + EF + FD is leſs than FDT + FET or two ſemicircles.

Cor. 2. *A great circle is the neareſt diſtance of any two points upon a ſphere.* For it is a line of the leaſt curvature that can be drawn, or deflects leaſt from a right line.

PROP. XIV.

In any triangle equal ſides are oppoſite to equal angles. 32.

1. Let AC = BC, let CD biſect the angle ACB, then in the triangles ACD, CDB, there are two ſides and the included angle at C reſpectively equal; therefore by Prop. XII. the ∠ A = ∠ B.

2. If ∠ A = ∠ B, make AE = BE, then by Caſe 1. the ∠ EAB = ∠ B = ∠ CAB by ſuppoſition; therefore E falls upon C, and AC is = BC.

Cor. 1. *An equilateral triangle is alſo equi-angular, and the contrary.*

Cor. 2. *In an iſoſceles triangle, a perpendicular let fall on the baſe, biſects it and the vertical angle: et contra.*

Suppoſe the convexity of the triangle CDB to lie the contrary way; then becauſe the angles at D are equal; if CDB be laid upon CDA, DB will fall

upon

FIG. 32. upon DA; and becauſe $CB = CA$, and $\angle A = \angle B$ (by this prop.) therefore the point B will fall upon A; and the triangles will coincide, then will $AD = DB$, and $\angle$s at C will be equal.

Again, If $AD = DB$, or $\angle ACD = \angle BCD$; then by Prop. XII. the angles at D are equal, or $CD \perp AB$.

PROP. XV.

31. *In any ſpherical triangle, the greateſt ſide is oppoſed to the greateſt angle; and the leaſt to the leaſt.*

1. Let $\angle C > B$, and make $\angle BCD = B$; then will $CD = BD$ by Prop. XIV. but $CD + AD$ or $AB > AC$ by Prop. XIII. For the ſame reaſon if $B > A$, then $AC > CB$.

2. Let $AB > AC$, then angle $C > B$; for if C be equal or leſs than B, then AB is equal or leſs than CA by Caſe 1. which is contrary to the ſuppoſition.

PROP. XVI.

33. *If one ſide* DA *of a triangle* DAB *be produced, and if the ſum of the other ſides* $DB + BA$ *be greater, equal, or leſs than a ſemicircle; the internal angle at the baſe* D, *is accordingly greater, equal, or leſs than the outward oppoſite angle* BAC.

For if $DB + BA$ be $> =$ or $<$ than DC, then BA is $> =$ or $<$ than BC; and by Prop. XV. the angle C or D is $> =$ or $<$ than BAC, reſpectively.

Cor. 1. *In an iſoſceles triangle, as one of the equal ſides is greater, equal, or leſs than a quadrant; the angle at the baſe is accordingly greater, equal, or leſs than a right angle.*

Cor. 2. *In an iſoſceles triangle, either ſide is of the ſame affection as its oppoſite or adjacent angle.* FIG.

PROP. XVII.

In any ſpherical triangle, if the ſum of any two ſides be greater, equal, or leſs than a ſemicircle; the ſum of the oppoſite angles is accordingly greater, equal, or leſs than two right angles. 33.

For by Prop. XVI. if DB + BA be greater, equal, or leſs than DC, then ∠ D is greater, equal, or leſs than BAC, and D + DAB greater, equal, or leſs than DAB + BAC, or two right angles.

Cor. 1. *In any triangle half the ſum of any two ſides is of the ſame affection, as half the ſum of their oppoſite angles.*

Cor. 2. *Hence it follows, that if in a triangle* ACB, *there be given two ſides* AC, CB, *and an oppoſite angle* A; *and if* CD *be made* = CB, *then you will have two triangles* ACB, ACD *containing the things given, and therefore the other oppoſite angle may be either* B *or* ADC; *and conſequently in both of them the ſum of the ſides* AC, BC; *or* AC, DC; *are of the ſame affection (with reſpect to* 180°*), as the ſum of the oppoſite angles* A *and* B, *or of* A *and* ADC. Therefore, whenever the ſum of the angles A and B are of the ſame affection as the ſum of A and the ſupplement of B, then it is doubtful whether the other angle B is acute or obtuſe, becauſe it may be either. But when theſe are of different affections, that value only of B (whether acute or obtuſe) muſt be taken, which, when added to A, their ſum is of the ſame affection, as the ſum of the oppoſite ſides. 34.

Cor. 3. *And for the ſame reaſon, when two angles* A *and* B, *and an oppoſite ſide* CB *are given: if there* 35.

FIG. 35. *be two triangles* ACB, DCB *that contain the ſame data, it will be doubtful whether the other oppoſite ſide be* AC *or its ſupplement* CD *(for* CD = CE, *the angles* A, D, E *being equal). For it may be either of them.* Therefore we muſt always take that value of AC which added to CB, their ſum may be of the ſame affection (with reſpect to 180), as the ſum of the oppoſite angles. And if both ſums *(viz.* of AC or its ſupplement, added to CB*)* be of one and the ſame affection; then it is doubtful. Alſo if CI be ⊥ AB, it will be ambiguous whether AI or its ſup. EI = DI be the ſegment of the baſe, and whether ACI or its ſup. ECI or DCI be the angle at the vertex.

Note, The caſes mentioned in the two laſt corol. contain the ſix ambiguous caſes of oblique triangles. For when two ſides and an angle oppoſite are given; the other three things will have different values, according as an arch or its ſupplement is taken for the other oppoſite angle. And if two angles and an oppoſite ſide be given; according as an arch or its ſupplement is taken for the other oppoſite ſide; the baſe and its oppoſite angle will have different values.

33. Cor. 4. *If in a right-angled triangle, there be given an angle and its oppoſite ſide, there will be two triangles* DBA, CBA *that have the ſame data, and therefore the other three things are doubtful. Whence the hypothenuſe may be either* DB *or its ſup.* BC. If DB be the hypothenuſe, then the baſe is DA and ∠ oppoſite DBA. If BC is the hyp. then AC is the baſe and ABC the op. angle, theſe three being the ſup. of the others. And theſe are called the three ambiguous caſes in right-angled triangles; when an angle and its op. leg is given, to find the reſt.

After the ſame manner in a quadrantal triangle, if a ſide and the oppoſite angle be given; the angle oppoſite to *to the quadrantal ſide may be either that angle* DAB, *or its ſupplement* BAC. And therefore the three things ſought are ambiguous, and each of them may be either an arch or its ſupplement. FIG. 33.

Cor. 5. From what has been ſaid it follows, that *in two ſpherical triangles, if two ſides and an oppoſite angle, be reſpectively equal in both triangles; and the other oppoſite angles in both triangles, of the ſame kind, either both acute or both obtuſe; then theſe two triangles will be equal in all reſpects.*

And if two angles and an oppoſite ſide be reſpectively equal in both; and the other oppoſite ſides, of the ſame kind: then the triangles are equal in all reſpects.

PROP. XVIII.

In any triangle ABC, *the ſum of the three angles is greater than two right angles.* 30.

Let FDE be the ſupplemental triangle to ABC, by Prop. X. then by Cor. 1. Prop. XIII. $DF + FE + ED$ is leſs than a circle; that is, the ſupplements of A, B, and C is leſs than 2 ſemicircles, or $180 - A + 180 - B + 180 - C < 360$; ſubtract 2 ſemicircles, and then $180 - A - B - C < 0$; add $A + B + C$, then $180 < A + B + C$, or $A + B + C > 180$.

Cor. 1. *The ſum of the three angles of any triangle is leſs than ſix right angles. For every angle muſt be leſs than 2 right angles, otherwiſe it would be one continued arch.*

Cor. 2. *In any triangle* DAB, *the external angle* BAC *is leſs than the ſum of the interior oppoſite angles* D *and* B. 33.

FIG. 33. Cor. 3. *The ſum of any two angles is greater than the ſupplement of the third angle.*

30. Cor. 4. *The ſum of any two angles — the third (or any angle and the difference of the other two) is leſs than two right angles.*

For FD is leſs than FE + ED, by Prop. XIII, that is, 180 — B, is leſs than 180 — A + 180—C; that is, 180 + A + C — B is leſs than 180 + 180, or A + C — B leſs than 180.

Cor. 5. *In a right-angled ſpherical triangle, the ſum of the oblique angles is greater than one, and leſs than three right angles. By this prop. and Cor.* 4.

PROP. XIX.

28. *If the angles at the baſe of a ſpherical triangle be of the ſame affection, the perpendicular on the baſe will fall within the triangle; if of different affection, it falls without.*

In the triangle GAB, whoſe baſe is GB and its pole P, ſince the perpendicular muſt paſs thro' the pole P, by Prop. I. Cor. 1. let it interſect the baſe in D, C, then the angles PGB and PBG are right, and therefore AGB and ABG are both obtuſe, when the perpendicular CD falls between them. Likewiſe the angles PFE, and PEF are both right; therefore in the triangle FAE the angles F and E are both acute, when AC falls within. But in the triangle BAE the perpendicular falls without, and the ∠ ABE is acute and AEB obtuſe. And the ſame will hold for any other triangle that can be drawn.

36. Cor. *If the two leaſt ſides of a ſpherical triangle,* AC, BC *be of the ſame affection; the perpendicular*

lar CD *upon the other* AB, *will fall within the triangle.* FIG. 36.

Let AB be the greateſt ſide, make BE = BC, and AF = AC, then by Cor. 2. Prop. XVI. the ∠ BEC is of the ſame affection as CB, and AFC of the ſame affection as AC; but AC, BC are of the ſame affection, and therefore the angles EFC, FEC. Therefore by this prop. the perpendicular CD falls within the triangle ECF or ACB.

If AB be leſs than AC, CB, but greater than their ſupplements (they being of the ſame affection), the perpendicular will fall within. For this and the adjoining triangle (whoſe common baſe is AB) have the ſame perpendicular. See Cor. 2. Prop. IX.

SCHOLIUM.

Some other properties of perpendiculars may be ſeen in the corollaries to Prop. VIII.

PROP. XX.

In any right-angled ſpherical triangle, each of the oblique angles is of the ſame affection as the oppoſite ſide. 37.

If the angle at A be right, and AC leſs than a quadrant, produce AC to D, that AD may be a quadrant, and draw BD; then D is the pole of BA, and conſequently ABD a right angle, and therefore ABC is leſs than a right angle. But if AE be greater than a quadrant; then its oppoſite angle ABE is greater than the right angle ABD. And if AD be a quadrant, the angle ABD is a right angle.

On the contrary, if ABD is a right angle, then D is the pole of AB, and AD a quadrant; but if ABC be leſs or ABE greater than a right angle; then

FIG. then the oppoſite ſide AC will be leſs, or AE greater
37. than the quadrant AD.

PROP. XXI.

38. *In a right-angled ſpherical triangle* BAD, *if the legs be of the ſame affection, the hypothenuſe* BA, *will be leſs than a quadrant; if of different affection, it will be greater: and the contrary.*

Let D be the right angle, and produce DB, DA to C; and make DP, DE quadrants, then P is the pole of DB, and E of DA, and PB a quadrant. Then in the triangle DBA, if DB, DA be leſs than a quadrant, then BA will be leſs than the quadrant BP, by Prop. VIII. And in the right-angled triangle BAC, if BC, AC, be greater than a quadrant; then alſo BA is leſs than BP.

But in the triangle DBF, where DB is leſs and DF greater than a quadrant; as likewiſe in the triangle BCF, the hypothenuſe BF will be greater than the quadrant BP, by the ſame Prop. VIII.

On the contrary, if the hypothenuſe BA be leſs than a quadrant BP, then DB, DA are both leſs; or elſe CB, CA both greater than a quadrant. But in the triangle DBF or CBF, where the hypothenuſe BF is greater than the quadrant BP; DB, DF or elſe CB, CF are of different affections.

Cor. 1. *As the two oblique angles are of the ſame or different affection; the hypothenuſe will be accordingly leſſer or greater than a quadrant: et contra, by Prop.* XX. *and* XXI.

Cor. 2. *As the hypothenuſe and one ſide (or its oppoſite angle) is of the ſame or different affection; the other ſide (and its oppoſite angle) will be leſs or greater than a quadrant.*

Cor.

Cor. 3. *As the hypothenuſe is leſſer or greater than a quadrant, each leg will be ſimilar or diſſimilar to its adjacent angle: and the contrary.* FIG. 38.

PROP. XXII.

If a triangle CIA *have one acute angle, it alſo hath one ſide leſs than a quadrant.* 39.

Let ACI be the acute angle, and in the triangle CBA, let C be a right angle; if either A fall between P and C, or B between Q and C, the prop. is manifeſt; for then either CA or CB is leſs than a quadrant. Let therefore CB, CA be greater than quadrants, then by Prop. XXI. BA is leſs than a quadrant, therefore in the acute angled triangle ACI, AI is ſtill leſs than a quadrant.

Cor. *If a triangle have one ſide greater than a quadrant, it alſo hath one obtuſe angle.*

For if a triangle hath one ſide greater than a quadrant, its ſupplemental triangle (by Prop. X.) hath one acute angle; and therefore by this prop. it hath a ſide alſo leſs than a quadrant. And if ſo, then (by Prop. X.) the former has an angle greater than a right angle.

PROP. XXIII.

If a triangle has two ſides leſs than quadrants, it has alſo an acute angle. 39.

For in the triangle DBA, if BD, DA be leſs than quadrants, the ſum of the oppoſite angles is leſs than 2 right angles, by Prop. XVII. therefore at leaſt one of the angles is leſs than a right angle.

Cor. *If a triangle* ACB *has two obtuſe angles, it has alſo one ſide greater than a quadrant.*

For

FIG. 39. For by Prop. XVII. if two angles be greater than two right angles, then the ſum of their oppoſite ſides is greater than a ſemicircle; therefore at leaſt one of them is greater than a quadrant.

PROP. XXIV.

36. *If the three angles of a ſpherical triangle be acute; each ſide will be leſs than a quadrant.*

For the perpendicular CD from any angle will fall within the triangle by Prop. XIX. and ſince, in the right-angled triangles ACD, and DCB, the angles A and ACD, and B and BCD are all acute, therefore by Cor. 1. Prop. XXI. the hypothenuſes AC, BC are leſs than a quadrant. And by the ſame reaſoning AB is leſs than a quadrant.

30. Cor. 1. *If the three ſides of a ſpherical triangle* DEF *be greater than quadrants, the three angles will be obtuſe.*

For then the three angles A, C, B of the ſupplemental triangle ACB, will be acute, and therefore the ſides leſs than quadrants, by this prop. And therefore by Prop. X. the angles of the triangle DEF will be obtuſe.

Cor. 2. *In a triangle* DEF, *if two angles* D, E *be obtuſe, and one* F *acute; the ſides are of the ſame affection as the oppoſite angles.*

For in the adjoining triangle DET all the angles are acute by Cor. 2. Prop. IX; and all the ſides leſs than quadrants, by this prop. and therefore in the triangle DEF; DF, EF are greater than quadrants.

Cor. 3. *In a triangle, if two ſides be leſſer, and one greater than a quadrant; the angles will be of the ſame affection as the oppoſite ſides.* This follows from Cor. 2. of this, and Prop. X.

PROP.

PROP. XXV.

FIG. 39.

In a ſpherical triangle, when two ſides are of the ſame affection, and the included angle acute; then the third ſide is leſs than a quadrant. But if the two ſides are of different affection, and the included angle obtuſe; the third ſide is greater than a quadrant.

1. In the triangle BDA or BCA, right-angled at D and C, if the legs are of the ſame affection, the hypothenuſe BA is leſs than a quadrant, (by Prop. XXI.) but if the angle at D or C be leſs than 90, as ACI, ADI; then AI is ſtill leſs.

2. Suppoſe the legs to be of different affection, then by (Prop. XXI.) the hyp. BA will be greater than a quadrant, and much more will BA be greater than a quadrant, if the angle at C and D be obtuſe.

Cor. *When two angles are of the ſame affection, and the included ſide more than a quadrant; then the third angle is obtuſe. But if two angles are of different affection, and the included ſide leſs than a quadrant; the third angle is acute.*

For by Prop. X. its ſupplemental triangle comes under the caſe of this prop.

* * *

SECT. III.

The proportions for calculating the ſides and angles of ſpherical triangles; with ſome other things relating thereto.

PROP. XXVI.

FIG. 40. *In any right-angled ſpherical triangle* ABC, *it is,*
As radius:
To ſine of the hypothenuſe BC : :
So ſine of any angle C :
To ſine of its oppoſite ſide AB.

Let the triangle BAC be right-angled at A; draw AG, CG to the center G of the ſphere. From B let fall BI ⊥ to AG, and thro' I draw IH ⊥ to GC; and draw BH, which will be in the plane of the circle BC. Since the plane ABG is ⊥ ACG, therefore BI is ⊥ ACG, and CH ⊥ to the plane BIH. Then BI is the ſine of BA, and BH the ſine of BC, and by Cor. 4. Prop. IV. ∠ BHI = ſpherical angle ACB.

This done, in the right-angled plain triangle BIH, it will be (by Prop. III. B. II.) rad. : BH : : S. H : BI; that is, rad. : S. BC : : S. C : S. AB.

40. Cor. 1. *In right-angled triangles that have equal angles at the baſe; the ſines of the hypothenuſes are as the ſines of the perpendiculars.*

For if ∠ C be given, the S. BC is to the S. AB in a given ratio.

Cor. 2. *In right-angled triangles having equal hypothenuſes; the ſines of the perpendiculars are as the ſines of their oppoſite angles.* For then BC is given.

Cor.

FIG. 41.

Cor. 3. *In a right-angled triangle* ABF, 2 *coſ. hyp.* AF × *rad.* = *coſ. ſum of the ſides* AB, BF + *coſ. their difference.* For in the triangle EDF, rad. : S. EF : : S. E : S. DF, that is, rad. : coſ. BF : : coſ. AB : coſ. AF, whence rad. × coſ. AF = coſ. AB × coſ. BF = (by Sch. 4. I.) coſ. $\overline{AB + BF}$ + coſ. $\overline{AB - BF}$: × into ½ radius.

PROP. XXVII.

In any right-angled triangle ABC, *as* 40.
Radius :
Sine of one ſide AC : :
Tan. angle adjacent C :
Tan. oppoſite ſide AB.

Suppoſing the ſame conſtruction as in the laſt prop. then in the right angled plane triangle HIG it will be (by Prop. III. B. II.) rad. : S. AGC or AC : : IG : IH. And in the right-angled triangle BIG (right-angled at I) rad. : tan. BGI or AB : : IG : IB. And in the right-angled plane triangle BIH, tan. BHI or C : rad. : : IB : IH. Therefore, *ex equo*, tan. C : tan. AB : : IG : IH; that is, rad. : S. AC : : tan. C : tan. AB.

Cor. 1. *In right-angled triangles having equal angles at the baſe; the ſines of the baſes are as the tangents of the perpendiculars.*

For S. AC is to tan. AB in a given ratio.

Cor. 2. *In right-angled triangles having equal baſes; the tangents of the perpendiculars are as the tangents of the oppoſite angles.*

For if AC be given, theſe are in a given ratio.

PROP.

PROP. XXVIII.

FIG. 41. *In any right-angled or quadrantal triangle; the ſine of middle part and radius, are reciprocally proportional to the tangents of the extremes conjunct, and to the coſines of the extremes disjunct.*

This is the Lord *Neper*'s theorem, and is demonſtrated by induction thus.

Produce the ſides AB, AF, BF to quadrants, and deſcribe the arch CDE. Then ſince the arches AC, AD, BE, CE are quadrants, the angles B, C, D will be right; and ED = complement of DC = comp. A; and FD = comp. AF; and EF = comp. FB. Alſo the coſine of the complement is the ſine, and the cotan. of the comp. is the tangent.

Now the middle part muſt be either a leg, the comp. of an angle, or the comp. of the hypothenuſe.

1. Suppoſe a leg as AB be middle part, then BF and comp. ∠ A are extremes conjunct. But by Prop. XXVII. rad. : S.AB : : tan. A : tan. FB. Or thus S.AB : tan. FB : : (rad. : tan. A : :) cotan. A : radius, by Cor. 2. Prop. 1. Book I.

2. Let comp. of an angle A be middle part, then AB and comp. AF are extremes conjunct; and in the triangle DEF, by Prop. XXVII. rad. : S.ED : : tan. E : tan. DF; or S.ED : tan. DF : : (rad. : tan. E : :) cotan. E : radius, by Cor. 2. Prop. 1. I. that is, coſ. A : cotan. AF : : tan. AB : rad.

3. Let the comp. hypothenuſe AF be middle part, then comp. A and comp. F are extremes conjunct. Therefore in the triangle DEF, rad. : S. DF : : tan. F : tan. ED; or S.DF : tan. ED (: : rad. : tan F.) : : cotan. F : radius; that is, coſ. AF : cotan. A : : cotan F : radius.

Therefore

FIG. 41.

Therefore radius × S. middle part = rectangle of the tangents of the extremes conjunct. Again,

1. Let a leg AB be middle part, then comp. F, and comp. AF are extremes disjunct, therefore by Prop. XXVI. rad : S.AF : : S.F : S.AB.

2. Let comp. of an angle A be middle part; then FB and comp. F are extremes disjunct: therefore in the triangle DEF, by Prop. XXVI. rad. : S.EF : : S.F : S.ED; that is, rad. : coſ. BF : : S.F : coſ. A.

3. Let comp. hyp. AF be middle part; then AB, FB are extremes disjunct. And in the triangle DEF, by Prop. XXVI. rad. : S.EF : : S.E : S.DF; that is, rad. : coſ. BF : : coſ. AB : coſ. AF.

Therefore radius × ſine of middle part = rectangle of the coſines of the extremes disjunct.

And in a quarantal triangle, its ſupplemental one, by Prop. X. will be a right-angled triangle. And ſince this prop. holds in that right-angled triangle, it will equally hold in the quadrantal triangle, wherein the parts are the ſupplements of the others; becauſe arches and their ſupplements have the ſame ſines, coſines, tangents, and cotangents.

Cor. 1. *If a perpendicular* FI *be let fall on the baſe of a ſpherical triangle, the coſines of the angles at the baſe are as the ſines of the angles at the vertex.* 42.

For in the right-angled triangles AFI, BFI, it is coſ. A : S.AFI : : (coſ. FI : rad. : :) coſ. FBI : S.BFI.

Cor. 2. *The coſines of the ſides are as the coſines of the ſegments of the baſe.*

For coſ. AI : coſ. AF : : (rad. : coſ. FI : :) coſ. BI : coſ. BF.

Cor. 3. *The ſines of the ſegments of the baſe are as the cotangents of the angles at the baſe.*

FIG. 42. For cotan. A : S. AI :: (rad. : tan. FI ::) cot. IBF : S. IB.

Cor. 4. *The cotangents of the ſides, are as the coſines of the angles at the vertex.*

For cotan. AF : coſ. AFI :: (rad. : tan. FI ::) cotan. BF : coſ. BFI.

Cor. 5. *The tangents of the ſegments of the baſe are as the tangents of the angles at the vertex.*

For tan. AFI : tan. AI :: (rad. : S. FI ::) tan. BFI : tan. BI.

SCHOLIUM.

This prop. will reſolve all right-angled and quadrantal triangles; and alſo all oblique ones, (except where 3 ∠s or 3 ſides are given,) by letting fall a perpendicular dividing it into two right-angled triangles.

PROP. XXIX.

43. *In any ſpherical triangle, the ſines of the ſides are proportional to the ſines of their oppoſite angles.*

On AB, AF let fall the perpendiculars FH, BI; then by Prop. XXVI.

S. AF : S. FH :: *rad.* : S. A
and S. FH : S. FB :: S. B : *rad.* therefore,
ex equo, S. AF : S. FB :: S. B : S. A.

again S. AB : S. BI :: *rad.* : S. A
and S. BI : S. BF :: S. F : *rad.* and,
ex equo, S. AB : S. BF :: S. F : S. A.

44. Cor. 1. *If a great circle be drawn from the vertex of a triangle to cut the baſe; the rectangles of the ſines, of the ſides and of the vertical angles; are directly as the ſines of the ſegments of the baſe.*

For

For $S.\,CDA : S.\,CA :: S.\,CAD : S.\,CD = \frac{S.\,CA \times S.\,CAD}{S.\,CDA}$, and $S.\,CDA$ or $BDA : S.\,AB :: S.\,DAB : S.\,DB = \frac{S.\,AB \times S.\,DAB}{S.\,CDA}$; therefore $S.\,CD : S.\,DB :: S.\,CA \times S.\,CAD : S.\,BA \times S.\,DAB$. FIG. 44.

Cor. 2. *If the vertical angle of a triangle be biſected; the ſines of the ſegment of the baſe are as the ſines of the ſides;* $S.\,CD : S.\,BD :: S.\,AC : S.\,AB$.

PROP. XXX.

In a ſpherical triangle, if a perpendicular be let fall on the baſe, it is 42.

As the ſine of the ſum of the ſides, AF, BF :
Sine of their difference : :
Cot. of ½ *ſum of the angles at the vertex,* AFI, BFI :
Tan. of half their difference.

For by Cor. 4. Pr. XXVIII. cotan. AF : cotan. BF : : coſ. AFI : coſ. BFI; or by Cor 4. Pr. I. B. I. tan. AF : tan. BF : : coſ. BFI : coſ. AFI. And by compoſition and diviſion, tan. AF + tan. BF : tan. AF — tan. BF : : coſ. BFI + coſ. AFI : coſ. BFI — coſ. AFI; that is, by Cor 4. and 2 Pr. VII. B. I. $S.\,\overline{AF + BF} : S.\,\overline{AF - BF} ::$ cotan. $\frac{AFI + BFI}{2}$: tan. $\frac{AFI - BFI}{2}$.

Cor. 1. *If the perpendicular falls within,* $\frac{AFI + BFI}{2}$ = *half the angle at the vertex* F : *if it falls without* $\frac{AFI - BFI}{2}$ = *half the vertical angle.*

FIG. 42. Cor. 2. *As ſine of the ſum of the ſides* :
Sine of difference of the ſides : :
Cotan. half the vertical angle :
Tan. half difference, or half ſum, of the vertical angles, according as the perpendicular falls within or without.

52. Cor. 3. *In a right-angled ſpherical triangle* ABF,
S. ſum of the hyp. and one ſide, AF, BF :
S. of their difference : :
Radius ſquare :
Tang. ſquare of half the contained angle F.

For in this caſe the ∠ F is both the ſum of the angles, and the difference of the angles. And cot. $\frac{1}{2}$ F : tan. $\frac{1}{2}$ F : : $\frac{\text{Rad.}^{2}}{\text{Tan.}\ \frac{1}{2}\ \text{F}}$: tan. $\frac{1}{2}$ F : : rad.^{2} : tan.^{2} of $\frac{1}{2}$ F.

PROP. XXXI.

42. *Let a perpendicular fall on the baſe of a triangle; then*
Tan. half the ſum of the ſides, AF, BF :
Tan. half their difference : :
Tan. half the ſum of the angles at the baſe, B, A :
Tan. half their difference.

For by Prop. XXIX. S. AF : S. BF : : S. B : S. A. And by compoſition and diviſion, S. AF + S. BF : S. AF — S. BF : : S. B + S. A : S. B — S. A; that is, by Cor 1. Pr. VII. B. I. tan. $\frac{\text{AF} + \text{BF}}{2}$: tan. $\frac{\text{AF} - \text{BF}}{2}$: : tan. $\frac{\text{B} + \text{A}}{2}$: tan. $\frac{\text{B} - \text{A}}{2}$.

PROP. XXXII.

FIG. 42.

Let a perpendicular fall on the baſe of a ſpherical triangle; then

Cotan. half the ſum of the angles at the baſe, A, B :
Tan. half their difference : :
Tan. $\frac{1}{2}$ *ſum of the angles at the vertex,* AFI, BFI :
Tan. half their difference.

For by Cor. 1. Prop. XXVIII. coſ. A : coſ. B :: S. AFI : S. BFI. And coſ. A + coſ. B : coſ. A — coſ. B :: S. AFI + S. BFI : S. AFI — S. BFI; that is, by Cor. 1. and 2. Prop. 7. B. I. cotan. $\frac{B+A}{2}$: tan. $\frac{B-A}{2}$: : tan. $\frac{AFI+BFI}{2}$: tan. $\frac{AFI-BFI}{2}$.

Note, If the perpendicular falls within, $\frac{AFI+BFI}{2}$ = half the vertical angle F; but $\frac{AFI-BFI}{2}$, if it fall without.

Cor. *Cotan. half the ſum of the angles at the baſe*;
Tan. half their difference : :
Tan. half the vertical angle :
Tan. half the difference, or half the ſum of the vertical angles, as the perpendicular falls within or without.

This is plain when the perpendicular falls within; and when it falls without, + BFI becomes — BFI, and the contrary.

PROP. XXXIII.

FIG. 42. *Suppoſe a perpendicular to fall on the baſe of a ſpherical triangle, then as*

Tan. half ſum of the ſegments of the baſe AI, BI :
Tan. half the ſum of the ſides, AF, BF : :
Tan. half difference of the ſides AF, BF :
Tan. $\frac{1}{2}$ *diff. of the ſegments of the baſe,* AI, BI.

For by Cor. 2. Prop. XXVIII. coſ. BF : coſ. AF : : coſ. BI : coſ. AI. And coſ. BF + coſ. AF : coſ. BF — coſ. AF : : coſ. BI + coſ. AI : coſ. BI — coſ. AI ; that is, by Cor. 2. Prop. VII. B. I. cotan. : $\frac{AF+BF}{2}$: tan. $\frac{AF-BF}{2}$: : cotan. $\frac{AI+BI}{2}$: tan. $\frac{AI-BI}{2}$.

Or by Cor. 4. Prop. I. B. I. tan. $\frac{AI+BI}{2}$: tan. $\frac{AF+BF}{2}$: : (cotan. $\frac{AF+BF}{2}$: cotan. $\frac{AI+BI}{2}$: :) tan. $\frac{AF-BF}{2}$: tan. $\frac{AI-BI}{2}$.

Here as the perpendicular falls within or without, AI + BI or AI — BI will be the baſe.

Cor. 1. *As tan. half the baſe :*
Tan. half ſum of the ſides : :
Tan. half difference of the ſides :
Tan. half the alternate baſe.

Where the alternate baſe is half the difference, or half the ſum of the ſegments of the baſe, according as the perpendicular falls within or without.

Cor. 2. *In a right-angled triangle ; the rectangle of the tan. of half the hypothenuſe + half one leg, and the*

the tan. half the hypothenuſe — half that leg = *tan. ſquare of half the other leg.* For then BI = 0.

PROP. XXXIV.

Let fall a perpendicular upon the baſe of a triangle, then 42.

Sine of the ſum of the angles at the baſe B, A :
Sine of their difference : :
Tan. half ſum of the ſegments of the baſe, AI, BI :
Tan. half their difference.

For by Cor. 3. Prop. XXVIII. cotan. A : cotan. B : : S. AI : S. BI, or tan. B : tan. A : : S. AI : S. BI. And tan. B + tan. A : tan. B — tan. A : : S. AI + S. BI : S. AI — S. BI; that is, (by Cor. 4 and 1. Pr. VII. B. I.) S. $\overline{B + A}$: S. B — A : : tan. $\frac{AI + BI}{2}$: tan. $\frac{AI - BI}{2}$.

According as the perpendicular falls within or without, $\frac{AI + BI}{2}$, or $\frac{AI - BI}{2}$ will be equal to half the baſe. Whence

Cor. 1. *Sine of the ſum of the angle at the baſe* :
Sine of their difference : :
Tan. half the baſe :
Tan. half the alternate baſe. (See Cor. 1. Pr. 33.)

For when the perpendicular falls without, + BI becomes — BI, and the contrary.

PROP. XXXV.

Let fall a perpendicular on the baſe, then as
Sine of the ſum of the ſegments of the baſe, AI, BI :
Sine of their difference : :
Sine of the ſum of the angles at the vertex, AFI, BFI :
Sine of their difference.

FIG. 42. For by Cor. 5. Prop XXVIII. tan. AI : tan. BI :: tan. AFI : tan. BFI. And tan. AI + tan. BI : tan. AI — tan. BI :: tan. AFI + tan. BFI : tan. AFI — tan. BFI; that is, by Cor. 4. Pr. VII. B. I. S. $\overline{\text{AI}+\text{BI}}$: S. $\overline{\text{AI}-\text{BI}}$ :: S. $\overline{\text{AFI}+\text{BFI}}$: S. $\overline{\text{AFI}-\text{BFI}}$. Here, as the perpendicular falls within or without; AI + BI, or AI — BI is the baſe; and AFI + BFI, or AFI — BFI, the vertical angle. Whence

Cor. *As ſine of the baſe :*
Sine of the vertical angle ::
So ſine of the diff. ſegments of the baſe :
Sine of the diff. angles at the vertex, when the perpendicular falls within ::
Or ſine of the ſum of the ſegments of the baſe :
Sine of the ſum of the vertical angles, when the perpendicular falls without.

PROP. XXXVI.

42. *As coſine of half the ſum of two ſides,* AF, BF :
Coſine of half their difference ::
Cotan. half the included angle AFB :
Tan. half ſum of the oppoſite angles A, ABF.

Put AF + BF = P, AF — BF = p, ABF + A = Q, ABF — A = q, AI + BI = R, AI — BI = r, AFI + BFI = V, AFI — BFI = v.

Then ſince the tangents are reciprocally as the cotangents, we ſhall have, by

Prop. 30. S. P : S. p :: *cotan.* $\frac{1}{2}$ V : T. $\frac{1}{2}v$.
31. *Cot.* $\frac{1}{2}$ P : *cot.* $\frac{1}{2}p$ :: T. $\frac{1}{2}q$: T. $\frac{1}{2}$ Q.
32. T. $\frac{1}{2}q$: T. $\frac{1}{2}v$ :: *cot.* $\frac{1}{2}$ V : T. $\frac{1}{2}$ Q.

And compounding theſe ratios, and expunging equal quantities,

$S.\,P \times \text{cot.}\,\frac{1}{2}P : S.\,p \times \text{cot.}\,\frac{1}{2}p :: \text{cot. ſquare}\,\frac{1}{2}V : \text{tan. ſquare}\,\frac{1}{2}Q$. But by ſch. Pr. 2. B. I. $\frac{2\,\overline{\text{coſ.}}^{2}\,\frac{1}{2}P}{\text{cot.}\,\frac{1}{2}P} = S.\,P$, or $\overline{\text{coſ.}}^{2}\,\frac{1}{2}P = \frac{S.P \times \text{cot.}\,\frac{1}{2}P}{2}$, and $\overline{\text{coſ.}}^{2}\,\frac{1}{2}p = \frac{S.\,p \times \text{cot.}\,\frac{1}{2}p}{2}$, therefore coſ. ſquare $\frac{1}{2}P$: coſ. ſquare $\frac{1}{2}p$ :: cot. ſquare $\frac{1}{2}V$: T. ſquare $\frac{1}{2}Q$; and coſ. $\frac{1}{2}P$: coſ. $\frac{1}{2}p$ :: cot. $\frac{1}{2}V$: tan. $\frac{1}{2}Q$.

FIG. 42.

This is plain when the perpendicular falls within; and when it falls without, inſtead of the third proportion, ſubſtitute this (by Cor. Pr. XXXII.) T. $\frac{1}{2}q$: T. $\frac{1}{2}V$ :: cot. $\frac{1}{2}v$: T. $\frac{1}{2}Q$, and you will have at laſt this; coſ. $\frac{1}{2}P$: coſ. $\frac{1}{2}p$ :: cot. $\frac{1}{2}v$: tan. $\frac{1}{2}Q$.

Cor. 1. *As ſine of half the ſum of the ſides* :
Sine of half their difference ::
Cotan. half the included angle :
Tan. half difference of the oppoſite angles.

For by the former proportions, a little varied,

S. P : S. p :: cot. $\frac{1}{2}V$: T. $\frac{1}{2}v$,
T. $\frac{1}{2}P$: T. $\frac{1}{2}p$ :: T. $\frac{1}{2}Q$: T. $\frac{1}{2}q$,
T. $\frac{1}{2}Q$: T. $\frac{1}{2}v$ :: cot. $\frac{1}{2}V$: T. $\frac{1}{2}q$.

Then proceeding as before, it will be found (by Sch. 2. Prop. 2. B. I.) that

S. $\frac{1}{2}P$: S $\frac{1}{2}p$ :: cot. $\frac{1}{2}V$: T. $\frac{1}{2}q$, when the perpendicular falls within; and when it falls without, inſtead of the third proportion put this (by Cor. Prop. XXXII.) cot. $\frac{1}{2}V$: cot. $\frac{1}{2}v$ :: cot. $\frac{1}{2}Q$: T. $\frac{1}{2}q$. and you will get at laſt S. $\frac{1}{2}P$: S. $\frac{1}{2}p$ :: cot. $\frac{1}{2}v$: tan. $\frac{1}{2}q$.

Cor. 2. *Coſ. half ſum of two ſides* :
Coſ. half their difference ::

Tan.

FIG.

Tan. half difference of their oppoſite angles :
Tan. half difference, or half ſum of the vertical angles ; according as the perpendicular, on the third ſide, falls within or without.

This follows from the preſent prop. and cor. Prop. XXXII.

PROP. XXXVII.

42. *As coſ. half ſum of two angles,* A, ABF :
Coſ. half their difference : :
Tan. half the included ſide AB :
Tan. half ſum of the oppoſite ſides, AF, BF.

Suppoſing as in the laſt prop. then by

Prop. 31. $\text{Cot.}\,\frac{1}{2}Q : \text{cot.}\,\frac{1}{2}q :: T.\,\frac{1}{2}p : T.\,\frac{1}{2}P$,
33. $T.\,\frac{1}{2}p : T.\,\frac{1}{2}r :: T.\,\frac{1}{2}R : T.\,\frac{1}{2}P$,
34. $S.\,Q : S.\,q :: T.\,\frac{1}{2}R : T.\,\frac{1}{2}r$.

But (by Sch. Prop. II. B. I.) $\frac{1}{2}\overline{S.Q} \times \text{cot.}\,\frac{1}{2}Q = \overline{\text{coſ.}^2}\,\frac{1}{2}Q$, and $\frac{1}{2}S.\,q \times \text{cot.}\,\frac{1}{2}q = \overline{\text{coſ.}^2}\,\frac{1}{2}q$, therefore by compounding,

$\overline{\text{Coſ.}^2}\,\frac{1}{2}Q : \overline{\text{coſ.}^2}\,\frac{1}{2}q :: T^2.\,\frac{1}{2}R : T^2.\,\frac{1}{2}P$.
Or $\text{coſ.}\,\frac{1}{2}Q : \text{coſ.}\,\frac{1}{2}q :: T.\,\frac{1}{2}R : T.\,\frac{1}{2}P$.

But when the perpendicular falls without, inſtead of the third proportion, put this (by Cor. Prop. XXXIV.) $S.\,Q : S.\,q :: T.\,\frac{1}{2}r : T.\,\frac{1}{2}R$. Whence at laſt you will have this,

$\text{Coſ.}\,\frac{1}{2}Q : \text{coſ.}\,\frac{1}{2}q :: T.\,\frac{1}{2}r : T.\,\frac{1}{2}P$.

Cor. 1. *Sine half ſum of two angles,* A, ABF :
Sine of half their difference : :
Tan. half the included ſide, AB :
Tan. half difference of the oppoſite ſides, AF, BF.

For by varying the terms of the foregoing proportions, it will be,

T.

$$\begin{aligned} &\text{T}.\tfrac{1}{2}Q : \text{T}.\tfrac{1}{2}q :: \text{T}.\tfrac{1}{2}P : \text{T}.\tfrac{1}{2}p, \\ &\text{T}.\tfrac{1}{2}P : \text{T}.\tfrac{1}{2}r :: \text{T}.\tfrac{1}{2}R : \text{T}.\tfrac{1}{2}p, \\ &\text{S}.Q : \text{S}.q :: \text{T}.\tfrac{1}{2}R : \text{T}.\tfrac{1}{2}r. \end{aligned}$$ FIG. 42.

But (by Sch. Prop. II. Book I.) $\tfrac{1}{2}\text{S}.Q \times \text{T}.\tfrac{1}{2}Q = \text{S}.^2\,\tfrac{1}{2}Q$, and $\tfrac{1}{2}\text{S}.q \times \text{T}.\tfrac{1}{2}q = \text{S}^2.\,\tfrac{1}{2}q$, therefore by compounding, $\text{S}.\tfrac{1}{2}Q : \text{S}.\tfrac{1}{2}q :: \text{T}.\tfrac{1}{2}R : \text{T}.\tfrac{1}{2}p$.

If the perpendicular fall without, inſtead of the third proportion put this (by Cor. Prop. XXXIV.) $\text{S}.Q : \text{S}.q :: \text{T}.\tfrac{1}{2}r : \text{T}.\tfrac{1}{2}R$, which will give this proportion, $\text{S}.\tfrac{1}{2}Q : \text{S}.\tfrac{1}{2}q :: \text{T}.\tfrac{1}{2}r : \text{T}.\tfrac{1}{2}p$.

Cor. 2. *Coſ. ½ ſum of the angles at the baſe*, A, ABF :
Coſ. half their difference : :
Tan. half diff. oppoſite ſides, AF, BF :
Tan. half diff. or half ſum, of the ſegments of the baſe; according as the perpendicular falls within or without.

This follows from the preſent prop. and Cor. 1. Prop. XXXIII.

P R O P. XXXVIII.

In any ſpherical triangle AFB; *the rectangle of the ſines of any two ſides × coſ. included angle + the rectangle of their coſines × radius = coſ. of the third ſide × radius ſquare; that is,* 42.

S.AB × S.FB × *coſ.* B + *coſ.* AB × *coſ.* FB × *rad.* = *coſ.* AF × *rad*².

Let $a = \text{S. AB}$, $m = \text{coſ. AB}$, $s = \text{S.B}$.
$b = \text{S. FB}$, $n = \text{coſ. FB}$, $c = \text{coſ. B}$, $\tau = \text{cot. B}$.
$r = \text{rad.}$ $p = \sqrt{rrnn + bbcc}$.

By Prop. 1. I. $n : b :: r : \text{tan. FB} :: \text{cot. FB} : r :: $ (by Prop. XXVIII.) $c : \text{tan. BI} = \dfrac{cb}{n}$. And by Sch. Prop. 1. I. ſine of $\text{BI} = \dfrac{rcb}{p}$, and coſ. $\text{BI} = \dfrac{rrn}{p}$;

FIG. 42. $\frac{rrn}{p}$; and by Cor. Pr. 6. I. coſ. AI $= \frac{abc + mnr}{p}$, but by Cor. 2. Prop. XXVIII. $\frac{rrn}{p} : \frac{abc + mnr}{p} :: n : \text{coſ. AF} = \frac{abc + mnr}{rr}$.

Cor. 1. *The coſine of any angle of a ſpherical triangle is equal to the coſine of the oppoſite ſide × radius ſquare — the rectangle of the coſines of the including ſides × radius, and the whole divided by the rectangle of the ſines of the including ſides.*

For $\text{coſ. B} = \frac{\text{coſ. AF} \times rr - \text{coſ. AB} \times \text{coſ. FB} \times r}{\text{S. AB} \times \text{S. FB}}$.

Cor. 2. *Cotan.* A $= \frac{\text{S. AB} \times \textit{coſ.}\ \text{FB} \times \textit{rad.} - \text{S. FB} \times \textit{coſ.}\ \text{AB} \times \textit{coſ.}\ \text{B}}{\text{S. B} \times \text{S. FB}}$.

For by Prop. 6. I. S. AI $= \frac{anr - bcm}{p}$, and by Cor. 3. Prop. XXVIII. I. $\frac{rcb}{p} : \frac{anr - bcm}{p} :: \tau : \text{cot. A} = \frac{anr - bcm}{rcb}\tau = \frac{anr - mbc}{bs}$, becauſe $\frac{\tau}{c} = \frac{r}{s}$ by Schol. Prop. 1. I.

Cor. 3. $\frac{\text{S. } \overline{\text{AB} - \text{FB}} \times rr + \text{S. FB} \times \textit{coſ.}\ \text{AB} \times \textit{verſ.}\ \text{B}}{\text{S. B} \times \text{S. FB}}$ = *cotan.* A.

For ſince coſ. B $= r -$ verſ. B, therefore by Cor. 2. cot. A $= \frac{anr - mbr + mb \times \text{verſ. B}}{bs}$; but by Pr. 6. I. $an - bm = \text{S. } \overline{\text{AB} - \text{FB}} \times r$, whence the Cor. is evident.

SCHOLIUM.

If the angle ABF be greater than a right angle, then its coſine c will be negative. FIG. 42.

PROP. XXXIX.

The rectangles of the ſines of two angles × coſine of the included ſide divided by radius ſquare — the rectangle of their coſines divided by radius, is equal to the coſine of the third angle. 45.

that is, $\frac{\text{S. A} \times \text{S.B} \times \text{coſ. AB}}{rr} - \frac{\text{coſ. A} \times \text{coſ. B}}{r} =$ coſ. F.

Let fall the perpendicular BR, and put
S. A$=a$, coſ. A$=m$, cot. A$=\tau$, coſ. AB$=c$, rad.$=r$.
S.B$=b$, coſ.B$=n$, $q=\sqrt{\tau\tau+cc}$;

then by Prop. XXVIII. cot. ABR $= \frac{rc}{\tau}$, and (by Sch. 1. I.) its ſine $= \frac{r\tau}{q}$, and its coſine $= \frac{rc}{q}$. And by Prop. 6. I. S. RBF $= \frac{bc - n\tau}{q}$; and by Cor. 1. XXVIII. $\frac{r\tau}{q} : \frac{bc-n\tau}{q} :: m : \text{coſ. F} = \frac{bc-n\tau}{r\tau}m = \frac{bcm}{r\tau} - \frac{mn}{r} = \frac{abc}{rr} - \frac{mn}{r}$; becauſe $\frac{m}{\tau} = \frac{a}{r}$, by Sch. 1. I.

Cor. 1. *The coſine of any ſide is equal to the rectangle of the coſines of the including angles × radius + coſ. oppoſite angle × radius ſquare, and the whole divided by the rectangles of the ſines of the including angles.*

For c or the *coſ.* AB $= \frac{mnr + rr \times \text{coſ. F}}{ab}$.

Cor.

FIG. 45. Cor. 2. *Cot.* FB $= \frac{\text{S. B} \times \mathit{cot.}\,\text{A}}{\text{S. AB}} + \frac{\mathit{coſ.}\text{B} \times \mathit{cot.}\,\text{AB}}{r}$.

For let $s =$ S. AB, $t =$ cotan. AB, then by Cor. 1. Prop. 6. I. coſ. RBF $= \frac{b\tau + nc}{q}$, and by Cor. 4. Prop. XXVIII. $\frac{rc}{q} : \frac{b\tau + nc}{q} :: t :$ cot. BF $= \frac{b\tau + nc}{rc} t = \frac{b\tau}{s} + \frac{nt}{r}$, becauſe $ts = cr$. Or cot. BF $= \frac{rbm + nac}{sa}$, becauſe $\tau = \frac{mr}{a}$, and $t = \frac{cr}{s}$, by Schol. Prop. 1. I.

Cor. 3. *Cotan.* FB $=$

$$\frac{r^2 \times \text{S.}\,\overline{\text{A} + \text{B}} - \text{S. A} \times \mathit{coſ.}\text{B} \times \mathit{verſ.}\,\text{AB}}{\text{S. A} \times \text{S. AB}}.$$

For let $v =$ verſ. AB $= r - c$, and $c = r - v$, then cotan. FB $= \frac{rbm + nac}{sa} = \frac{rbm + ran - anv}{sa}$; but (by Prop. 5. I.) $mb + an = r \times \text{S.}\,\overline{\text{A} + \text{B}}$, whence cot. BF $= \frac{rr \times \text{S.}\,\overline{\text{A} + \text{B}} - anv}{as}$.

PROP. XL.

47. *Let a, s, b, p, f, d be the ſines, and m, c, n, i, g, e the coſines* } *of the angles and ſides as mark'd in the figure. And*
$hh = bb - aa = mm - nn$.
and $kk = 1 - bbpp = 1 - aadd$.
Radius $= 1$. Then

Sine of $\angle$ F $= \frac{hh}{bim - aen} = \frac{bim + aen}{kk}$.

For

FIG. 47.

For 1. $bif - ng = m$. By Prop. XXXIX.
2. $aef - mg = n$. By Prop. XXXIX.
3. $\frac{n}{b} = \frac{pe - dig}{df}$. By Cor. 2. Pr. XXXVIII.

For by Pr. 1. I. $\frac{b}{n} =$ cotan. B. From the two first equations $g = \frac{bif - m}{n} = \frac{aef - n}{m}$, whence $bifm - m^2 = aefn - nn$, and $f = \frac{m^2 - n^2}{bim - aen}$.

Again, by the third equation $dnf = bpe - bdig = ade - bdig$, because (by Prop. XXIX.) $bp = ad$; and $nf = ae - big$; whence $g = \frac{ae - nf}{bi} = \frac{bif - m}{n}$, and $aen - fn^2 = b^2i^2f - bim$, whence $f = \frac{aen + bim}{n^2 + bbii} = \frac{aen + bim}{1 - bbpp}$; because $ii = 1 - pp$.

Cor. 1. *Cof.* F $= \frac{abei - mn}{kk} = \frac{aem - bin}{bim - aen}$.

For by the 1st and 3d equations $f = \frac{ng + m}{bi} = \frac{ae - big}{n}$, and $nng + mn = aebi - bbiig$; whence $g = \frac{aebi - mn}{nn + bbii} = \frac{aebi - mn}{1 - bbpp}$.

Again, by the 1st and 2d equations $f = \frac{m + ng}{bi} = \frac{n + mg}{ae}$, and $aem + aeng = bin + bimg$, whence $g = \frac{aem - bin}{bim - aen}$.

Cor.

FIG. 47. Cor. 2. *Cotan.* F $= \frac{ame - bin}{bb} = \frac{e \times S.2A - i \times S.2B}{cof.2A - cof.2B}$.

For multiplying the 1ſt equation by n, and the 2d by m, we have $mn = binf - nng = aefm - mmg$, and $\overline{m^2 - n^2}.g = \overline{aem - bin}.f$, and $\frac{g}{f} = \frac{aem - bin}{m^2 - n^2} = \frac{2aem - 2bin}{2b^2 - 2a^2}$; but (by Prop. 2. I.) $2am =$ S. 2A, and $2bn =$ S. 2B, alſo $2aa =$ verſ. 2A, and $2b^2 =$ verſ. 2B, and $2bb - 2aa =$ verſ. 2B — verſ. 2A $=$ coſ. 2A — coſ. 2B, and $b^2 - a^2 = m^2 - n^2$, for $bb + n^2 = 1 = m^2 + a^2$.

SCHOLIUM.

Any of the quantities a, b, p, d, or m, n, i, e, may be expunged by help of the equation $bp = ad$, or the tangents or cotangents ſubſtituted for them.

PROP. XLI.

47. *Suppoſe as in the laſt Prop. and let*
$ll = dd - pp = ii - ee$,
$tt = 1 - bbpp = 1 - aadd$,
Radius = 1. *Then*
Sine of AB $= \frac{ll}{mid - pen} = \frac{mid + pen}{tt}$.

For 1. $pns + ci = e$, by Prop. XXXVIII.
2. $mds + ce = i$, by Prop. XXXVIII.
3. $\frac{i}{p} = \frac{bm + anc}{as}$, by Cor. 2. Pr. XXXIX.

By the two firſt equations, $c = \frac{e - pns}{i} = \frac{i - mds}{e}$, and $ee - pens = ii - imds$, and $imds - pens = ii - ee$, whence $s = \frac{ii - ee}{dim - pen}$.

Alſo

Alſo by the third equation $asi = pbm + pnac =$ (ſince $ad = bp$) $adm + pnac$, that is, $si = dm + pnc$, whence $c = \frac{si - dm}{pn} = \frac{e - pns}{i}$, and $si^2 - dim = pen - p^2n^2s$, whence $s = \frac{pen + dim}{i^2 + p^2n^2} = \frac{pen + dim}{1 - ppbb}$. FIG. 47.

Cor. 1. *Coſ.* AB $= \frac{ei - mndp}{tt} = \frac{pin - dem}{pen - dim}$.

For by 1ſt and 3d equations, $s = \frac{e - ci}{pn} = \frac{dm + cpn}{i}$, and $ei - ci^2 - pndm + p^2n^2c$, whence $c = \frac{ei - pdmn}{i^2 + p^2n^2} = \frac{ei - pdmn}{1 - bbpp}$.

Alſo by the 1ſt and 2d equations $s = \frac{e - ci}{pn} = \frac{i - ce}{md}$, and $mde - mdci = pni - pnec$, whence $c = \frac{dem - pin}{dim - pen}$.

Cor. 2. *Cotan.* AB $= \frac{med - pin}{ll} = \frac{m \times S.2AF - n \times S.2BF}{cof.\ 2BF - cof.\ 2AF}$. 47.

For multiplying the 1ſt equation by i, and the 2d by e, we have $pnsi + cii = ei = mdse + cee$, and $cii - cee = mdes - pins$, whence $\frac{c}{s} = \frac{med - pin}{ii - ee} = \frac{2med - 2pin}{2dd - 2pp}$; but by Prop. 2. I. $2ed = S.2AF$, and $2pi = S.2BF$, and $2dd =$ verſ. 2AF, and $2p^2 =$ verſ. 2BF, and $2dd - 2pp =$ verſ. 2AF — verſ. 2BF = coſ. 2BF — coſ. 2AF.

SCHOLIUM.

FIG. 47. Any of the quantities a, d, b, p; or m, e, i, n, may be extirminated by help of the equation $ad = bp$, or the tangents or cotangents ſubſtituted inſtead of them.

PROP. XLII.

46. *In any ſpherical triangle,* ABF,
As rectangle of the ſines of the ſides, AB, FB :
Radius ſquare : :
Verſ. baſe — verſ. difference of the ſides :
Verſ. included angle B : :
Coſ. difference of the ſides — coſ. baſe :
Verſ. included angle B : :
As ſine of one ſide :
To coſecant of the other.

Let $a =$ S.AB, $m =$ coſ. AB, $c =$ coſ. B. $v =$ verſ B.
$b =$ S.FB, $n =$ coſ. FB, $d =$ coſ. AF, rad. $= r$.

Then (by Cor. 2. Prop. XXXVIII.) $\frac{dr^2 - mnr}{ab} = c = r - v$, and $v = \frac{abr - drr + mnr}{ab}$; but (by Cor. 1. Prop. 6. I.) $ab + mn = r \times \text{coſ.}\ \overline{\text{AB} - \text{FB}}$. Therefore $v = \frac{rr \times \text{coſ.}\ \overline{\text{AB} - \text{FB}} - drr}{ab} = \frac{\text{verſ. AF} - \text{verſ.}\ \overline{\text{AB} - \text{FB}}}{ab} \times rr$. Laſtly, S.FB $= \frac{rr}{\text{coſec. FB}}$, and S.AB × S.FB : rr : : S.AB × $\frac{rr}{\text{coſec. FB}}$: rr : : S.AB : coſec. FB.

Cor. 1. *Rectangle of the ſines of the ſides* :
S. half ſum × S. half diff. of the baſe and diff. ſides : :
And verſ. ſum of the ſides — verſ. their difference :
Verſ. baſe — verſ. difference of the ſides : :
And

And coſ. difference — coſ. ſum of the ſides : FIG. 46.
Coſ. diff. ſides — coſ. baſe : :
So twice radius :
Verſ. included angle, B.

For let the greater ſide AB $= a$, leſſer BF $= e$, baſe AF $= b$, $a+e=z$, $a-e=d$, $v=$ verſ. B.

Then by this Prop. $S.a\times S.e : \overline{\text{verſ.}\ b-\text{verſ.}\ d}\times r :: r : v$. But (by Cor. 3. Pr. 3. I.) $\frac{1}{2}r\times\overline{\text{verſ.}\,b-\text{verſ.}\,d} = S.\frac{b+d}{2}\times S.\frac{b-d}{2}$, and (by Cor. 2. Prop. 3. I.) $S.a\times S.e=\frac{1}{2}r\times\overline{\text{coſ.}\ d-\text{coſ.}\ z}$. Whence you have theſe two proportions,

$$S.a\times S.e : S.\frac{b+d}{2}\times S.\frac{b-d}{2} :: 2r : v.$$

And $\text{verſ.}\ z-\text{verſ.}\ d : \text{verſ.}\ b-\text{verſ.}\ d :: 2r : v$.
And the diff. of the verſed ſines is = diff. coſines.

Cor. 2. *Rectangle of the ſines of the ſides* :
Radius ſquare : :
Verſ. ſum of the ſides — verſ. baſe :
Verſ. ſup of the included angle B.

For (by Cor. 1.)
$\text{verſ.}\ z-\text{verſ.}\ d : \text{verſ.}\ b-\text{verſ.}\ d :: 2r : v$.
And by diviſion,
$\text{verſ.}\,z-\text{verſ.}\,d : \text{verſ.}\,z-\text{verſ.}\,b :: 2r : 2r-v$,
Or $\text{verſ.}\,z-\text{verſ.}\,d : 2r :: \text{verſ.}\,z-\text{verſ.}\,b : 2r-v$.
And $\frac{\text{verſ.}\,z-\text{verſ.}\,d}{2}\times r : rr :: \text{verſ.}\,z-\text{verſ.}\,b : 2r-v$.
that is (by Cor. 2. Prop. 3. I.) $S.a\times S.e : rr :: \text{verſ.}\ z-\text{verſ.}\,b : 2r-v$, the verſ. ſup. B. and the diff. verſed ſines = diff. coſines. Whence,

Cor. 3. *Verſ. ſum of the ſides — verſ. diff. ſides* :
Verſ. ſum ſides — verſ. baſe : :
Twice radius :
Verſ. ſupplement of the included angle B.

FIG. 46. Cor. 4. *As rectangle of the ſines of the ſides :*
Radius ſquare : :
Or as ſum of the ſines × diff. ſines, of $\frac{1}{2}$ *ſum and* $\frac{1}{2}$ *diff. ſides :*
Radius ſquare : :
So $\text{S.}\overline{\tfrac{1}{2}\,\text{baſe} + \tfrac{1}{2}\,\text{diff. ſides}} \times \text{S.}\overline{\tfrac{1}{2}\,\text{baſe} - \tfrac{1}{2}\,\text{diff. ſides}}$:
Sine ſquare of half the included angle : :
So $\text{S.}\tfrac{1}{2}\,\text{ſum of the 3 ſides} \times \text{S.}\overline{\tfrac{1}{2}\,\text{ſum 3 ſides} - \text{baſe}}$:
Coſine ſquare of half the included angle B.

Let V = ſup. B, $s = \text{S.}\frac{1}{2}\text{B}$, $c = \text{coſ.}\,\frac{1}{2}\text{B}$. The reſt as before, then (by Prop. 2. I.) $v = \frac{2ss}{r}$, and $\text{V} = \frac{2cc}{r}$, and (by Prop. 4. I.) $\frac{\text{verſ.}\,b - \text{verſ.}\,d}{2} \times r = \text{S.}\frac{b+d}{2} \times \text{S.}\frac{b-d}{2}$, and $\frac{\text{veſ.}\,z - \text{verſ.}\,b}{2} \times r = \text{S.}\frac{z+b}{2} \times \text{S.}\frac{z-b}{2}$; and (by Cor. 2. Prop. 4. I.) $\text{S.}a \times \text{S.}e = \overline{\text{S.}\frac{a+e}{2} + \text{S.}\frac{a-e}{2}} \times \overline{\text{S.}\frac{a+e}{2} - \text{S.}\frac{a-e}{2}} = \overline{\text{S.}\tfrac{1}{2}z + \text{S.}\tfrac{1}{2}d} \times \overline{\text{S.}\tfrac{1}{2}z - \text{S.}\tfrac{1}{2}d}$. Now by this Prop. $\text{S.}a \times \text{S.}e : rr :: \text{verſ.}\,b - \text{verſ.}\,d : v$ or $\frac{2ss}{r} :: \frac{\text{verſ.}\,b - \text{verſ.}\,d}{2} \times r$ or $\text{S.}\frac{b+d}{2} \times \text{S.}\frac{b-d}{2} : ss$.

Again, $a + e + b = z + b$, and $\frac{a+e+b}{2} - b = \frac{z-b}{2}$, and by Cor. 2. $\text{S.}a \times \text{S.}e : rr :: \text{verſ.}\,z - \text{verſ.}\,b : \text{V}$ or $\frac{2cc}{2} :: \frac{\text{verſ.}\,z - \text{verſ.}\,b}{2} \times r$ or $\text{S.}\frac{z+b}{2} \times \text{S.}\frac{z-b}{2} : cc$. Hence alſo, ſince $d = a - e$,

Cor. 5. *As rectangle of the ſines of the ſides :*
Radius ſquare : :
So S.$\frac{1}{2}$ *ſum of 3 ſides — one ſide, × S.*$\frac{1}{2}$ *ſum — other ſide :*
Sine ſquare of half the included angle.

Cor.

Cor. 6. *From half the ſum of the three ſides, ſubtract the baſe, and two ſides ſeparately; then* FIG. 46.

As S. half ſum of the three ſides × S. firſt difference :
Rectangle of the ſines of the 2^d and 3^d differences : :
So radius ſquare :
Tan. ſquare of half the included angle.

For by Cor. 4. $S.\frac{z+b}{2} \times S.\frac{z-b}{2} : S.\frac{b+d}{2} \times S.\frac{b-d}{2} : : (cc : ss : :)\, rr$: tan. ſquare of half the angle, by Prop. 1. I.

SCHOLIUM.

Any of theſe compound proportions may eaſily be reſolved into two ſimple ones. For example, in Cor. 4. ſay, $S.a : r : : S.\frac{b+d}{2}$: to the ſine of a fourth P. And $S.e : r : S.\frac{b-d}{2}$, Q; then $P \times Q = ss$ required. And the ſame may be done for the next prop. and its corollaries.

PROP. XLIII.

In any ſpherical triangle, 42.
As rectangle of the ſines of two angles, A, B *:*
Radius ſquare : :
Verſ. ſum of the including angles — verſ. ſup. 3^d angle :
Verſ. included ſide : :
And ſo verſ. third angle — verſ. ſup. ſum of the including angles :
Verſ. included ſide : :
And ſo coſ. diff. of the including angles + coſ. 3^d angle :
Verſ. ſup. included ſide : :
And ſo verſ. ſup. 3^d angle — verſ. diff. including angles :
Verſ. ſup. included ſide : :
And ſo verſ. ſup. diff. including angles — verſ. 3^d angle :
Verſ. ſup. included ſide, AB.

FIG. 42. For let B be the greater angle, A the leſſer, AB the included ſide, and put

a=S.A, m=coſ. A, c=coſ. AB, v=verſ. AB,
b=S.B, n=coſ. B, d=coſ. F, V=verſ. ſup. AB,

then (by Cor. 1. Pr. XXXIX.) $\frac{mnr + drr}{ab} = c = r - v$, therefore $v = \frac{abr - mnr - drr}{ab}$; but (by Cor. 1. Pr. 5. I.) $\frac{mn - ab}{r} =$ coſ. $\overline{A + B}$ = (by Sch. 1. I.) r — verſ. $\overline{A + B}$ = verſ. ſup. $\overline{A + B} - r$, and $d = r$ — verſ. F = verſ. ſup. F — r, whence

$$v = \frac{\text{verſ. } \overline{A + B} - \text{verſ. ſup. F}}{ab} rr =$$

$$\frac{\text{verſ. F} - \text{verſ. ſup. } \overline{A + B}}{ab} rr.$$

Again, $V = r + c = \frac{abr + mnr + drr}{ab}$. But (by Cor. 1. Pr. 6. I.) $\frac{ab + mn}{r} =$ coſ. $\overline{B - A} = r$ — verſ. $\overline{B - A}$ = verſ. ſup. $\overline{B - A} - r$. Therefore

$$V = \frac{\text{coſ. } \overline{B - A} + \text{coſ. F}}{ab} \times rr =$$

$$\frac{\text{verſ. ſup. F} - \text{verſ. } \overline{B - A}}{ab} rr =$$

$$\frac{\text{verſ. ſup. } \overline{B - A} - \text{verſ. F}}{ab} rr.$$

Cor. 1. *As verſ. ſum of two angles — verſ. diff.* :
To twice radius : :
Or as coſ. difference of two angles — coſ. ſum :
To twice radius : :
So verſ. ſum of the including angles — verſ. ſup. third angle :
Verſ. included ſide : :

So

FIG. 42.

So verſ. third angle — verſ. ſup. ſum of the including angles :
Verſ. included ſide : :
And ſo coſ. diff. including angles + coſ. third angle :
Verſ. ſup. included ſide : :
And ſo verſ. ſup. third angle — verſ. diff. including angles :
Verſ. ſup. included ſide : :
And ſo verſ. ſup. diff. incl. angles — verſ. third angle :
Verſ. ſup. included ſide, AB.

For (by Cor. 2. Pr. 3. I.) $S.A \times S.B = \frac{\text{coſ.}\ \overline{B-A} - \text{coſ.}\ \overline{B+A}}{2} \times r = \frac{\text{verſ.}\ \overline{B+A} - \text{verſ.}\ \overline{B-A}}{2} \times r$. And all the reſt follows from this prop.

Cor. 2. *As rectangle of the ſines of two angles,* A, B :
Radius ſquare : :
So S. $\frac{1}{2}$ *ſum of the including* ∠*s* + $\frac{1}{2}$ *ſup.* 3^d ∠ × *S.* $\frac{1}{2}$ *ſum including* ∠*s* — $\frac{1}{2}$ *ſup.* 3^d ∠ :
Sine ſquare of half the included ſide : :
And ſo S. half ſum of : $\overline{3^d \angle, \text{ and ſup. ſum incl. } \angle s}$ × *S.* $\frac{1}{2}$ *their diff.* :
Sine ſquare of half the included ſide : :
So S. $\frac{1}{2}$ *ſup.* 3^d ∠ + $\frac{1}{2}$ *diff. incl.* ∠*s* × *S.* $\frac{1}{2}$ *ſup.* 3^d ∠ — $\frac{1}{2}$ *diff. incl.* ∠*'s* :
Coſ. ſquare of half the included ſide : :
So S. $\frac{1}{2}$ *ſup. diff. incl.* ∠*s* + $\frac{1}{2}$ *the* 3^d ∠ × *S.* $\frac{1}{2}$ *ſup. diff. incl.* ∠*s* — $\frac{1}{2}$ *the* 3^d ∠ :
Coſ. ſquare of half the included ſide, AB.

For ſince the diff. coſ. = diff. verſed ſines, therefore (by Cor. 1. Pr. 3. I.) $\frac{\text{verſ.}\ \overline{B+A} - \text{verſ. ſup. } F}{2} \times r = S.\ \frac{B+A+\text{ſup. } F}{2} \times S.\ \frac{B+A-\text{ſup. } F}{2}$.

FIG. 42.

And $\frac{\text{verſ. F} - \text{verſ. ſup. } \overline{B + A}}{2} \times r =$

$S.\frac{F + \text{ſup. } \overline{B + A}}{2} \times S.\frac{F - \text{ſup. } \overline{B + A}}{2}.$

And $\frac{\text{verſ. ſup. F} - \text{verſ. } \overline{B - A}}{2} \times r =$

$S.\frac{\text{ſup. F} + B - A}{2} \times S.\frac{\text{ſup. F} + A - B}{2}.$

And $\frac{\text{verſ. ſup. } \overline{B - A} - \text{verſ. F}}{2} \times r =$

$S.\frac{\text{ſup. } \overline{B - A} + F}{2} \times S.\frac{\text{ſup. } \overline{B - A} - F}{2}.$

And (by Pr. 2. I.) verſ. AB × $\frac{1}{2}r$ = ſine ſquare of $\frac{1}{2}$AB, and verſ. ſup. AB × $\frac{1}{2}r$ = coſ. ſquare of $\frac{1}{2}$AB, then all the reſt follows from the preſent prop.

Cor. 3. *Rectangle of the ſines of two angles,* A, B :
Radius ſquare : :
Or *ſum of the ſines* × *diff. ſines, of* $\frac{1}{2}$ *ſum and* $\frac{1}{2}$ *diff.*
Radius ſquare : : [*angles* :
So *coſ.* $\frac{1}{2}$ *ſum* × *coſ.* $\frac{1}{2}$ *diff. of the* 3d ∠ *and ſum incl. an-*
Sine ſquare of half the included ſide : : [*gles* :
So *coſ.* $\frac{1}{2}$ *ſum* × *coſ.* $\frac{1}{2}$ *diff. of* 3d ∠, *and diff. incl. an-*
Coſ. ſquare of half the included ſide, AB. [*gles* :

For (by Cor. 2. Pr. 4. I.) $S.A \times S.B = S.\frac{B + A}{2}$
$+ S.\frac{B - A}{2} \times S.\frac{B + A}{2} - S.\frac{B - A}{2}$; and

S.

$$\text{S.}\,\frac{B+A+180-F}{2}\times\text{S.}\,\frac{B+A-180+F}{2}=$$ FIG. 42.

$$\text{S.}\,90+\frac{B+A-F}{2}\times\text{S.}\,\frac{B+A+F}{2}-90=$$

$$\text{coſ.}\,\frac{F-A-B}{2}\times\text{coſ.}\,180-\frac{B+A+F}{2}=$$

$$\text{coſ.}\,\frac{F-A-B}{2}\times\text{coſ.}\,\frac{F+A+B}{2}.$$

Again, $\text{S.}\,\frac{180-F+B-A}{2}\times$

$$\text{S.}\,\frac{180-F+A-B}{2}=\text{S.}\,90-\frac{F+A-B}{2}\times$$

$$\text{S.}\,90-\frac{F+B-A}{2}=\text{coſ.}\,\frac{F-B+A}{2}\times$$

$\text{coſ.}\,\frac{F+B-A}{2}$. The reſt follows from Cor. 2.

Cor. 4. From half the ſum of the three angles, ſubtract each angle at the baſe ſeparately, and then the third angle. Then

As rectangle of the coſines of the two firſt differences :
Rectangle of the coſ. third difference and coſ. $\frac{1}{2}$ *ſum* [*of three angles* : :
So radius ſquare :
Tan. ſquare of half the included ſide or baſe, AB.

This follows from Cor. 3. becauſe coſ. : ſine : : rad. : tangent. See ſchol. Prop. XLII.

PROP. XLIV.

If the vertical angle F *of a triangle* AFB *be biſected, it will be*, 48.

As twice the cot. of the biſecting line FI :
Sum of the cotangents of the ſides AF, BF : :
Radius :
Coſ. half the vertical angle F.

Pro-

FIG. 48. Produce FA, FI, FB to quadrants, and to the pole F draw the great circle LTR, and produce BA to interſect it in R. And let tan. AS $= b$, tan. IT $= c$, tan. BL $= d$, S.RT $= y$, coſ. $\frac{1}{2}$F $= x$. Then the angles at S, T, L are right. And by Cor. 1. Prop. XXVII. $c : b :: y : \frac{by}{c} =$ S.RS. And $c : d :: y : \frac{dy}{c} =$ S.RL. And ſince ST $=$ TL; therefore (by Prop. 3. I.) $\frac{by + dy}{c} r = 2x \times y$, or $\frac{b + d}{c} \times r = 2x$, that is, $2cx = \overline{b + d} \times r$.

Cor. *As twice radius :*
Tan. biſecting arch ::
Sum of the cotan. ſides :
Coſ. half the vertical angle F.

PROP. XLV.

48. *If an arch be drawn from the vertex to the middle of the baſe of a triangle; then the ſines of the vertical angles are reciprocally as the ſines of the ſides, and directly as the ſines of the angles at the baſe.*

For (by Cor. 1. Prop. XXIX.) S.AFI × S.AF = S.BFI × S.BF, becauſe AI = BI. Therefore S.AFI : S.BFI : : S.BF : S.AF : : (by Pr. XXIX.) S.A : S.B.

PROP. XLVI.

49. *As one right angle :*
To the angle intercepted between two great circles ::
So the area of a great circle of the ſphere :
Lunular area contained between theſe great circles.

Let

Let ABC, ADC be the two great circles; BDF a great circle, whoſe poles are A and C. Divide the circle BDF into an infinite number of equal parts, through all which draw great circles paſſing through A and C, which will divide the ſurface of the ſphere into the ſame number of equal parts, ſimilar and equal to one another; ſince their baſes (in the circle BF) and ſides are all equal. Therefore as the number of the parts in the whole circumference, to the number of parts in BD, that is, as the whole circumference, to the arch BD : : ſo the ſurface of the ſphere, to the area ABCDA; or, as 4 right angles : angle BAD : : ſurface of the ſphere : area ABCDA. But (by geometry) the ſurface of the ſphere = 4 great circles. Therefore as 4 right angles : angle BAD : : 4 great circles : area ABCDA; or as one right angle : one great circle : : angle BAD : area ABCDA. FIG. 49.

Cor. *As* 4 *right angles* :
Intercepted angle BAD : :
So ſurface of the ſphere :
Surface intercepted between theſe two great circles : :
And ſo ſolidity of the ſphere :
Solidity contained between theſe great circles.

For the ſolidities are as the ſurfaces, becauſe they may be reſolved into pyramids of equal height.

PROP. XLVII.

In any ſpherical triangle, G; 50.
As two right angles :
Exceſs of the three angles above two right angles : :
Area of a great circle of the ſphere :
Area of the triangle.

Produce all the ſides to ſemicircles, then the oppoſite angles are equal (by Cor. 1. Prop. IV.); and (by

FIG. 50. (by Prop. IX.) triangle H = triangle G, alſo half the ſurface of the ſphere = G + R + S + T; let a, b, c be the three angles, then (by Cor. Prop. XLVI.),

180°: a : : $\frac{1}{2}$ ſurface ſphere : G + R,
180 : b : : $\frac{1}{2}$ ſurface ſphere : G + S,
180 : c : : $\frac{1}{2}$ ſurface ſphere : H + T or G + T,

therefore 180 : $a + b + c$: : $\frac{1}{2}$ ſurface ſphere : 3G + R + S + T, and by diviſion, 180 : $a + b + c -$ 180 : : $\frac{1}{2}$ ſurface ſphere : 2G : : $\frac{1}{4}$ ſurface ſphere, or the area of a great circle : G.

Cor. 1. *As* 1 : 57.2957795 : : *the* 3 *angles* — 2 *right angles* : *area, in ſquare degrees.*

For area of a great circle = $\frac{1}{2}$ circumference × radius = 180 × radius, and radius = 57.2957795 &c.

Cor. 2. *As* 180 : 3 *angles* — 180 : : $3.1416 \times \overline{\textit{rad.}}^2$: *area of the triangle.*

For $3.1416rr$ = area of the circle, whoſe radius is r.

Cor. 3. *In any ſpherical polygon, put* n = *number of ſides*, $A = 180° \times \overline{n-2}$. Then I ſay,

As 2 *right angles or* 180° :
Sum of all the angles of the polygon — A : :
So the area of a great circle of the ſphere :
Area of the polygon.

Suppoſe the polygon to be divided into as many triangles as it has ſides, by arches drawn from a point within it. Then ſince (by this Prop.) it is in any one triangle, as 180 : a great circle : : ſum of its angles — 180 : its area; therefore by compoſition it will be, 180 : a great circle : : ſum of all the angles internal and external — $n \times 180$: ſum of all the areas of the triangles; but the ſum of the internal angles =

$= 360$ or 2×180, therefore 180 : great circle : : FIG.
ſum of all the angles of the polygon — A (or $\overline{n-2} \times 180$) : area of the triangle.

PROP. XLVIII.

If a ſphericnl triangle ABG *be inſcribed in a leſſer circle, ſo as that the baſe* AG *paſs through the pole* D *of that circle; then the angle at the vertex* ABG *will be equal to the ſum of the angles at the baſe* A, G; *and the contrary.* 71.

For draw BD to the pole D. Then ſince AD, DB, DG are all equal, the angle $A = ABD$, and the $\angle G = DBG$; and therefore the ſum $A + G = ABG$.

And if the $\angle B = A + G$, then $AD = DB = DG$. For if DB be greater or leſſer than AD or DG, then A will be greater or leſſer than ABD, and G greater or leſſer than DBG; and therefore $A + G$ accordingly greater or leſſer than ABG.

PROP. XLIX.

If the ſum of the two ſides (AB, BG) *of the ſpherical triangle* ABG, *be a ſemicircle, and the included angle* B *be biſected; the oppoſite ſide* AG *will alſo be biſected, and the biſecting arch* BD *will be a quadrant.* 72.

For produce BA, BD, BG to E, then BE is a ſemicircle $= AB + BG$; and $EG = AB$. Alſo angle $ABD = DBG = DEG$ (by Prop. V.) and $\angle BAD = EGD$ (by Prop. XVI.); and the alternate angles BDA and EDG are equal; therefore in the two triangles BAD, and EGD, all the angles are reſpectively equal; whence (by Prop. XI.) $AD = DG$, and $BD = DE$, half a ſemicircle.

PROP.

PROP. L.

FIG. 73. *If from the vertex of any ſpherical triangle* ABG, *two arches,* BD, BE *be drawn to cut the baſe* AG, *making the angles* ABD, GBE *equal; it will be,*

As ſquare of the ſine of AB :
To ſquare of the ſine of BG : :
So rectangle of the ſines of AD, AE :
To rectangle of the ſines of GE, GD.

For ſince the ∠ ABD = GBE, therefore ABE = DBG. And (by Cor. 1. Prop. XXIX.)

S.AB × S.ABD : S.BG × S.DBG or ABE : : S.AD : S.DG.

And S.AB × S.ABE : S.BG × S.EBG or ABD : : S.AE : S.EG. Therefore

$\overline{\text{S.AB}}^2 : \overline{\text{S.BG}}^2 :: \text{S.AD} \times \text{S.AE} : \text{S.DG} \times \text{S.EG}$.

PROP. LI.

74. *In any ſpherical triangle* ABG, *if two angles* A, G *be biſected by the arches* AD, GD; *and if an arch* BDE *be drawn from the third angle* B, *to the point of concourſe* D, *it will alſo biſect that angle.*

For (by Cor. 2. Prop. XXIX.) S.BD : S.DE : : S.BG : S.GE : : S.BA : S.AE; and alternately S.BG : S.BA : : S.GE : S.AE : : S.BG × S.GBE : S.BA × S.ABE (by Cor. 1. ib.) therefore ∠ GBE = ABE.

Cor. *Hence, three arches biſecting the three angles of a ſpherical triangle, interſect each other in one point.*

PROP. LII.

FIG. 74. *In any ſpherical triangle* ABG, *if two arches* AC, GF *be drawn from two angles* A, G, *to biſect the oppoſite ſides* BG, AB; *then the arch* BDE, *drwn from the third angle* B *through the point of concourſe* D, *will alſo biſect the third ſide* AG.

For by Prop. XXIX. and Cor. 1. we ſhall have
$S.AG \times S.AGD = S.AD \times S.ADF = S.BD \times S.BDF$, becauſe $AF = FB$. Alſo
$S.AG \times S.GAD = S.GD \times S.GDC = S.BD \times S.BDC$, ſince $BC = GC$. Therefore
$S.GAD : S.AGD :: S.BDC$ or $S.ADE : S.BDF$ or GDE.
But $S.AE : S.EG :: S.AD \times S.ADE : S.GD \times S.GDE :: S.AD \times S.GAD : S.GD \times S.AGD$. But in the triangle ADG, $S.AD \times S.GAD = S.GD \times S.AGD$; therefore $AE = EG$.

Cor. *Three arches drawn from the three angles of a ſpherical triangle to biſect the oppoſite ſides, will meet in one point.*

PROP. LIII.

75. *In any ſpherical triangle* ABG, *if three arches be erected perpendicularly upon the middle of the three ſides; they will interſect in one point.*

Let CD, FD be two perps. draw GD, BD, AD; then in the right-angled triangles CDG, CDB; $BC = CG$, and CD common; therefore $GD = DB$. Alſo in the right-angled triangles BDF, ADF, we have $BF = AF$, and FD common; therefore $AD = DB = GD$. Therefore the triangle ADG is iſoſceles, and therefore the arch DE, which biſects the baſe AG, is perpendicular to it.

PROP.

PROP. LIV.

FIG. 76. *In any ſpherical triangle* ABG, *if two arches be drawn from two angles* A, G, *perpendicular to the oppoſite ſides* GB, AB; *and if the arch* BDE *be drawn from the third angle* B *through the point of interſection* D, *it will be perpendicular to the third ſide* AG.

The ratio of S.AD to S.DE is compounded of S.AD to S.DC, S.DC : to S.DB, and S.DB to S.DE. And by Prop. XXIX. Cor. 1. in the triangles AGC, CBD, BGE, it is,

S.AD : S.DC : : S.AG × S.AGD : S.CG × S.CGD, and
S.DC : S.DB : : S.CBD : radius 1.
S.DB : S.DE : : S.BG × S.BGD : S.EG × S.EGD.

and throwing out the equal ratios S.BGD, S.CGD, and S.AGD, S.EGD, then

S.AD : S.DE : : S.AG × S.BG × S.CBD : S.CG × S.EG.

But in the triangle BGE, S.BG × S.CBD or S.GB × S.GBE = S.GE × S.GED.

Therefore S.AD : S.DE : : S.AG × S.GE × S.GED : S.CG × S.EG : : S.AG × S.GED : S.CG.

But S.AG : S.CG : : 1 rad. : S.CAG, in the triangle ACG; therefore S.AD : S.DE : : S.GED : S.CAG or EAD.

But in the triangle ADE; S AD : S.DE : : S.AED : S.EAD.

Therefore S.GED : S.EAD : : S.AED : S.EAD, or S.GED : S.AED : : S.EAD : S.EAD, therefore GED = AED, or BDE perpend. to AG.

Cor. *Three perpendiculars let fall from the three angles to the oppoſite ſides, all meet in one point.*

PROP. LV.

In the right-angled ſpherical triangle ABG; *if from the right angle* B, *two arches* BD, BE, *be drawn, making equal angles with the ſide* BA; *then will* S.GD : S.GE : : S.AD : S.AE. FIG. 77.

For ſince the ∠ ABG is right, and EBA = ABD, the ∠ EBG + DBG = 2 right angles, or EBG = ſup. of DBG. Therefore in the two triangles GBD, GBE, it is S.GD : S.DB : : S.GBD or GBE : S.G : : S.GE : S.EB, by Prop. XXIX. and alternately, S.GD : S.GE : : S.DB : S.EB : : S.DA : S.EA, by Cor. 1. Prop. XXIX.

PROP. LVI.

In any triangle LMN, *if any ſide* NM *be produced, and the arch* EOA *be drawn to interſect the three ſides of the triangle, in the points* A, O, E; *it will be*, S.LA : S.AN : : S.LO × S.EM : S.OM × S.EN. 78.

For in the triangle AEN, S.A : S.EN : : S.E : $\text{S.AN} = \frac{\text{S.E} \times \text{S.EN}}{\text{S.A}}$, and in the triangle LAO, S.A : S.LO : : S.O : $\text{S.LA} = \frac{\text{S.O} \times \text{S.LO}}{\text{S.A}}$. Therefore S.LA : S.AN : : S.LO × S.O : S.EN × S.E : : S.LO × S.EM : S.EN × S.OM, in the triangle OEM.

PROP. LVII.

FIG. 65. *If an arch* EOA *be drawn to interſect the three ſides of a triangle* LMN *in the points* A, O, E; *any ſide thereof being produced; then will* S.NL : S.NA : : S.LM × S.EO : S.AE × S.OM.

For in the triangle NLM; S.N : S.LM : : S.M : S.NL $= \frac{\text{S.LM} \times \text{S.M}}{\text{S.N}}$. And in the triangle AEN; S.N : S.AE : : S.E : S.AN $= \frac{\text{S.AE} \times \text{S.E}}{\text{S.N}}$. Whence S.NL : S.AN : : S.LM × S.M : S.AE × S.E : : S.LM × S.EO : S.AE × S.OM; becauſe S.M : S.E : : S.EO : S.OM, in the triangle OEM.

SECT. IV.

The ſolution of all the caſes of ſpherical triangles.

By help of the propoſitions delivered in the laſt ſection, all the caſes of ſpherical triangles may be reſolved. I ſhall give the canons for the ſolution of each caſe in particular, both in right-angled triangles, and in oblique triangles; after I have ſhewn, in general, how they all may be reſolved by the foregoing propoſitions. FIG.

Every triangle has ſix parts, and any three being given, the reſt may be found.

The ſolution of right-angled triangles.

All the caſes of right-angled and quadrantal triangles may be reſolved by the XXVIth and XXVIIth propoſitions; or by Prop. XXVIII. alone.

I.

The ſolution of any caſe may be performed by the XXVIth or XXVIIth Prop. one of the two; either immediately in the triangle itſelf, or elſe in another right-angled triangle, formed by producing all its ſides to quadrants; wherein the parts of the triangle are either the ſame, or the complements of thoſe in the firſt. Thus let ABF be the propoſed triangle; 41. produce the ſides either through A or F, ſo that AC, AD, BE, and conſequently CE, may be quadrants. Then you have two right-angled triangles, ABF, EDF. In the figure ABCDEF, mark what is given, and its complement, with a daſh (/), and what is ſought, and its complement, with a cypher (o). Then examine the two right-angled triangles ABF, EDF; and obſerve in which of them, *an*

FIG. 41. *angle and the hypothenuſe, or an angle and its oppoſite ſide*, do not enter the queſtion, (for there are always two parts out of the queſtion) ; and that triangle will afford the ſolution, by applying Prop. XXVII. in the former caſe, and Prop. XXVI. in the latter. If any ſide be greater than a quadrant, then a quadrant muſt be cut off, and the ſame rule applied.

Quadrantal triangles may be reſolved by the ſame propoſitions, if you reduce them to right-angled triangles, by producing their oblique ſides to quadrants, or cutting off a quadrant if they be bigger. As in the quadrantal triangle AFE, produce AF, EF to quadrants, as AD, EB, and draw AB, ED round the poles E, A ; then you will have two right-angled triangles ABF, EDF, to be reſolved by the former rule. Take an example or two for right-angled triangles.

Ex. 1.

51. Let the hypothenuſe AF and an angle F be given, and the oppoſite ſide AB required. I mark AF, F as given, and AB as ſought, then ſince there is an angle A and its oppoſite ſide FB out of the queſtion ; therefore I need produce the ſides no further, for I have the ſolution in the triangle ABF, by Prop. XXVI. as rad. : S.AF : : S.F : S.AB required.

Ex. 2.

41. Let the hyp. AF, angle A be given ; to find the other angle F. I produce the ſides through F, and mark AF, FD, A, DC, ED as given ; and F as ſought. Then I ſee, that in the triangle EFD, the angle E and hypothenuſe EF are not marked, and therefore by Prop. XXVII. S.FD : rad. : : tan. ED : tan. angle F ; that is, coſ. AF : rad. : : cotan. A : tan. F required.

II.

Right-angled or quadrantal triangles may alſo be reſolved by Prop. XXVIII. obſerving if middle part

part be ſought, to begin with radius; and if one extreme be ſought, to begin with the other extreme. FIG.

Ex. 1.

Given the hyp. AF, and angle F, to find the ſide AB; what is given and ſought being marked as in the figure. Then AB is middle part, and AF, F, extremes disjunct, therefore rad. : coſ. comp. AF :: coſ. comp. F : S.AB; that is, rad. : S.AF :: S.F : S.AB ſought. 51.

Ex. 2.

Given the hyp. AF, and an angle A, to find the angle F. Here AF is middle part, and A, F extremes conjunct. Therefore tan. comp. A : rad. :: S. compl. AF : tan. comp. F, or cotan. A : rad. :: coſ. AF : cotan. F. 41.

Ex. 3.

In the quadrantal triangle AFE, let ſide AF, angle A, be given; to find the angle F. Here AF is middle part, and A and F are extremes conjunct; therefore, 41.

Tan. EAF : rad. :: coſ. AF : cotan. F.

Laſtly, the affection of the angle or ſide ſought will be known by Prop. XX. and Prop. XXI. and its corollaries; except it be ambiguous, and then it may be either acute or obtuſe.

The ſolution of oblique ſpherical triangles.

Oblique triangles may be reſolved ſeveral ways, either by letting fall a perpendicular, or without it. In any triangle, mark what is given with a daſh (/), and what is required with a cypher (0), to diſtinguiſh them.

I.

Oblique triangles may be reſolved by the rules of right-angled triangles, together with the corollaries

laries of Prop. XXVIII. with Prop. XXIX, XXXII, XXXII; by letting fall a perpendicular, which either divides the triangle into two right-angled triangles; or makes two right-angled triangles, by adding a right-angled triangle to it.

In letting fall the perpendicular, let it fall from the end of a given ſide, and oppoſite to a given angle; and if the three things given be adjoining to one another, let it alſo fall from the end of the ſide required, or oppoſite to the angle required; and then you will have enough given in one of the right-angled triangles, to find any of its unknown parts: obſerving to find ſuch a part (which muſt be either a baſe or vertical angle), as either of itſelf, or when added to or ſubtracted from ſome part given, the proportion to find the thing ſought may come under ſome of the corollaries of Prop. XXVIII.

II.

All the caſes of oblique triangles may alſo be reſolved by Prop. XXVII, XXIX, XXXII, and XXXIII, alone, by letting fall a perpendicular according to the former directions, when there is occaſion. For the two laſt caſes will be reſolved by Prop. XXXIII. and XXXII, and all the reſt by Prop. XXVII. and XXIX. and they are eaſily remembered, for three of them are the ſame with thoſe in plain trigonometry, for reſolving the like caſes. Only there will be more operations in the right-angled triangles, than if the corollaries of Prop. XXVIII. were made uſe of.

III.

All the caſes of oblique triangles (except the 1ſt, 4th, and two laſt) may alſo be reſolved by Prop. XXVIII. alone, by letting fall a perpendicular dividing the whole into two right-angled triangles. In the firſt triangle you have two things given to find a third, which muſt be either the baſe or vertical angle.

angle. In the ſecond triangle you have one thing given, or at leaſt you can find one thing, after the firſt operation in the firſt triangle is over; and in the ſecond triangle there is alſo the thing ſought. Then in theſe two triangles you have either both the baſes, or elſe both the vertical angles. Therefore, from theſe, to find the thing required, compare theſe three things in the ſecond triangle, viz. the perpendicular, the thing given, and the thing ſought, and find which is middle part, and whether the other two be extremes conjunct or disjunct, in order to know whether you muſt uſe ſines or tangents. Do the ſame thing in the firſt triangle, with the three correſponding parts; and (ſetting aſide the perpendicular, which is an extreme in both triangles); then the middle part and its extreme in the firſt triangle, will be proportional to the middle part and its extreme in the ſecond triangle.

FIG. 53.

For example. Given FB, F and B; to find AF. In the firſt triangle BFI, as cotan. B : rad. : : coſ. FB : cotan. BFI; then you have AFI. Then in the triangle AFI, the angle AFI is middle part, and AF, FI extremes conjunct. Likewiſe in the triangle BFI; BFI is middle part, and BF, FI, extremes conjunct. Therefore coſ. BFI (middle part) : coſ. AFI (middle part) : : cotan. BF (extreme conjunct) : cotan. AF (extreme conjunct.)

The reaſon of this operation will appear thus: let p = perpendicular, m = middle part, a = the other extreme, in the firſt triangle. M = middle part, A = extreme in the ſecond triangle. Then by Prop. XXVIII. $pa = mr$, and $pA = Mr$, therefore $\frac{p}{r} = \frac{m}{a} = \frac{M}{A}$, or $m : a :: M : A$.

IV.

All oblique triangles may alſo be reſolved without letting fall a perpendicular, and theſe five propoſitions, FIG. 53. poſitions, the XXIX, XXXVI, XXXVII, XL, XLI, and their corollaries, will reſolve them all. Thus, Caſe 1 and 4 are reſolved by Prop. XXIX. Caſe 2 and 6 by Prop. XXIX, and Cor. 1. Prop. XXXVII. Caſe 3 and 5 by Prop. XXIX, and Cor. 1. Prop. XXXVII. Caſe 7 by Prop. XXXVI, and Cor. 1. Caſe 8 by Prop. XXXVI, and Cor. 1. and Prop. XXIX. Caſe 9 by Prop. XXXVII, and Cor. 1. Caſe 10 by Prop. XXXVII, and Cor. 1. and Prop. XXIX. Caſe 11 by Prop. XLII, or its corollaries. Caſe 12 by Prop. XLIII, or its corollaries.

Laſtly, the affection of the angle or ſide ſought may be gathered from Prop. XVII, and its corollaries. And the falling of the perpendicular, from Prop. XIX, and Cors. Prop. VIII.

Here follow the 16 caſes of right-angled triangles, and the 12 of oblique. In right-angled triangles, I ſhall give only the ſolution by the tables, omitting the algebraic ſolution; for that is very eaſily had by putting letters inſtead of the quantities, and tranſforming them by Sch. Prop. 1. I. if there is occaſion.

In oblique triangles you have both; where note, that natural ſines, tangents, &c. muſt be uſed in the algebraic way. I refer to the caſes in right-angled triangles, for the determination of the ſpecies of the angle or ſide ſought, in the method of letting fall a perpendicular.

The rule for working any log. proportion in ſpherical trigonometry, is the ſame as in plain trigonometry, viz. *Add the logarithms of the ſecond and third terms together, and from the ſum ſubtract the log. of the firſt term; and there remains the log. of the fourth term ſought.*

Note, All proportions of ſpherical triangles agree with plain triangles, taking the ſides inſtead of ſines and tangents of the ſides. But then neither coſines, cotangents, coſecants, or verſed ſines of the ſides muſt be concerned. And radius can only come in when two terms are ſides.

Right-angled ſpherical triangles.

CASE I.

Given the *hypothenuſe* AF, and an *angle* A; to FIG.
find the *ſide adjacent* AB. 52.

Rad. : *coſ.* angle A : : *tan. hyp.* AF : *tan. adjacent ſide* AB.

If the angle and hyp. (A, AF) are of the ſame affection, AB is leſs than a quadrant; if of different affection, more.

CASE II.

Given the *hyp.* AF, and *angle* A; to find the opposite *ſide* BF. 52.

Rad. : S. *hyp.* AF : : S. *an angle* A : S. *oppoſite ſide* BF.

If the angle A be acute, BF is leſs than 90; if obtuſe, more.

CASE III.

Given the *hyp.* AF, and an *angle* A; to find the other *angle* F. 52.

Rad. : *coſ. hyp.* AF : : *tan. an angle* A : *cotan. other angle* F.

When the hyp. and angle (AF and A) are of the ſame affection, F is acute; if of different affection, obtuſe.

CASE IV.

Given the *hyp.* AF, and a *leg* AB; to find the adjacent *angle* A. 52.

Rad. : *cot. hyp.* AF : : *tan. a ſide* AB : *coſ. adjacent angle* A.

If the hyp. and leg (AB, AF) be of the ſame affection, A is acute; if of different affection, obtuſe.

CASE V.

FIG. 52. Given the *hyp.* AF, and one *ſide* AB; to find the oppoſite *angle* F.

S. *hyp.* AF : *rad.* : : S. *one ſide* AB : S. its *oppoſite angle* F.

If the ſide (AB) is leſs than 90, F is acute; if more than 90, obtuſe.

CASE VI.

52. Given the *hyp.* AF, and a *leg* AB; to find the other *leg* BF.

Coſ. one leg AB : *coſ. hyp.* AF : : *rad.* : *coſ. other leg* BF.

If the hyp. and leg (AB, AF) be of one affection, BF is leſs than 90; if of different affection, more.

CASE VII.

52. Given a *ſide* AB, and its adjacent *angle* A; to find the oppoſite *ſide* FB.

Rad. : S. *one ſide* AB : : *tan. adjacent angle* A : *tan. oppoſite ſide* FB.

If the angle (A) be acute, BF is leſs than 90; if obtuſe, greater.

CASE VIII.

52. Given a *ſide* AB, and its adjacent *angle* A; to find the oppoſite *angle* F.

Rad. : S. *an angle* A : : *coſ. its adjacent ſide* AB : *coſ. its oppoſite angle* F.

If the ſide (AB) is leſs than 90, F is acute; if more, obtuſe.

CASE

CASE IX.

Given a *ſide* AB, and *angle adjacent* A; to find the *hyp.* AF. FIG. 52.

Coſ. an angle A : *rad.* : : *tan. adjacent ſide* AB : *tan. hyp.* AF.

If the angle and ſide (A and AB) be of the ſame affection, AF is leſs than 90; if different, more.

CASE X.

Given a *ſide* BF, and oppoſite *angle* A; to find the *ſide* adjacent AB. 52.

Rad. : *cotan. an angle* A : : *tan. op. ſide* BF : S. *adjacent ſide* AB.

Here AB is ambiguous, that is, it may be either leſſer or greater than 90.

CASE XI.

Given a *ſide* BF, and oppoſite *angle* A; to find the *angle* adjacent F. 52.

Coſ. one ſide BF : *coſ. op. angle* A : : *rad.* : S. *adjacent angle* F, ambiguous.

CASE XII.

Given a *ſide* BF, and *opp. angle* A; to find the *hyp.* AF. 52.

S. *an angle* A : S. *op. ſide* BF : : *rad.* : S. *hyp.* AF, ambiguous.

CASE XIII.

Given the *ſides* AB, FB, to find an *angle* A. 52.

S. *adjacent ſide* AB : *rad.* : : *tan. other ſide* FB : *tan. op. angle* A.

If the oppoſite ſide (BF) is leſs than 90, A is acute; if more, obtuſe.

CASE

CASE XIV.

FIG. 52. Given the *ſides* AB, FB; to find the *hyp.* AF.

Rad. : *coſ. one leg* FB : : *coſ. other leg* AB : *coſ. hyp.* AF.

If the ſides (AB, FB) are of one affection, AF is leſs than a quadrant; if of different affection, more.

CASE XV.

52. Given the *angles* A, F; to find a *ſide* BF.

S. *adjacent angle* F : *rad.* : : *coſ. other angle* A : *coſ. op. ſide* BF.

If the oppoſite angle (A) be acute, BF is leſs than a quadrant; if obtuſe, it is greater.

CASE XVI.

52. Given the *angles* A, F; to find the *hyp.* AF.

Tan. one angle F : *cotan. other angle* A : : *rad.* : *coſ. hyp.* AF.

If the angles (A, F) be of the ſame affection, AF is leſs than a quadrant; if of different affection, it is greater.

Oblique

Oblique ſpherical triangles.

CASE I.

Given two *ſides* AF, BF, and an oppoſite *angle* FIG.
A ; to find the other oppoſite *angle* B. 54.
55.

I. *Logarithmically, by the table of artificial ſines,* &c. 56.

S. *one ſide* FB : S. *op. angle* A : : S. other *ſide* AF : S. *its op. angle* B.

Then $\frac{A+B}{2}$ is of the ſame affection as $\frac{AF+BF}{2}$.

And if $\frac{A+B}{2}$ be of the ſame affection as $\frac{A+\text{ſup.}B}{2}$, then B is ambiguous.

II. *Algebraically, by the table of natural ſines,* &c.

Let $d=$ S.AF, $p=$ S.BF, $a=$ S.A ; then S.B $=\frac{ad}{p}$.

CASE II.

Given two *ſides* AF, BF, and an oppoſite *angle* 54.
A ; to find the included *angle* F. 55.
56.

I. *Logarithmically.*

Let fall a perpendicular from F, (beyond B, fig. 56.) Then by Caſe 3. right ∠ triangles, R : *coſ.* AF : : *tan.* A : *cotan.* AFI.
And *cotan.* AF : *coſ.* AFI : : *cotan.* BF : *coſ.* BFI.

Then according as BF and angle FAI are of the ſame or different affection, BFI is acute or obtuſe.

If

FIG. If $AFI > BFI$, then $AFI - BFI = F$ } if
54. If $AFI + BFI < 180$, then $AFI + BFI = F$ } both,
55. then F is ambiguous.
56.

Or thus,

Find the angle B *by Caſe* 1. *then*

$$S.\frac{AF \backsim BF}{2} : S.\frac{AF + BF}{2} :: \tan.\frac{B \backsim A}{2} : \cot.\tfrac{1}{2}F.$$

II. *Algebraically.*

47. *Let* a, d, p *be the ſines, and* m, e, i *the coſines of* A, AF, BF. *And* $b = \frac{ad}{p}$ = S.B, n = *coſ.* B, *and* $hh = bb - aa, = mm - nn$, $kk = 1 - bbpp = 1 - aadd$, *rad.* = 1.

Then $S.F = \frac{hh}{bim - aen} = \frac{bim + aen}{kk}$.

Alſo ſee Cor. 1, 2. *Prop.* XL.

CASE III.

54. Given two *ſides* AF, BF, and an oppoſite *angle*
55. A; to find the third *ſide* AB.
56.

I. *Logarithmically.*

Let fall a perp. from F, then *by Caſe* I. *right* ∠ *triangles*,

Rad. : *coſ.* A : : *tan.* AF : *tan.* AI.

Then *coſ.* AF : *coſ.* AI : : *coſ.* BF : *coſ.* BI.

Then if BF and angle FAI are of one affection, BI is leſs than a quadrant; if of different affection, more.

FIG.

$$\left.\begin{array}{l} \text{If } AI > BI,\ AI - BI = AB, \\ \text{If } AI + BI < 180,\ AI + BI = AB, \end{array}\right\} \text{if both,}$$ 54.

then AB is ambiguous. 55. 56.

Or thus.

Find the angle B *by Caſe* I. *then*

$$S.\frac{B \backsim A}{2} : S.\frac{B + A}{2} :: tan.\frac{AF \backsim BF}{2} : tan.\tfrac{1}{2}AB.$$

II. *Algebraically.*

Let a, d, p *be the ſines, and* m, e, i *the coſines of* A, AF, BF, $b = \frac{ad}{p} = S.B$, $n = cof.\,B$, *and* $ll = dd - pp = ii - ee$, $tt = 1 - bbpp = 1 - aadd$, *radius* $= 1$. *Then*

$$S.AB = \frac{ll}{mid - pen} = \frac{mid + pen}{tt}.$$

See alſo Cor. 1, 2. *Prop.* XLI.

CASE IV.

Given two *angles* A, B, and an oppoſite *ſide* BF; to find the other oppoſite *ſide* AF. 54. 55. 56.

I. *Logarithmically.*

Let fall a perp. from F, then S.A : S.BF : : S.B : S.AF.

$\frac{AF + BF}{2}$ is of the *ſame affection* as $\frac{A + B}{2}$. If $\frac{BF + AF}{2}$ be of the *ſame affection* as $\frac{BF + \text{ſup.}AF}{2}$, then AF is ambiguous.

II.

II. *Algebraically.*

FIG. 54. 55. 50. Let $a =$ S.A, $b =$ S.B, $p =$ S.BF, *then* S.AF $= \frac{pb}{a}$.

CASE V.

54. 55. 56. Given two angles A, B, and an oppoſite *ſide* BF; to find the included *ſide* AB.

I. *Logarithmically.*

Let fall a perp. from F, then by Caſe I. right ∠ triangles, *rad.* : *tan.* BF : : *coſ.* B : *tan.* BI.

Then *cotan.* B : *cotan.* A : : S.BI : S.AI.

If A, B be of the *ſame affection*, AI + BI = AB. If BI + AI, and BI + ſup. AI be each $<$ 180, (AI and conſequently) AB is ambiguous.

If A and B are of *different affection*, AI − BI = AB. If both AI and ſup. AI be $>$ BI, then AI and AB are ambiguous.

Or thus,

47. *Find* AF *by Caſe* IV. *then* S. $\frac{B \backsim A}{2}$: S. $\frac{B+A}{2}$: : *tan.* $\frac{AF \backsim BF}{2}$: *tan.* $\frac{1}{2}$AF.

II. *Algebraically.*

Let a, b, p *be the ſines*, m, n, i *the coſines of* A, B, BF; $d = \frac{pb}{a} =$ S.AF, $e =$ coſ. AF, $ll = dd - pp = ii - ee$, $tt = 1 - bbpp = 1 - aadd$, rad. = 1. Then

S.AB

FIG.

S.AB $= \frac{ll}{mid - pen} = \frac{mid + pen}{tt}$. 47.

See alſo Cor. 1, 2. *Prop. XLI.*

CASE VI.

Given two *angles* A and B, and an oppoſite *ſide* BF; to find the third *angle* F. 54. 55. 56.

I. *Logarithmically.*

Let fall a perp. from F; then by Caſe 3. right ∠ triangles, *rad.* : *tan.* B :: *coſ.* BF : *cotan.* BFI.

Then *coſ.* B : S.BFI : *coſ.* A : S.AFI.

If A and B are of the *ſame affection*, BFI + AFI = F. If both BFI + AFI and BFI + ſup. AFI be < 180, then (AFI and) F is ambiguous.

If A and B are of *different affection*, AFI — BFI = F, if both AFI and ſup. AFI > BFI, then (AFI and) F is ambiguous.

Or thus,

Find AF by Caſe 4. then S. $\frac{AF \backsim BF}{2}$: 47.

S. $\frac{AF + BF}{2}$ :: *tan.* $\frac{B \backsim A}{2}$: *cotan.* $\frac{1}{2}$ F.

II. *Algebraically.*

Let a, b, p be the *ſines*; m, n, i the *coſines* of A, B, FB; $d = \frac{pb}{a} =$ S.AF, $e =$ *coſ.* AF; $hh = bb - aa = mm - nn$, $kk = 1 - bbpp = 1 - aad$, rad. $= 1$. 47.

Then S.F $= \frac{hh}{bim - aen} = \frac{bim + aen}{kk}$.

Alſo ſee Cor. 1, 2. Prop. XL.

CASE VII.

FIG. 54. 55. 56. Given two *ſides* AF, AB, and the included *angle* A; to find an oppoſite *angle* B.

I. *Logarithmically.*

Let fall a perp. from F ; then by Caſe 1 right ∠ triangles, *rad.* : *coſ.* A : : *tan.* AF : *tan.* AI, and AB $\backsim$ AI = IB.

Then S.AI : S.BI : : *cotan.* A : *cotan.* B.

According as AI is < or > than AB, B and A are of the ſame or different affection.

Or thus,

$$\textit{Coſ.}\ \frac{AB+AF}{2} : \textit{coſ.}\ \frac{AB \backsim AF}{2} :: \textit{cotan.}\ \tfrac{1}{2}A : \textit{tan.}\ \frac{F+B}{2}.$$

$$\text{And S.}\ \frac{AB+AF}{2} : \text{S.}\ \frac{AF \backsim AB}{2} :: \textit{cotan.}\ \tfrac{1}{2}A : \textit{tan.}\ \frac{F \backsim B}{2}.$$

Then $\frac{F+B}{2} + \frac{F \backsim B}{2}$ = *angle oppoſite* to the greater ſide.

And $\frac{F+B}{2} - \frac{F \backsim B}{2}$ = *angle op.* to the leſſer ſide.

Note, $\frac{F+B}{2}$ is of the ſame affection as $\frac{AF+AB}{2}$.

II. *Algebraically.*

47. Let s, a, d be the *ſines*, c, m, e the *coſines* of AB, A, AF ; z = *cotan.* AB, t = *cotan.* AF, τ = *cotan.* A, *rad.* = 1. Then

Cotan.

FIG.

$$Cotan.\ B = \frac{se - dcm}{da} = \frac{s}{a}\,t - c\tau.$$

$$Cotan.\ F = \frac{dc - sem}{as} = \frac{d}{a}\,z - e\tau.$$

CASE VIII.

Given two *ſides* AF, AB, and the included *angle* 54.
A; to find the third *ſide* FB. 55.
56.

I. *Logarithmically.*

Let fall a perp. from F; then by Caſe 1. right ∠s,
Rad. : *coſ.* A : : *tan.* AF : *tan.* AI,
And AB $\backsim$ AI = BI.
Then coſ. AI : *coſ.* AF : : *coſ.* BI : *coſ.* BF.
As BI and A are of the ſame or different affection, BF is < or > than a quadrant.

Or thus,

Find the ∠ B *by Caſe* 7, *then* S.B : S.AF : : S.A : S.BF, *by Caſe* 4.

II. *Algebraically.*

Let s, d *be the ſines, and* c, e *the coſines of* AB, 47.
AF; m = *coſ.* A, v = *verſ.* A; D = *coſ.* $\overline{AB \backsim AF}$;
rad. = 1. *Then coſ.* FB = $sdm + ce$ = D — sdv.

CASE IX.

Given two *angles* A and F, and the included *ſide* 54.
AF; to find an oppoſite *ſide* FB. 55.

I. *Logarithmically.* 56.

Let fall a perp. from F, then by caſe 3^d right ∠ triangles,
Rad. : *tan.* A : : *coſ.* AF : *cotan.* AFI;
And AFB $\backsim$ AFI = BFI; then,

FIG. *Coſ.* AFI : *coſ.* BFI : : *cotan.* AF : *cotan.* BF.
54. As BFI and A are of the ſame or different affection,
55. BF is $<$ or $>$ than a quadrant.
56.

Or thus,

$$\text{Coſ.}\,\frac{F+A}{2} : \text{coſ.}\,\frac{F\backsim A}{2} :: \tan.\tfrac{1}{2}AF : \tan.\,\frac{AB+BF}{2},$$

$$\text{And S.}\,\frac{F+A}{2} : \text{S.}\,\frac{F\backsim A}{2} :: \tan.\tfrac{1}{2}AF : \tan.\,\frac{AB\backsim BF}{2}.$$

Then $\frac{AB+BF}{2} + \frac{AB\backsim BF}{2} =$ *ſide* oppoſite to the greater angle.

And $\frac{AB+BF}{2} - \frac{AB\backsim BF}{2} =$ *ſide* opp. to the leſſer angle.

Note, $\frac{AF+BF}{2}$ is of the ſame affection as $\frac{F+A}{2}$.

II. *Algebraically.*

47. *Let* a, d, f *be the ſines, and* m, e, g *the coſines of* A, AF *and* F, $\tau =$ *cotan.* A, $t =$ *cotan.* AF, $y =$ *cotan.* F, *radius* $= 1$, then

$$\text{Cotan. BF} = \frac{f\tau + ge}{d} = \frac{f\tau}{d} + gt.$$

$$\text{And cotan. AB} = \frac{ay + me}{d} = \frac{ay}{d} + mt.$$

CASE X.

54. Two *angles* A and F, and the included *ſide* AF,
55. being given; to find the third *angle* B.
56.

I. *Loga-*

I. *Logarithmically.*

Let fall a perp. from F, then by Caſe 3. right angles, FIG.
Rad. : *tan.* A : : *coſ.* AF : *cotan.* AFI, 54.
And $AFB \backsim AFI = BFI$; then 55.
S.AFI : S.BFI : : *coſ.* A : *coſ.* B. 56.

As AFI is $<$ or $>$ than AFB, B and A are of the ſame or different affection.

Otherwiſe,

Find FB *by Caſe* 9, *then* B *by Caſe* 1.

II. *Algebraically.*

Let a, f *be the ſines, and* m, g *the coſines of* A *and* F, $e =$ *coſ.* AF, *rad.* $= 1$; *then* 47.

Coſ. B $= afe - mg$. *Alſo ſee Prop. XLIII.*

CASE XI.

Given the three *ſides*, to find an *angle* A. 54. 55. 56.

I. *Logarithmically.*

Let fall a perp. from F, then

Tan. $\frac{1}{2}$AB : *tan.* $\frac{AF + FB}{2}$: : *tan.* $\frac{AF \backsim FB}{2}$: *tan.* an arch Q.

If $\frac{1}{2}AB > Q$, the perpendicular falls within; if leſs, without.

Then $\frac{1}{2}AB + Q =$ *greater ſegment* AI or BR, and $\frac{1}{2}AB \backsim Q =$ *leſſer ſegment* BI or AR; then

If $AB + FB < 180$, the perpendicular falls neareſt the leſſer ſide.

If $AF + FB > 180$, the perpendicular falls neareſt the greater ſide.

Then by Caſe 4. *right* ∠s, *rad.* : *tan.* AI : : *cotan.* AF : *coſ.* A.

Or thus,

FIG. 54. 55. 56.

$$\text{S.AB} \times \text{S.AF} : \textit{rad. ſquare} : : \text{S.}\,\frac{\text{BF}+\text{AB}-\text{AF}}{2} \times \text{S.}\,\frac{\text{BF}+\text{AF}-\text{AB}}{2} : \text{S. ſquare}\,\tfrac{1}{2}\text{A}.$$

$$\textit{Or}\ \text{S.AB}\times\text{S.AF} : \textit{rad. ſquare} : : \text{S.}\,\frac{\text{BF}+\text{AB}+\text{AF}}{2} \times \text{S.}\,\frac{\text{BF}+\text{AB}+\text{AF}}{2} - \text{BF} : \textit{coſ. ſquare}\,\tfrac{1}{2}\text{A}.$$

See alſo Prop. XLII. and Cor.

II. *Algebraically.*

47. *Let* d, s be the *ſines*, and e, c the *coſines of* AF, AB; $i = $ *coſ.* FB, $z = $ *coſ.* $\overline{\text{AB}+\text{AF}}$, $x = $ *coſ.* $\overline{\text{AB} \backsim \text{AF}}$, *radius* $= 1$,

$$\textit{Then coſ.}\ \text{A} = \frac{i - ce}{ds},\ \textit{and verſ.}\ \text{A} = \frac{x - i}{x - z} \times 2.$$

CASE XII.

54. 55. 56. The three *angles* being given; to find a *ſide* AF.

I. *Logarithmically.*

As A, B are of the ſame or different affection; the perpendicular from F falls within or without.

$$\textit{Cot.}\,\frac{\text{A}+\text{B}}{2} : \textit{tan.}\,\frac{\text{A} \backsim \text{B}}{2} : : \textit{tan.}\,\tfrac{1}{2}\text{F} : \textit{tan. an arch}\,\text{Q}.$$

Then $\tfrac{1}{2}\text{F} + \text{Q} =$ *greater angle at the vertex* AFI or BFR.

And $\tfrac{1}{2}\text{F} \backsim \text{Q} =$ *leſſer angle at the vertex* BFI or AFR;

Then if $\text{A}+\text{B} < 180$, the perpendicular falls neareſt the leſſer ſide or greater angle.

If

If A + B > 180, the perpendicular falls neareſt the greater ſide or leſſer angle. FIG. 54.

Then by Caſe 16. *right angles,* 55.

Tan. AFI : *cotan.* A : : *rad.* : *coſ.* AF. 56.

Or thus,

Let z = ſum, D = difference of A and F.

Then S.A × S.F : *rad. ſquare* : : *coſ.* $\frac{B + Z}{2}$ × *coſ.* $\frac{B \backsim Z}{2}$: *ſine ſquare* $\frac{1}{2}$AF.

Or S.A × S.F : *rad. ſquare* : : *coſ.* $\frac{B + D}{2}$ × *ſoſ.* $\frac{B \backsim D}{2}$: *coſ. ſquare* $\frac{1}{2}$AF.

Alſo ſee Prop. XLIII. and Cor.

II. *Algebraically.*

Let a, f *be the ſines, and* m, g *the coſines of* A *and* F, $n =$ *coſ.* B, $z =$ *coſ.* $\overline{F + A}$, $x =$ *coſ.* $\overline{F \backsim A}$, *rad* = 1. *Then* 47.

$$\text{Cos. AF} = \frac{mg + n}{af}, \text{ and vers. AF} = \frac{n + z}{z - x} \times 2.$$

SCHOLIUM.

Any of the caſes of right-angled triangles is reſolved at one operation; but in obliques, all but the 1, 4, 11 and 12 require two operations, but then they find two things for theſe two operations, without a perpendicular. Let me only add, that the directions in general for performing the arithmetical work in ſpherics, is the ſame as in plain triangles.

FIG. *The ſolution of all the caſes of ſpherical triangles in numbers.*

Right-angled triangles.

CASE I.

79. Given hyp. AF = 78 20 } required the adjacent
angle A = 27 43 } ſide AB.

Rad. —— —— 10.
Coſ A (27 43) —— 9.9470700
Tan. AF (78 20) —— 10.6851149
Tan. AB (76 52) —— 10.6321849

which is leſs than a quadrant, becauſe A and AF are of the ſame affection.

CASE II.

Given hyp. AF = 78 20 } required the oppoſite
angle A = 27 43 } ſide BF.

Rad. —— —— 10.
S.AF (78 20) —— 9.9909338
S.A (27 43) —— 9.6675459
S.BF (27 06) —— 9.6584797

which is leſs than 90, becauſe A is acute.

CASE III.

Given hyp. AF = 78 20 } required the other FIG.
angle A = 27 43 } ∠ F. 79.

Rad. — — 10.
Coſ. AF (78 20) — 9.3058189
Tan. A (27 43) — 9.7204759
Cotan. F (83 56) — 9.0262948

which is acute, becauſe AF and A are of the ſame affection.

CASE IV.

Given the hyp. AF = 78 20 } required the ∠ 80.
a leg AB = 76 52 } adjacent A.

Rad. — — 10.
Cot. AF (78 20) — 9.3148851
Tan. AB (76 52) — 10.6320468
Coſ. A (27 45) — 9.9469319

which is acute, becauſe AB, AF are of the ſame affection.

CASE V.

Given hyp. AF = 78 20 } required the oppoſite 80.
leg AB = 76 52 } ∠ F.

S.AF (78 20) — 9.9909338
Rad. — — 10.
S.AB (76 52) — 9.9884894
S.F (83 56) — 9.9975556

which is acute, becauſe AB is leſs than 90.

CASE VI.

FIG. 80. Given hyp. AF = 78 20 ⎫ required the other leg
leg AB = 76 52 ⎭ BF.

Coſ. AB (76 52) — 9.3564426
Coſ. AF (78 20) — 9.3058189
Rad. —— —— 10.
Coſ. BF (27 08) — 9.9493763

which is leſs than 90, becauſe AB, AF are of one affection.

CASE VII.

81. Given a ſide AB = 117 34 ⎫ required oppoſite
adj. ∠ A = 31 51 ⎭ ſide BF.

Rad. —— —— 10.
S.AB (117 34) — 9.9476655
Tan. A (31 51) — 9.7932560
Tan. FB (28 51) — 9.7409215

which is leſs than 90, becauſe A is acute.

CASE VIII.

81. Given a ſide AB = 117 34 ⎫ required op. ∠ F.
adj. ∠ A = 31 51 ⎭

Rad. —— —— 10.
S.A (31 51) — 9.7223848
Coſ. AB (117 34) — 9.6653749
Coſ. F (104 08) — 9.3877597

which is obtuſe, becauſe AB is greater than 90.

CASE IX.

Given a ſide AB = 117 34 } required hyp. AF. FIG. 81.
adj. ∠ A = 31 51 }

Coſ. A (31 51) — 9.9291289
Rad. —— —— 10.
Tan. AB (117 34) — 10.2822906
Tan. AF (113 55) — 10.3531617

which is greater than 90, becauſe A and AB are of different affection.

CASE X.

Given ſide BF = 28 51 } required ſide adj. AB. 82.
op. ∠ A = 31 51 }

Rad. —— —— 10.
Cot. A (31 51) — 10.2067440
Tan. BF (28 51) — 9.7410662
S.AB (62 28 or 117 32) — 9.9478102

AB may be either leſſer or greater than 90.

CASE XI.

Given ſide BF = 28 51 } required adj. ∠ F. 82.
op. ∠ A = 31 51 }

Coſ. BF (28 51) —— 9.9424476
Coſ. A (31 51) —— 9.9291289
Rad. —— — 10.
S.F (75 53 or 104 7) —— 9.9866813

which may be either acute or obtuſe.

CASE XII.

FIG. 82. Given a ſide BF = 28 51 } required hyp. AF.
op. ∠ A = 31 51

S.A (31 51) —— 9.7223848
S.BF (28 51) —— 9.6835137
Rad. —— —— 10.
S.AF (66 07 or 113 53) —— 9.9611289

which may be either greater or leſs than 90.

CASE XIII.

83. Given, leg AB = 76 52 } required the ∠ A.
leg FB = 27 06

S.AB (76 52) — 9.9884894
Rad. —— —— 10.
Tan. BF (27 06) — 9.7090374
T. op. ∠ A (27 43) — 9.7205480

which is acute, becauſe BF is leſs than 90.

CASE XIV.

83. Given the leg AB = 76 52 } required the hyp. AF.
leg FB = 27 06

Rad. —— —— 10.
Coſ. FB (27 06) — 9.9494938
Coſ. AB (76 52) — 9.3564426
Coſ. AF (78 20) — 9.3059364

which is leſs than a quadrant, becauſe AB, BF are of one affection.

CASE XV.

Given the ∠ A = 31 51, ∠ F = 104 08 } required a leg BF. FIG. 84.

S.F (104 08)	9.9866509
Rad.	10.
Coſ. A (31 51)	9.9291289
Coſ. op. ſide BF (28 51)	9.9424780

which is leſs than 90, becauſe A is acute.

CASE XVI.

Given ∠ A = 31 51, ∠ F = 104 08 } required hyp. AF. 84.

Tan. F (104 08)	10.5989422
Cot. A (31 51)	10.2067440
Rad.	10.
Coſ. AF (113 55)	9.6078018

which is greater than 90, becauſe A, F are of different affection.

Oblique triangles.

CASE I.

FIG. 85.

$$\left.\begin{array}{r} \text{Given AF} = 79\ \ 13 \\ \text{BF} = 62\ \ 42 \\ \angle\text{A} = 50\ \ 12 \end{array}\right\} \text{required } \angle \text{ B.}$$

S.FB (62 42) —— 9.9487147

S.A (50 12) —— 9.8855215

S.AF (79 13) —— 9.9922626

19.8777841

$$\text{S.B} \left\{\begin{array}{l} 58\ \ 08 \text{ or} \\ 151\ \ 52 \end{array}\right\} — 9.9290694$$

which is ambiguous, becauſe A + B, and A + ſup. B, are each of them leſs than 180.

CASE II.

85.

$$\left.\begin{array}{r} \text{Given AF} = 79\ \ 13 \\ \text{BF} = 62\ \ 42 \\ \angle\text{A} = 50\ \ 12 \end{array}\right\} \text{required F.}$$

Draw FI perp. on AB, then

Rad. —— 10.

Coſ. AF (79 13) —— 9.2720635

Tan. A (50 12) —— 10.0792671

Cot. AFI (77 21) —— 9.3513306

Then

Cot. AF (79 13) —— 9.2798009

Coſ. AFI (77 21) —— 9.3404338

Cot. BF (62 42) —— 9.7127662

19.0532000

Coſ. BFI (53 36) —— 9.7733991

which is acute, becauſe BF and A are of the ſame affection. Then AFI ± BFI = F = 130 57 or 23 45, ambiguous.

CASE

CASE III.

FIG. 85.

$$\text{Given}\ \left.\begin{array}{r} AF = 79\ \ 13 \\ BF = 62\ \ 42 \\ \angle A = 50\ \ 12 \end{array}\right\}\ \text{required } AB.$$

Draw FI perp. on AB, then

Rad.	10.
Coſ. A (50 12)	9.8062544
T.AF (79 13)	10.7201991
T.AI (73 26)	10.5264535

Then

Coſ. AF (79 13)	9.2720635
Coſ. AI (73 26)	9.4550441
Coſ. BF (62 42)	9.6614810
	19.1165251
Coſ. BI (45 39)	9.8444616

which is leſs than 90, becauſe BF and A are of one affection. Then AI ∓ BI = AB = 27 47 or 119 5, ambiguous.

CASE IV.

86.

$$\text{Given}\ \left.\begin{array}{r} A = 50\ \ 12 \\ B = 58\ \ 08 \\ BF = 62\ \ 42 \end{array}\right\}\ \text{required } AF.$$

S.A (50 12)	9.8855215
S.BF (62 42)	9.9487147
S.B (58 08)	9.9290504
	19.8777651
S.AF $\left\{\begin{array}{cc} 79 & 12 \\ 100 & 48 \end{array}\right\}$	9.9922436

AF is ambiguous, becauſe BF + AF, and BF + ſup. AF, are both leſs than 180.

CASE

CASE V.

FIG. 86.

Given A = 50 12, B = 58 08, BF = 62 42 } required AB.

Draw FI perp. to AB, then

Rad.	—— ——	10.
T.BF	(62 42) ——	10.2872338
Coſ. B	(58 08) ——	9.7225881
T.BI	(45 39) ——	10.0098219

Then

Cot. B	(58 08) ——	9.7935378
Cot. A	(50 12) ——	9.9207329
S.BI	(45 39) ——	9.8543564
		19.7750893
S.AI	{ 73 25 106 35 } ——	9.9815515

then AB (= AI + BI) = 119 04 or 152 14, ambiguous.

CASE VI.

86.

Given A = 50 12, B = 58 08, BF = 62.42 } required F.

Draw FI perp. on AB; then

Rad.	—— ——	10.
Tan. B	(58 08) —	10.2064622
Coſ. BF	(62 42) —	9.6614810
Cot. BFI	(53 35) —	9.8679432

Then

Then

FIG. 86.

Coſ. B (58 08)	—	9.7225881
S.BFI (53 35)	—	9.9056454
Coſ. A (50 12)	—	9.8062544
		19.7118998
S.AFI (77 21 or 102 39)	—	9.9893117

Then F (= BFI + AFI) = 130 56 or 156 14, ambiguous.

CASE VII.

87.

Given AF = 79 13
AB = 119 05
A = 50 12
} required B.

Draw FI perp. on AB; then

Rad.	—	—	10.
Coſ. A (50 12)		—	9.8062544
T. AF (79 13)		—	10.7201991
T. AI (73 26)		—	10.5264535

Then AB — AI = IB = 45 39.

S.AI (73 26)	—	9.9815870
S.BI (45 39)	—	9.8543564
Cot. A (50 12)	—	9.9207329
		19.7750893
Cot. B (58 08)	—	9.7935023

which is acute, becauſe B is of the ſame affection as A.

CASE VIII.

FIG. 87.

Given AF = 79 13
AB = 119 05 } required FB.
A = 50 12

Draw FI perp. to AB; then

Rad. —— —— 10.
Coſ. A (50 12) —— 9.8062544
T. AF (79 13) —— 10.7201991
T. AI (73 26) —— 10.5264535

Then AB — AI = BI = 45 39.

And coſ. AI (73 26) —— 9.4550441
Coſ. AF (79 13) —— 9.2720635
Coſ. BI (45 39) —— 9.8445018
19.1165653
Coſ. BF (62 40) —— 9.6615212

which is acute, becauſe BI and A are of the ſame affection.

CASE IX.

88.

Given A = 50 12
AFB = 130 56 } required FB.
AF = 79 13

Draw FI perp. to AB; then

Rad. —— —— 10.
T. A (50 12) —— 10.0792671
Coſ. AF (79 13) —— 9.2720635
Cot. AFI (77 21) —— 9.3513306

Then AFB — AFI = BFI = 53 35.

Coſ. AFI (77 21) —— 9.3404338
Coſ. BFI (53 35) —— 9.7735327
Cot. AF (79 13) —— 9.2798009
19.0533336
Cot. BF (62 42) —— 9.7128998

which is leſs than 90, becauſe BFI and A are of the ſame affection.

CASE

CASE X.

FIG. 88.

Given A = 50 12
AFB = 130 56 } required ∠ B.
AF = 79 13

Draw FI perp. AB; then

Rad. —— —— 10.
T. A (50 12) —— 10.0792671
Coſ. AF (79 13) —— 9.2720635
Cot. AFI (77 21) —— 9.3513306

Then AFB — AFI = BFI = 53 35.

S.AFI (77 21) —— 9.9893279
S.BFI (53 35) —— 9.9056454
Coſ. A (50 12) —— 9.8062544
19.7118998
Coſ. B (58 08) —— 9.7225719

which is acute, becauſe A is acute, and AFI leſs than AFB.

CASE XI.

89.

Given AF = 79 13
BF = 62 42 } required A.
AB = 119 05

Draw FI perp. to AB; then

T. $\frac{1}{2}$ AB (59 32$\frac{1}{2}$) —— 10.2305741
T. $\frac{AF+BF}{2}$ (70 57$\frac{1}{2}$) —— 10.4620031
T. $\frac{AF-BF}{2}$ (8 15$\frac{1}{2}$) —— 9.1617917
19.6237948
Tan. Q = 13 54 —— 9.3932207

Then $\frac{1}{2}$ AB + Q = AI = 73 26$\frac{1}{2}$.

And rad. —— 10.
Tan. AI (73 26$\frac{1}{2}$) —— 10.5267740
Cot. AF (79 13) —— 9.2798009
Coſ. A (50 10) —— 9.8065749

CASE XII.

FIG. 90.

Given A = 50 12
B = 58 08 } required AF.
F = 130 56

Draw FI perp. to AB; then

Cot. $\frac{1}{2}.\overline{A+B}$ (54 10) —— 9.8586019
Tan. $\frac{1}{2}.\overline{B-A}$ (3 58) —— 8.8409977
Tan. $\frac{1}{2}$ F (65 28) —— 10.3406267
19.1816244
Tan. Q (11 53) —— 9.3230225
Then $\frac{1}{2}$ F + Q = 77 21 = AFI.
And tan. AFI (77 21) —— 10.6488941
Cot. A (50 12) —— 9.9207329
Rad. —— —— 10.
Cos. AF (79 13) —— 9.2718388

The uſe of the following TABLES *of natural and artificial ſines and tangents.*

To find the ſine, tangent, &c. of any number of degrees and minutes; and the contrary.

RULE.

FIND the degrees at the head or foot of the table, and the minutes under or above the degrees, on the ſame ſide of the leaf; againſt which, you have the ſine or tangent in its proper column; that is, under the words *ſine*, *tang.* if the degrees are at top; and above the words *ſine*, *tang.* if the degrees are at the bottom of the table.

And on the contrary, having a ſine, or tangent, given; find it, or the neareſt to it, in its proper column, under (or above) the word *ſine* or *tang.* and you have the degrees at top (or bottom) of the table, and the minutes on the ſide, under (or above) theſe degrees.

If more exactneſs be required, as when there are odd ſeconds, you muſt find a proportional part of the difference of the next greater and leſſer ſine, or tangent, &c. which is to be added or ſubtracted, according as the ſine or tangent increaſes or decreaſes. And the like method with proportional parts muſt be uſed to find the odd ſeconds, when a ſine or tangent cannot be exactly found.

Note, *In theſe tables, if you find not the entire number againſt the minutes, as directed; look higher up for the reſt of it.*

Ex. 1.

Suppoſe an arch of 41° 27′ *be given*; the natural ſine will be .6619662, the nat. coſine .7495337, the nat. tangent .8831707, the nat. cotangent 1.1322839.

Likewiſe the log. ſine of the ſame arch is 9.8208358, log. coſine 9.8747912, log. tangent 9.9460447, log. cotan. 10.0539553.

And if an arch of 138° 33′ be propoſed, you will have the ſame ſine, tangent, &c.

Ex. 2.

To find the ſine, &c. of 68° 35′. The nat. ſine is .9309496, the log ſine is 9.9689262, and ſo of the reſt; and the ſame for 111° 25′.

Ex. 3.

What is the nat. ſine of 38° 12′ 25″? S. of 38° 12′ is .6184084; and S. 38° 13′ is .6186370; the difference is 2286; then ſay, as diff. arches 60″: diff. ſines 2286 : : 25″ : 952; which added to

$$\begin{array}{r} .6184084 \\ 952 \\ \hline \end{array}$$

gives .6185036 for the nat. ſine of 38° 12′ 25″.

Ex. 4.

What arch belongs to the nat. ſine .4586496? The arch required is either 27° 18′, or 152° 42′.

Ex. 5.

What is the arch belonging to the log. cotangent 9.5023625? The neareſt is 72° 22′, or 107° 38′. But

But if more exactneſs be required, you will find
Log. cotan. 72° 21′ is 9.5026721
Log. cotan. 72 22 is 9.5022347
the difference - 4374

And the difference between 9.5026721 and 9.5023625 is 3096; then ſay, as 4374 : 60″ :: 3096 : 42″, and then the arch is 72° 21′ 42″.

Ex. 6.

To find the nat. or log. ſine and tangent of any number of ſeconds under 60, *as ſuppoſe of* 37.

The nat. ſine or tangent of 1 ſecond is .000004848, which is nearly the ſame as the length of the arch of one ſecond; and the log. ſine or tangent of 1 ſecond is 4.6855749; therefore 37 × 000004848 = .0001794 is the nat. ſine or tangent of 37 ſeconds.

And to —— —— 4.6855749
Add log. 37 —— —— 1.5682017

The log. ſine or tan. of 37″ 6.2537766

To find the ſecant or verſed ſine.

This is eaſily done by the notes prefixed to the tables.

Ex. 1.

What is the nat. ſecant of 13° 24′?

The comp. of 13° 24′ is - 76° 36′
Half the comp. - - - - 38 18

Tan. 13° 24′ - - - - - - .2382336
Tan. 38 18 (half the comp.) - .7897524

The ſecant of 13° 24′ - - - 1.0279860

Ex. 2.

Required the nat. verſed ſine of 65° 45′ ?

Rad. - - -	1.0000000
Coſ. 65° 45′ -	0.4107189
Verſ. 65 45 -	.5892811

But if the arch is greater than 90°, you muſt add the coſine to 1.

Ex. 3.

What is the log. ſecant of 72° 14′?

	20.
Coſ. 72° 14′ - -	9.4845010
Secant of 72° 14′	10.5154990

Ex. 4.

To find the log. verſed ſine of 37° 53′?

Half the arch is - - - -	18° 56′½
S. 18° 56′ - - -	9.5111716
S. 18 57 - - -	9.5115397
S. 18 56½ - -	19.0227113
Subtr. -	9.6989700
Verſ. 37 53 - -	9.3237413

Note, Any proportion with log. ſines, &c. is wrought according to Ex. 8. of the uſe of logarithms; but with the nat. ſines, tangents, &c. you muſt multiply the 2d and 3d terms together, and divide by the firſt term, to get the fourth.

The nature and uſe of the TABLE *of Logarithms.*

THIS table contains the logarithms of all numbers in their natural order, from 1 to 10000. The firſt column contains the number, and the ſecond ſhews the correſponding logarithm.

The firſt figure (or that on the left-hand of the point) in any logarithm, is called the *index*, or *characteriſtic*; which, if it be affirmative, always ſhews how many places of integers the number has, being one more than the index. If the index be negative, it denotes what place of decimals the firſt figure of the number ſtands in; ſo the index always ſhews the diſtance of the firſt figure from the place of units. And though the index in this table is put along with the logarithm, yet that index muſt be varied, by the foregoing rule, according to the number of integers, or decimals in the number given, whatever number of places the whole conſiſts of. So that the index will be + or —, according as the number is an integer or fraction; whilſt the logarithm (without the index) is to be eſteemed affirmative.

Thus the log. of 301700 is	5.4795753
30170	4.4795753
3017	3.4795753
301.7	2.4795753
3.017	0.4795753
.03017	—2.4795753 &c.

When the number exceeds not four places of figures, their logarithms are had from the table by inſpection; but for larger numbers, where great exactneſs is required, the log muſt be found by the method of taking a proportional part. But the uſe

of

of the table will be ſhewn by the following examples.

Ex. 1.

A number being given leſs than 10000, *to find its logarithm.*

Seek the number in the column *Num.* and adjoining to it, in the column *Logar.* you have the logarithm of it.

Thus if the given number is 824, its log. is 2.9159272.

And if the number is 3307, its log. is 3.5194342.

Note, If the entire log. is not ſet againſt the number, you will find the reſt of it above.

Ex. 2.

A log. being given, to find the number anſwering.

Find the log. or the neareſt thereto, in the column *Logar.* and againſt it in the column *Num.* is the number thereof; and the index will ſhew how many places of integers it conſiſts of.

As if the log. be 1.8976271, the number is 79.

And if the log. be 3.9812507, the neareſt in the table is 3.9812295, whoſe number is 9577.

And if the log. be 5.6631353, the number is 460400.

Ex. 3.

To find the log. of a number of more than four places.

You muſt find a proportional part to be added to the log. of the next leſſer number. *Or thus,* take the difference of the logarithms of the next greater and leſſer numbers; and multiply it by the figures you have above four; and cut off ſo many places to the right-hand; the reſt muſt be added to the log. of the next leſſer number.

Let

Let the number be 705436. Then
N. 7054 —— log. 8484355
N. 7055 —— log. 8484970

The diff. 915
Multiply by 36

3690
1845

221|40

To .8484355
Add 221

8484576

Therefore 5.8484576 is the log. of 705436.

Ex. 4.

To find a number anſwering to any given log. which is not in the table.

Find the neareſt number, by Ex. 2, which will do for common uſe; but where more exactneſs is required, find a proportional part, *thus:* Take the difference of the given log. and the next leſs, that you can find in the table; likewiſe take the difference between the next leſſer and next greater log.; divide the firſt difference by the ſecond, and annex the quotient to the end of the number of the next leſſer log., and make it a proper number of integers, agreeable to the index.

Let the log. be 5.5386715 | leſſer log. | 5.5385737
given log. | 5.5386715

1 diff. 978

Leſſer log. 5.5385737 - N. 3456
Greater log. 5.5386994 - N. 3457

2d diff. 1257

1257)

1257)978.0(.778, the quotient.
8799 the number 3456.778

9810
8799

1011
1005

6

And 345677.8 is the number of the log. 5.5386715.

Ex. 5.

To multiply ſeveral numbers together.

This is done by the *addition* of their logarithms, thus: Set down the logs. of the numbers, and add them together; and when you come at the *indexes*, add the affirmatives, and what you carry, into one ſum; and the negatives, if any, into another; and take the difference of them, to which put the ſign of the greater; and this is the index, for the ſum of the logs; then find the number belonging to that log., and that will be the product.

Multiply 37 by 519.
N. 37 —— log. 1.5682017
N. 519 —— log. 2.7151674

Prod. 19203 —— 4.2833691

Multiply 7 and 50430 and .00681 together.
N. 7 —— log. 0.8450980
N. 50430 — log. 4.7026890
N. .00681 — log. —3.8331471

Product 2404 —— 3.3809341

Ex.

Ex. 6.

To divide one number by another.

This is done by *ſubtraction* of logarithms; ſubtract the log. of the diviſor from the log. of the dividend, for the log. of the quotient, borrowing 1 when the ſubtrahend is greater. As to the indices, change the ſign of the ſubtrahend, and follow the rule of addition, in Ex. 5.

Divide 3 by 8.
N. 3 —— log. 0.4771212
N. 8 —— log. 0.9030900

Quotient .375 — 1.5740312

Divide 3450 —— log. 3.5378191
by 23 —— log. 1.3617278

Quot. 150 —— 2.1760913

Divide .00734 —— log. —3.8656961
by 21.02 —— log. 1.3226327

Quot. .0003492 —— —4.5430634

Ex. 7.

To involve a given number to any power.

Multiplication of logarithms does this; for if you multiply the log. of the number by the index of the power, the product is the log. of the power. *Note*, When you come at the index of the log. if it be negative, take away from its product what you carried, and ſet down the remaining index negative.

To find the cube of 315 - log. 2.4983105
3

the cube 31250000 near, 7.4949315

To find the 5th power of .0837 - log. —2.9227255
5

5th power .000004108 —6.6136275

Ex.

Ex. 8.

To extract any root of a given number.

This is performed by *diviſion* of logarithms, thus: Divide the log. of the number by the index of the root: the quotient is the log. of the root. If the index is negative, you muſt increaſe it till it is diviſible (if not ſo already) and increaſe the log. alſo by the ſame number, before you begin your diviſion.

What is the ſquare root of 6889?

Index 2) log. 3.8381562
Root 83. 1.9190781

What is the 5th root of .007067?

5)—3.8492351 = —5.
——— +2.8492351
—1.5698470

Its number .3714, is the root.

Ex. 9.

To reſolve a queſtion by the direct rule of three.

Set the logs. of the numbers in order, one under another, and add the logs. of the 2d and 3d terms together, and from the ſum ſubtract the log. of the firſt; the remainder is the log. of the fourth.

The indirect or inverſe rule of three is wrought the ſame way, putting the third term in the place of the firſt.

If 317 Cwt. give 53 lb. what will 29¼ Cwt. give?

317 log. 2.5010593
53 1.7242759
29.25 1.4661259

3.1904018

4.89 0.6893425

Or 4 l. — 17 s. — 9½ d.

Ex.

Ex. 10.

Two numbers being given; to find a 3d, 4th, 5th, &c. in continual proportion.

Find as many logarithms in continual arithmetic proportion to the logarithms of the numbers given, thus:

Take the log. of the firſt term from the log. of the ſecond; and multiply the remainder by 2, 3, 4, &c. for the 3d, 4th, 5th, &c. proportional; (that is, by 1 leſs than the number denoting the proportional ſought) and to the product add the log. of the firſt term, gives the log. of the term required.

To find a 6th proportional to 37 and 47.

	47 log.	1.6720979
	37 log.	1.5682017
	Rem.	0.1038962
	Multiply	5
		0.5194810
	Add	1.5682017
The term required	122.4	2.0876827

To find a 4th proportional to 27 and 7.

	7 its log.	0.8450980
	27 log.	1.4313638
		—1.4137342
		3
		—2.2412026
		0.8450980
The term ſought	.122	—1.0863006

Ex. 11.

To find any number of mean proportional between two numbers given.

This is done by finding as many arithmetic means, between the logarithms of the numbers given, thus:

Take

Take the log. of the firſt number from the log. of the ſecond, and divide the remainder by 1 more than the number of mean proportionals: then the quotient added to the log. of the firſt term, gives the log. of the firſt mean; and continually adding the ſaid quotient, you get the logs. of the 2d, 3d, 4th mean, &c.

To find 5 means between 16 and 7093.

7093 —— log. 3.8508300
16 —— log. 1.2041200
6) 2.6467100
Quotient 0.4411183
Add 1.2041200
1 mean 44.18 . . . 1.6452383
Add .4411183
2 mean 122 . . . . 2.0863566
Add 4411183
3 mean 336.9 . . . . 2.5274749, &c.

Ex. 12.

To convert the common logs. into Naper's or hyperbolic logarithms.

Multiply the common log. by 2.30258509, the product is the hyp. log.

What is Naper's log. of 8?

Com. log. of 8 is 0.9030900
2.30258509
812781
451545
722472
451545
180618
270927
180618
2.0794415 the hyp. log. of 8.

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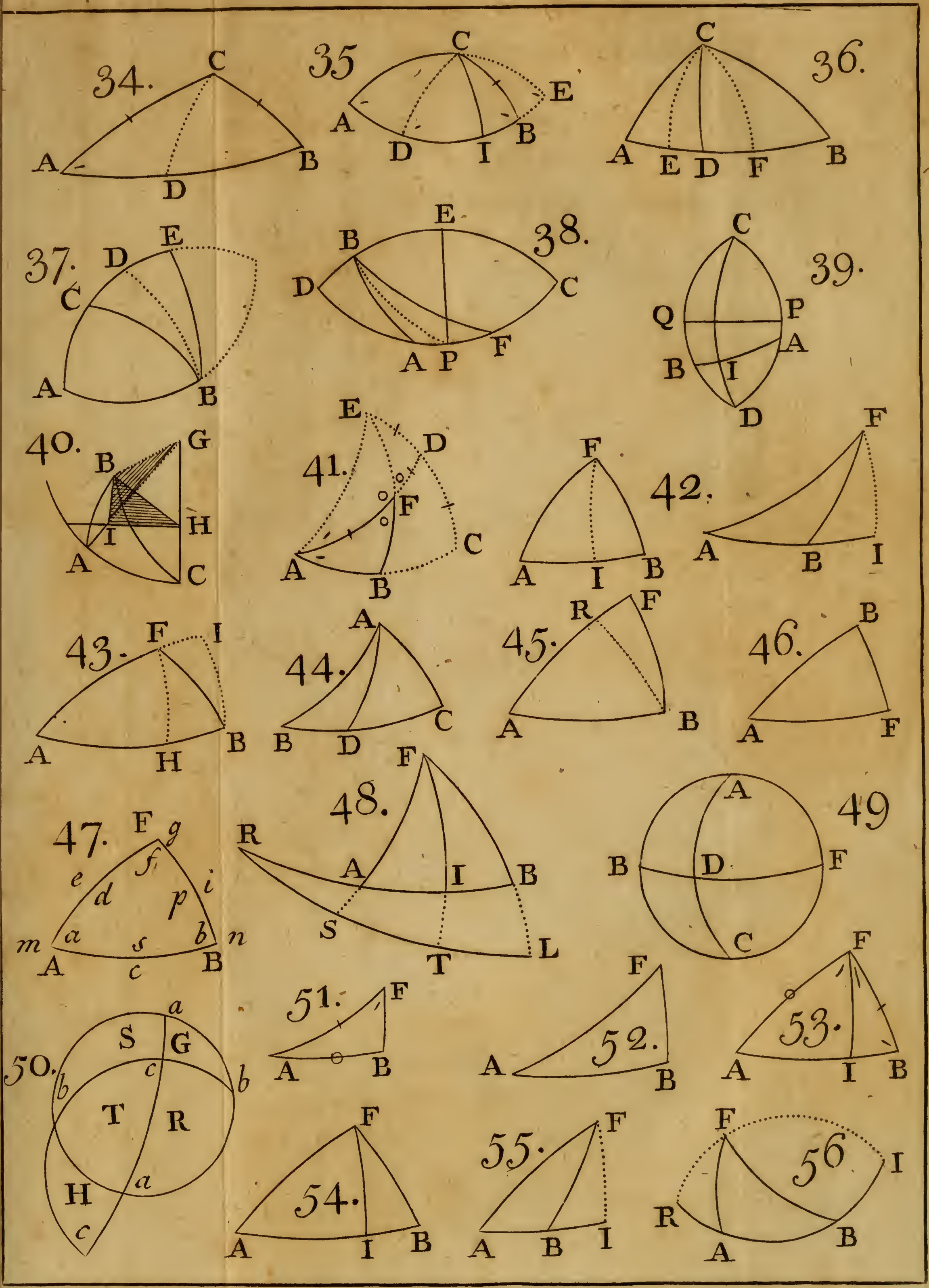
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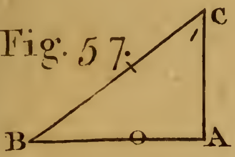
Pl. VI. *the End.*

Fig. 57.



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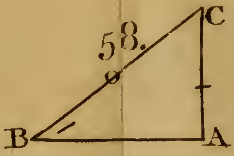

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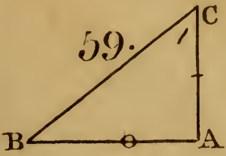

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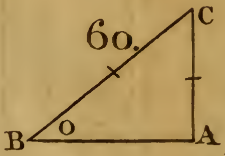

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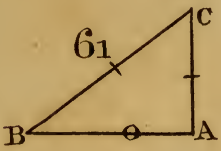

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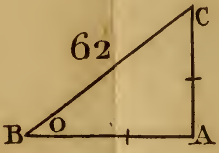

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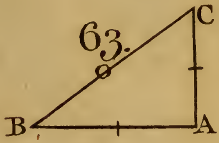

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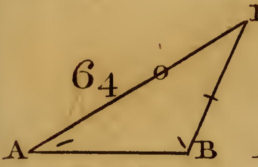

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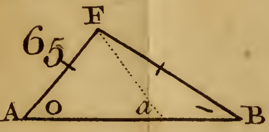

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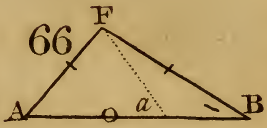

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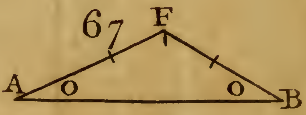

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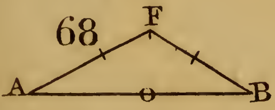

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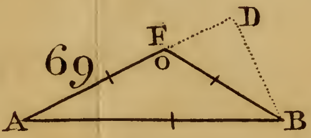

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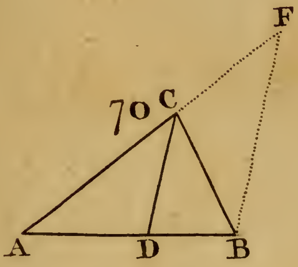

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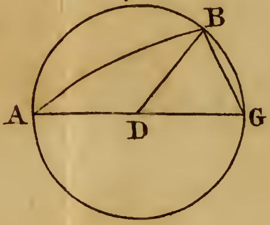

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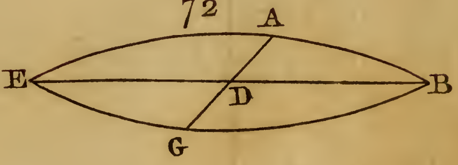

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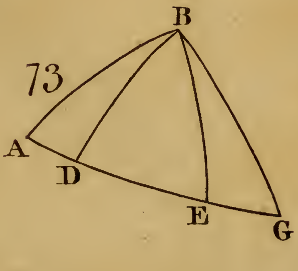

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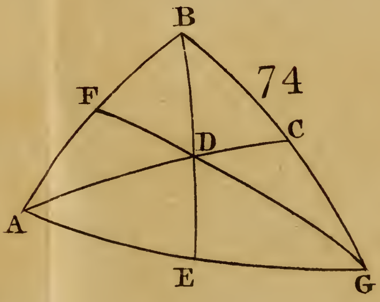

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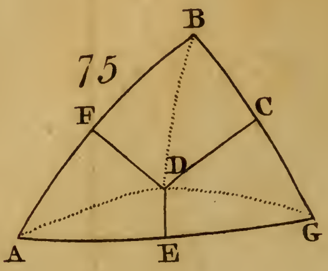
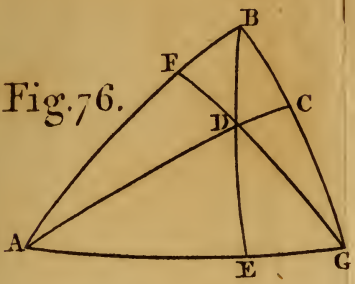


Pl. VII. *the End.*

Fig. 76.



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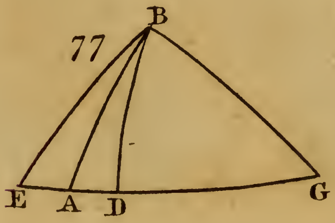

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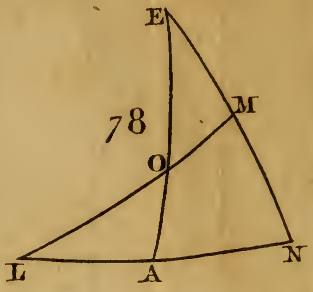

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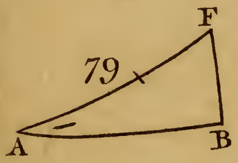

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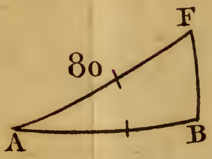

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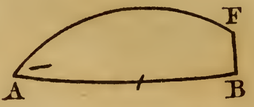

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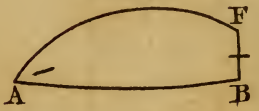

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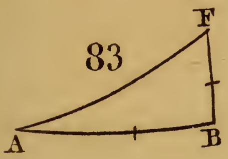

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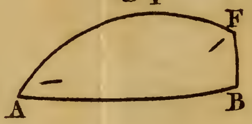

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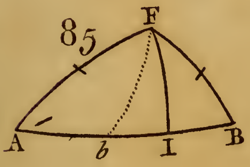

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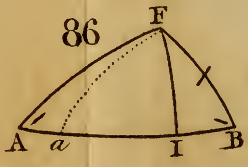

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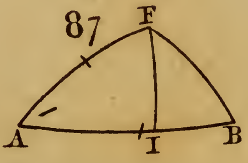

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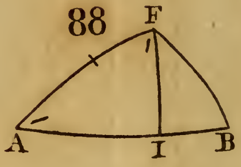

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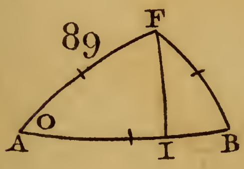
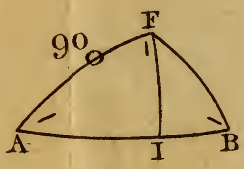


90



Pl. VIII. *the End*

A TABLE OF Natural Sines and Tangents, TO EVERY Degree and Minute of the Quadrant;

The Radius being 1.0000000;

From whence the Secants and Verſed Sines are eaſily found.

For,

Note 1. *Verſed Sine* $= 1 \mp$ *Coſine.*

2. *Secant* $=$ *Tangent* $+$ *Tan.* $\frac{1}{2}$ *Complement.*

If half the complement comes to half an odd minute; take a mean between the tangents on each ſide, or half the ſum of the next greater and leſſer tangents.

Min.	0	*Degrees*		179	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.0000000	1.0000000	0.0000000	Infinite	60
1	.0002909	·9999999	.0002909	3437·74667	59
2	5818	98	5818	1718.87319	58
3	8727	96	8727	1145·91530	57
4	.0011636	93	.0011636	859·43630	56
5	4544	89	4544	687·54887	55
6	7453	·9999985	7453	572·95721	54
7	.0020362	79	.0020362	491.10600	53
8	3271	73	3271	429·71757	52
9	6180	66	6180	381.97099	51
10	9089	58	9089	343·77371	50
11	.0031998	49	.0031998	312.52137	49
12	4906	39	4907	286.47773	48
13	7815	28	7816	264.44080	47
14	.0040724	17	.0040725	245·55198	46
15	3633	05	3633	229.18166	45
16	6542	.9999892	6542	214.85762	44
17	9451	78	9451	202.21875	43
18	.0052360	63	.0052360	190.98419	42
19	5268	47	5269	180.93220	41
20	8177	31	8178	171.88540	40
21	.0061086	13	.0061087	163.70019	39
22	3995	·9999795	3996	156.25908	38
23	6904	76	6905	149·46502	37
24	9813	56	9814	143·23712	36
25	.0072721	36	.0072723	137·50745	35
26	5630	14	5632	132.21851	34
27	8539	·9999692	8541	127·32134	33
28	.0081448	68	.0081450	122.77396	32
29	4357	44	4360	118.54018	31
30	7265	19	7269	114·58865	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	90	*Degrees*		89	

Min.	0	Degrees		179	
	Sine	Coſine	Tangent	Cotangent	
30	.0087265	.9999619	.0087269	114.58805	30
31	.0090174	.9999593	.0090178	110.89205	29
32	3083	66	3087	107.42648	28
33	5992	39	5996	104.17094	27
34	8900	11	8905	101.10690	26
35	.0101809	.9999482	.0101814	98.217943	25
36	4718	52	4724	95.489475	24
37	7627	21	7633	92.908487	23
38	.0110535	.9999389	.0110542	90.463336	22
39	3444	56	3451	88.143572	21
40	6353	23	6361	85.939791	20
41	9261	.9999289	9270	83.843507	19
42	.0122170	54	.0122179	81.847041	18
43	5079	18	5088	79.943430	17
44	7987	.9999181	7998	78.126342	16
45	.0130896	43	.0130907	76.390009	15
46	3805	04	3817	74.729165	14
47	6713	.9999065	6726	73.138991	13
48	9622	25	9635	71.615070	12
49	.0142530	.9998984	.0142545	70.153346	11
50	5439	42	5454	68.750087	10
51	8348	.9998899	8364	67.401854	9
52	.0151256	55	.0151273	66.105473	8
53	4165	11	4183	64.858008	7
54	7073	.9998766	7093	63.656741	6
55	9982	720	.0160002	62.499154	5
56	.0162890	673	.0162912	61.382905	4
57	5799	625	5821	60.305820	3
58	8707	576	8731	59.265872	2
59	.0171616	527	.0171641	58.261174	1
60	4524	477	4551	57.289962	0
	Coſine	Sine	Cotang.	Tangent	Min.
	90	Degrees		89	

Min.	1		Degrees		178
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.0174524	·9998477	.0174551	57·289962	60
1	7432	426	7460	56.350590	59
2	.0180341	374	.0180370	55·441517	58
3	3249	321	3280	54·561300	57
4	6158	267	6190	53·708587	56
5	9066	212	9100	52.882109	55
6	.0191974	157	.0192010	080673	54
7	4883	101	4920	51.303157	53
8	7791	044	7830	50.548506	52
9	.0200699	·9997986	.0200740	49.815726	51
10	3608	927	3650	103881	50
11	6516	867	6560	48.412084	49
12	9424	806	9470	47·739501	48
13	.0212332	745	.0212380	085343	47
14	5241	683	5291	46.448862	46
15	8149	620	8201	45·829351	45
16	.0221057	556	.0221111	226141	44
17	3965	491	4021	44.638596	43
18	6873	425	6932	066113	42
19	9781	359	9842	43·508122	41
20	.0232690	292	.0232753	42.964077	40
21	5598	224	5663	433464	39
22	8506	155	8574	41.915790	38
23	.0241414	085	.0241484	410588	37
24	4322	014	4395	40.917412	36
25	7230	·9996943	7305	435837	35
26	.0250138	871	.0250216	39·965460	34
27	3046	798	3127	505895	33
28	5954	724	6038	056771	32
29	8862	649	8948	38.617738	31
30	.0261769	573	.0261859	188459	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	91		*Degrees*		88

Min.	1	Degrees		178	
	Sine	Coſine	Tangent	Cotangent	
30	.0261769	·9996573	.0261859	38.188459	30
31	4677	496	4770	37·768613	29
32	7585	419	7681	357892	28
33	.0270493	341	.0270592	36.956001	27
34	3401	262	3503	562659	26
35	6309	182	6414	177596	25
36	9216	101	9325	35·800553	24
37	.0282124	019	.0282236	431282	23
38	5032	·9995936	5148	069546	22
39	7940	853	8059	34·715115	21
40	.0290847	769	.0290970	367771	20
41	3755	684	3882	027303	19
42	6662	598	6793	33·693509	18
43	9570	511	9705	366194	17
44	.0302478	424	.0302616	045173	16
45	5385	336	5528	32·730264	15
46	8293	247	8439	421295	14
47	.0311200	157	.0311351	118099	13
48	4108	066	4263	31.820516	12
49	7015	·9994974	7174	528392	11
50	9922	881	.0320086	241577	10
51	.0322830	788	2998	30·959928	9
52	5737	694	5910	683307	8
53	8644	599	8822	411580	7
54	.0331552	503	.0331734	144619	6
55	4459	406	4646	29.882299	5
56	7366	308	7558	624499	4
57	.0340273	209	.0340471	371106	3
58	3181	109	3383	122005	2
59	6088	009	6295	28.877089	1
60	8995	·9993908	9208	636253	0
	Coſine	Sine	Cotang.	Tangent	Min.
	91	Degrees		88	

Min.	2	Degrees		177	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.0348995	.9993908	.0349208	28.636253	60
1	.0351902	806	.0352120	399397	59
2	4809	703	5033	166422	58
3	7716	599	7945	27.937233	57
4	.0360623	495	.0360858	711740	56
5	3530	390	3771	489853	55
6	6437	284	6683	271486	54
7	9344	177	9596	056557	53
8	.0372251	069	.0372509	26.844984	52
9	5158	.9992960	5422	636690	51
10	8065	851	8335	431600	50
11	.0380971	740	.0381248	229638	49
12	3878	629	4161	030736	48
13	6785	517	7074	25.834823	47
14	9691	404	9988	641832	46
15	.0392598	290	.0392901	451700	45
16	5505	175	5814	264361	44
17	8411	060	8728	079757	43
18	.0401318	.9991944	.0401641	24.897826	42
19	4224	.827	4555	718512	41
20	7131	.709	7469	541758	40
21	.0410037	590	.0410383	367509	39
22	2944	470	3296	195714	38
23	5850	349	6210	026320	37
24	8757	228	9124	23.859277	36
25	.0421663	106	.0422038	694537	35
26	4569	.9990983	4952	532052	34
27	7475	859	7866	371777	33
28	.0430382	734	.0430781	213666	32
29	3288	608	3695	057677	31
30	6194	482	6609	22.903765	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	92	Degrees		87	

Min.	2	Degrees		177	
	Sine	Coſine	Tangent	Cotangent	
30	.0436194	.9990482	.0436609	22.903765	30
31	9100	355	9524	751892	29
32	.0442006	227	.0442438	602015	28
33	4912	098	5353	454096	27
34	7818	.9989968	8268	308097	26
35	.0450724	837	.0451182	163980	25
36	3630	705	4097	021710	24
37	6536	573	7012	21.881251	23
38	9442	440	9927	742569	22
39	.0462347	306	.0462842	605630	21
40	5253	171	5757	470401	20
41	8159	035	8673	336851	19
42	.0471064	.9988898	.0471588	204949	18
43	3970	761	4503	074664	17
44	6876	623	7419	20.945966	16
45	9781	484	.0480334	818828	15
46	.0482687	344	3250	693220	14
47	5592	203	6166	569115	13
48	8498	061	9082	446486	12
49	.0491403	.9987918	.0491997	325307	11
50	4308	775	4913	205553	10
51	7214	631	7829	087199	9
52	.0500119	486	.0500746	19.970219	8
53	3024	340	3662	854591	7
54	5929	193	6578	740291	6
55	8835	045	9495	627296	5
56	.0511740	.9986897	.0512411	515584	4
57	4645	748	5328	405133	3
58	7550	598	8244	295922	2
59	.0520455	447	.0521161	187930	1
60	3360	295	4078	081137	0
	Coſine	Sine	Cotang.	Tangent	Min.
	92	Degrees		87	

NATURAL SINES

Min.	3	*Degrees*		176	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.0523360	.9986295	.0524078	19.081137	60
1	6264	142	6995	18.975523	59
2	9169	.9985989	9912	871068	58
3	.0532074	835	.0532829	767754	57
4	4979	680	5746	665562	56
5	7883	524	8663	564473	55
6	.0540788	367	.0541581	464471	54
7	3693	209	4498	365537	53
8	6597	050	7416	267654	52
9	9502	.9984891	.0550333	170807	51
10	.0552406	731	3251	074977	50
11	5311	570	6169	17.980150	49
12	8215	408	9087	886310	48
13	.0561119	245	.0562005	793442	47
14	4024	081	4923	701529	46
15	6928	.9983916	7841	610559	45
16	9832	751	.0570759	520516	44
17	.0572736	585	3678	431385	43
18	5640	418	6596	343155	42
19	8544	250	9515	255809	41
20	.0581448	081	.0582434	169337	40
21	4352	.9982911	5352	083723	39
22	7256	741	8271	16.998957	38
23	.0590160	570	.0591190	915025	37
24	3064	398	4109	831915	36
25	5967	225	7029	749614	35
26	8871	051	9948	668112	34
27	.0601775	.9981876	.0602867	587396	33
28	4678	701	5787	507455	32
29	7582	525	8706	428279	31
30	.0610485	348	.0611626	349856	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	93	*Degrees*		86	

Min.	3	Degrees		176	
	Sine	Coſine	Tangent	Cotangent	
30	.0610485	.9981348	.0611626	16.349856	30
31	3389	170	4546	272174	29
32	6292	.9980991	7466	195225	28
33	9196	811	.0620386	118998	27
34	.0622099	630	3306	043482	26
35	5002	449	6226	15.968667	25
36	7905	267	9147	894545	24
37	.0630808	084	.0632067	821104	23
38	3711	.9979900	4988	748337	22
39	6614	715	7908	676233	21
40	9517	529	.0640829	604784	20
41	.0642420	343	3750	15.533981	19
42	5323	156	6671	463814	18
43	8226	.9978968	9592	394276	17
44	.0651129	779	.0652513	325358	16
45	4031	589	5435	257052	15
46	6934	398	8356	189349	14
47	9836	206	.0661278	122242	13
48	.0662739	014	4199	055723	12
49	5641	.9977821	7121	14.989784	11
50	8544	627	.0670043	924417	10
51	.0671446	432	2965	859615	9
52	4348	236	5887	795372	8
53	7251	039	8809	731679	7
54	.0680153	.9976842	.0681732	668529	6
55	3055	644	4654	605916	5
56	5957	445	7577	14.543833	4
57	8859	245	.0690499	482273	3
58	.0691761	044	3422	421230	2
59	4663	.9975842	6345	360696	1
60	7565	640	9268	300666	0
	Coſine	Sine	Cotang.	Tangent	Min.
	93	Degrees		86	

Min.	4	Degrees		175	
	Sine	*Cosine*	*Tangent*	*Cotangent*	
0	.0697565	.9975640	.0699268	14.300666	60
1	.0700466	437	.0702191	14.241134	59
2	3368	233	5115	14.182092	58
3	6270	028	8038	23536	57
4	9171	.9974822	.0710961	14.065459	56
5	.0712073	615	3885	07856	55
6	4974	407	6809	13.950719	54
7	7876	109	9733	13.894045	53
8	.0720777	.9973990	.0722657	37827	52
9	3678	780	5581	13.782068	51
10	6580	569	8505	26738	50
11	9481	357	.0731430	13.671856	49
12	.0732382	144	4354	17409	48
13	5283	.9972931	7279	13.563391	47
14	8184	717	.0740203	09799	46
15	.0741085	502	3128	13.456625	45
16	3986	286	6053	03867	44
17	6887	069	8979	13.351518	43
18	9787	.9971851	.0751904	13.299574	42
19	.0752688	632	4829	48031	41
20	5589	413	7755	13.196883	40
21	8489	193	.0760680	46127	39
22	.0761390	.9970972	3606	13.095757	38
23	4290	750	6532	45769	37
24	7190	527	9458	12.996160	36
25	.0770091	303	.0772384	46924	35
26	2991	079	5311	12.898058	34
27	5891	.9969854	8237	49557	33
28	8791	628	.0781164	01417	32
29	.0781691	401	4090	12.753634	31
30	4591	173	7017	06205	30
	Cosine	*Sine*	*Cotang.*	*Tangent*	
	94	Degrees		85	Min.

Min.	4	Degrees		175	
	Sine	Coſine	Tangent	Cotangent	
30	.0784591	.9969173	.0787017	12.706205	30
31	7491	.9968944	9944	12.659125	29
32	.0790391	715	.0792871	12390	28
33	3290	485	5798	12.565997	27
34	6190	254	8726	19942	26
35	9090	022	.0801653	12.474221	25
36	.0801989	.9967789	4581	28831	24
37	4889	555	7509	12.383768	23
38	7788	320	.0810437	39028	22
39	.0810687	085	3365	12.294608	21
40	3587	.9966849	6293	50506	20
41	6486	612	9221	06716	19
42	9385	374	.0822150	12.163236	18
43	.0822284	135	5078	20062	17
44	5183	.9965895	8007	12.077192	16
45	8082	655	.0830936	34622	15
46	.0830981	414	3865	11.992349	14
47	3880	172	6794	50370	13
48	6778	.9964929	9723	08682	12
49	9677	685	.0842653	11.867282	11
50	.0842576	440	5583	26167	10
51	5474	194	8512	11.785333	9
52	8373	.9963948	.0851442	44779	8
53	.0851271	701	4372	04500	7
54	4169	453	7302	11.664495	6
55	7067	204	.0860233	24761	5
56	9966	.9962954	3163	11.585294	4
57	.0862864	703	6094	46093	3
58	5762	452	9025	07154	2
59	8660	200	.0871956	11.468474	1
60	.0871557	.9961947	4887	30052	0
	Coſine	Sine	Cotang.	Tangent	
	94	Degrees		85	Min.

Min.	5	Degrees		174	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.0871557	.9961947	.0874887	11.430052	60
1	4455	693	7818	11.391885	59
2	7353	438	.0880749	53970	58
3	.0880251	182	3681	16304	57
4	3148	.9960926	6612	11.278885	56
5	6046	669	9544	41712	55
6	8943	411	.0892476	04780	54
7	.0891840	152	5408	11.168089	53
8	4738	.9959892	8341	31635	52
9	7635	631	.0901273	11.095416	51
10	.0900532	369	4206	59431	50
11	3429	107	7138	23676	49
12	6326	.9958844	.0910071	10.988150	48
13	9223	580	3004	52850	47
14	.0912119	315	5938	17775	46
15	5016	049	8871	10.882921	45
16	7913	.9957782	.0921804	48288	44
17	.0920809	515	4738	13872	43
18	3706	247	7672	10.779673	42
19	6602	.9956978	.0930606	45687	41
20	9499	708	3540	11913	40
21	.0932395	437	6474	10.678348	39
22	5291	165	9409	44992	38
23	8187	.9955892	.0942344	11841	37
24	.0941083	619	5278	10.578895	36
25	3979	345	8213	46151	35
26	6875	070	.0951148	13607	34
27	9771	.9954794	4084	10.481261	33
28	.0952666	517	7019	49112	32
29	5562	240	9955	17158	31
30	8458	.9953962	.0962890	10.385397	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	95	Degrees		84	

Min.	5	Degrees		174	
	Sine	Coſine	Tangent	Cotangent	
30	.0958458	.9953962	.0962890	10.385397	30
31	.0961353	682	5826	53827	29
32	4248	403	8763	22447	28
33	7144	122	.0971699	10.291255	27
34	.0970039	.9952840	4635	60249	26
35	2934	557	7572	29428	25
36	5829	274	.0980509	10.198789	24
37	8724	.9951990	3446	68332	23
38	.0981619	705	6383	38054	22
39	4514	419	9320	07954	21
40	7408	132	.0992257	10.078031	20
41	.0990303	.9950844	5195	48283	19
42	3197	555	8133	18708	18
43	6092	266	.1001071	9.9893050	17
44	8986	.9949976	4009	600724	16
45	.1001881	685	6947	310088	15
46	4775	393	9885	021125	14
47	7669	100	.1012824	9.8733823	13
48	.1010563	.9948806	5763	448166	12
49	3457	512	8702	164140	11
50	6351	217	.1021641	9.7881732	10
51	9245	.9947921	4580	600927	9
52	.1022138	624	7520	321713	8
53	5032	326	.1030460	044075	7
54	7925	027	3400	9.6768000	6
55	.1030819	.9946728	6340	493475	5
56	3712	428	9280	220486	4
57	6605	127	.1042220	9.5949022	3
58	9499	.9945825	5160	679068	2
59	.1042392	522	8101	410613	1
60	5285	218	.1051042	143645	0
	Coſine	Sine	Cotang.	Tangent	Min.
	95	Degrees		84	

Min.	6	Degrees		173	
	Sine	Cosine	Tangent	Cotangent	
0	.1045285	.9945218	.1051042	9.5143645	60
1	8178	.9944914	3983	9.4878149	59
2	.1051070	609	6924	614116	58
3	3963	303	9866	351531	57
4	6856	.9943996	.1062808	090384	56
5	9748	688	5750	9.3830663	55
6	.1062641	379	8692	572355	54
7	5533	069	.1071634	315450	53
8	8425	.9942759	4576	059936	52
9	.1071318	448	7519	9.2805802	51
10	4210	136	.1080462	553035	50
11	7102	.9941823	3405	301627	49
12	9994	509	6348	051564	48
13	.1082885	194	9291	9.1802838	47
14	5777	.9940879	.1092234	555436	46
15	8669	563	5178	309348	45
16	.1091560	246	8122	064564	44
17	4452	.9939928	.1101066	9.0821074	43
18	7343	609	4010	578867	42
19	.1100234	289	6954	337933	41
20	3126	.9938969	9899	098261	40
21	6017	648	.1112844	8.9859843	39
22	8908	326	5789	622668	38
23	.1111799	003	8734	386726	37
24	4689	.9937679	.1121679	152008	36
25	7580	354	4625	8.8918505	35
26	.1120471	028	7571	686206	34
27	3361	.9936702	.1130517	455103	33
28	6252	375	3463	225186	32
29	9142	047	6409	8.7996446	31
30	.1132032	.9935718	9356	768874	30
	Cosine	Sine	Cotang.	Tangent	Min.
	96	Degrees		83	

Min.	6	Degrees		173	
	Sine	*Cosine*	*Tangent*	*Cotangent*	
30	.1132032	.9935718	.1139356	8.7768874	30
31	4922	388	.1142303	542461	29
32	7812	058	5250	317198	28
33	.1140702	.9934727	8197	093077	27
34	3592	395	.1151144	8.6870088	26
35	6482	062	4091	648223	25
36	9371	.9933728	7039	427475	24
37	.1152261	393	9987	207833	23
38	5151	057	.1162935	8.5989290	22
39	8040	.9932720	5883	771838	21
40	.1160929	383	8831	555468	20
41	3818	045	.1171780	340172	19
42	6707	.9931706	4729	125943	18
43	9596	366	7678	8.4912772	17
44	.1172485	025	.1180628	700651	16
45	5374	.9930684	3578	489573	15
46	8263	342	6528	279531	14
47	.1181151	.9929999	9478	070515	13
48	4040	655	.1192428	8.3862519	12
49	6928	310	5378	655536	11
50	9816	.9928964	8328	449557	10
51	.1192704	617	.1201279	244577	9
52	5593	270	4230	040586	8
53	8481	.9927922	7181	8.2837579	7
54	.1201368	573	.1210132	635547	6
55	4256	223	3084	434485	5
56	7144	.9926872	6036	234384	4
57	.1210031	521	8988	035239	3
58	2919	169	.1221940	8.1837041	2
59	5806	.9925816	4893	639786	1
60	8693	462	7846	443464	0
	Cosine	*Sine*	*Cotang.*	*Tangent*	Min.
	96	Degrees		83	

NATURAL SINES

Min.	7		Degrees	172	
	Sine	Coſine	Tangent	Cotangent	
0	.1218693	.9925462	.1227846	8.1443464	60
1	.1221581	5107	.1230799	248071	59
2	4468	4751	3752	053599	58
3	7355	4394	6705	8.0860042	57
4	.1230241	4036	9658	667394	56
5	3128	3678	.1242612	475647	55
6	6015	3319	5566	284796	54
7	8901	2959	8520	094835	53
8	.1241788	2598	.1251474	7.9905756	52
9	4674	2236	4429	717555	51
10	7560	1874	7384	530224	50
11	.1250446	1511	.1260339	343758	49
12	3332	1147	3294	158151	48
13	6218	0782	6249	7.8973396	47
14	9104	0416	9205	789489	46
15	.1261990	0049	.1272161	606423	45
16	4875	.9919681	5117	424191	44
17	7761	9313	8073	242790	43
18	.1270646	8944	.1281029	062212	42
19	3531	8574	3986	7.7882453	41
20	6416	8203	6943	703506	40
21	9301	7831	9900	525366	39
22	.1282186	7459	.1292857	348028	38
23	5071	7086	5815	171486	37
24	7956	6712	8773	7.6995735	36
25	.1290841	6337	.1301731	820769	35
26	3725	5961	4689	646584	34
27	6609	5584	7648	473174	33
28	9494	5206	.1310607	300533	32
29	.1302378	4828	3566	128657	31
30	5262	4449	6525	7.5957541	30
	Coſine	Sine	Cotang.	Tangent	Min.
	97		Degrees	82	

Min.	7	Degrees		172	
	Sine	Coſine	Tangent	Cotangent	
30	.1305262	.9914449	.1316525	7.5957541	30
31	8146	4069	9484	787179	29
32	.1311030	3688	.1322444	617567	28
33	3913	3306	5404	448699	27
34	6797	2923	8364	280571	26
35	9681	2539	.1331324	113178	25
36	.1322564	2155	4285	7.4946514	24
37	5447	1770	7246	780576	23
38	8330	1384	.1340207	615357	22
39	.1331213	0997	3168	450855	21
40	4096	0609	6129	287064	20
41	6979	0221	9091	123978	19
42	9862	.9909832	.1352053	.73961595	18
43	.1342744	9441	5015	799909	17
44	5627	9051	7977	638916	16
45	8509	8659	.1360940	478610	15
46	.1351392	8266	3903	318989	14
47	4274	7872	6866	160047	13
48	7156	7478	9829	001780	12
49	.1360038	7083	.1372793	7.2844184	11
50	2919	6687	5757	687255	10
51	5801	.9906290	8721	530987	9
52	8683	5892	.1381685	375378	8
53	.1371564	5493	4650	220422	7
54	4445	5094	7615	066116	6
55	7327	4694	.1390580	7.1912456	5
56	.1380208	4293	3545	759437	4
57	3089	3891	6510	607056	3
58	5970	3488	9476	455308	2
59	8850	3084	.1402442	304190	1
60	.1391731	2680	5408	153697	0
	Coſine	Sine	Cotang.	Tangent	Min.
	97	Degrees		82	

Min.	8	Degrees		171	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.1391731	.9902680	.1405408	7.1153697	60
1	4612	2275	8374	003826	59
2	7492	1869	.1411341	7.0854573	58
3	.1400372	1462	4308	705934	57
4	3252	1054	7275	557905	56
5	6132	0645	.1420243	410482	55
6	9012	0236	3211	263662	54
7	.1411892	.9899826	6179	117441	53
8	4772	9415	9147	6.9971806	52
9	7651	9003	.1432115	826781	51
10	.1420531	8590	5084	682335	50
11	3410	8176	8053	538473	49
12	6289	7762	.1441022	395192	48
13	9168	7347	3991	252489	47
14	.1432047	6931	6961	110359	46
15	4926	6514	9931	6.8968799	45
16	7805	6096	.1452901	827807	44
17	.1440684	5677	5871	687378	43
18	3562	5257	8842	547508	42
19	6440	4837	.1461813	408196	41
20	9319	4416	4784	269437	40
21	.1452197	.9893994	7755	131227	39
22	5075	3571	.1470727	6.7993565	38
23	7953	3147	3699	856446	37
24	.1460830	2723	6671	719867	36
25	3708	2298	9644	583826	35
26	6585	1872	.1482617	448319	34
27	9463	1445	5590	313341	33
28	.1472340	1017	8563	178891	32
29	5217	0588	.1491536	044966	31
30	8094	0158	4510	6.6911562	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	98	Degrees		81	

Min.	8	Degrees		171	
	Sine	Coſine	Tangent	Cotangent	
30	.1478094	.9890158	.1494510	6.6911562	30
31	.1480971	.9889728	7484	778677	29
32	3848	9297	.1500458	646307	28
33	6724	8865	3433	514449	27
34	9601	8432	6408	383100	26
35	.1492477	7998	9383	252258	25
36	5353	7563	.1512358	121919	24
37	8230	7128	5333	6.5992080	23
38	.50 1106	6692	8309	862739	22
39	3981	6255	.1521285	733892	21
40	6857	5817	4261	605538	20
41	9733	.9885378	7238	477672	19
42	.1512608	4938	.1530215	350293	18
43	5484	4498	3192	223396	17
44	8359	4057	6189	096981	16
45	.1521234	3615	9147	6.4971043	15
46	4109	3172	.1542125	845581	14
47	6984	2728	5103	720591	13
48	9858	2283	8082	596070	12
49	.1532733	1838	.1551061	472017	11
50	5607	1392	4040	348428	10
51	8482	0945	7019	225301	9
52	.1541356	0497	9998	102633	8
53	4230	0048	.1562978	6.3980422	7
54	7104	.9879598	5958	858665	6
55	9978	9148	8938	737359	5
56	.1552851	8697	.1571919	616502	4
57	5725	8245	4900	496092	3
58	8598	7792	7881	376126	2
59	.1561472	7338	.1580862	256601	1
60	4345	6883	3844	137515	0
	Coſine	Sine	Cotang.	Tangent	Min.
	98	Degrees		81	

NATURAL SINES

Min.	9	*Degrees*		170	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.1564345	.9876883	.1583844	6.3137515	60
1	7218	6428	6826	018866	59
2	.1570091	5972	9808	6.2900651	58
3	2963	5515	.1592791	782868	57
4	5836	5057	5774	665514	56
5	8708	4598	8757	548588	55
6	.1581581	4138	.1601740	432086	54
7	4453	3677	4724	316007	53
8	7325	3216	7708	200347	52
9	.1590197	2754	.1610692	085106	51
10	3069	2291	3677	6.1970279	50
11	5940	1827	6662	855867	49
12	8812	1362	9647	741865	48
13	.1601683	0897	.1622632	628272	47
14	4555	0431	5617	515085	46
15	7426	.9869964	8603	402303	45
16	.1610297	9496	.1631589	289923	44
17	3167	9027	4576	177943	43
18	6038	8557	7563	066360	42
19	8909	8086	.1640550	6.0955174	41
20	.1621779	7615	3537	844381	40
21	4650	7143	6525	733979	39
22	7520	6670	9513	623967	38
23	.1630390	6196	.1652501	514 43	37
24	3260	5721	5489	405103	36
25	6129	5246	8478	296247	35
26	8999	4770	.1661467	187772	34
27	.1641868	4293	4456	079676	33
28	4738	3815	7446	5.9971957	32
29	7607	3336	.1670436	864614	31
30	.1650476	2856	3426	757644	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	99	*Degrees*		80	

	9	Degrees		170	
Min.	*Sine*	*Coſine*	*Tangent*	*Cotangent*	
30	.1650476	.9862856	.1673426	5.9757644	30
31	3345	2375	6416	651045	29
32	6214	1894	9407	544815	28
33	9081	1412	.1682398	438952	27
34	.1661951	0929	5389	333455	26
35	4819	0445	8381	228322	25
36	7687	.9859960	.1691373	123550	24
37	.1670555	9474	4365	019138	23
38	3423	8988	7358	5.8915084	22
39	6291	8501	.1700351	811386	21
40	9159	8013	3344	708042	20
41	.1682026	7524	6337	605051	19
42	4894	7034	9331	502410	18
43	7761	6544	.1712325	400117	17
44	.1690628	6053	5319	298172	16
45	3495	5561	8314	196572	15
46	6362	.9855068	.1721309	095315	14
47	9228	4574	4304	5.7994400	13
48	.1702095	4079	7300	893825	12
49	4961	3583	.1730296	793588	11
50	7828	3087	3292	693688	10
51	.1710694	2590	6288	594122	9
52	3560	2092	9285	494889	8
53	6425	1593	.1742282	395988	7
54	9291	1093	5279	297416	6
55	.1722156	0592	8277	199173	5
56	5022	0091	.1751275	101256	4
57	7887	.9849589	4273	003663	3
58	.1730752	9086	7272	5.6906394	2
59	3617	8582	.1760271	809446	1
60	6482	8077	3270	712818	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	99	Degrees		80	

Min.	10	Degrees		169	
	Sine	Coſine	Tangent	Cotangent	
0	.1736482	.9848077	.1763270	5.6712818	60
1	9346	7571	6269	616509	59
2	.1742211	7065	9269	520516	58
3	5075	6558	.1772269	424838	57
4	7939	6050	5269	329474	56
5	.1750803	5541	8270	234421	55
6	3667	5031	.1781271	139680	54
7	6531	4521	4272	045247	53
8	9395	4010	7274	5.5951121	52
9	.1762258	3498	.1790276	857302	51
10	5121	2985	3278	763786	50
11	7984	2471	6281	670574	49
12	.1770847	1956	9284	577663	48
13	3710	1440	.1802287	485052	47
14	6573	0924	5291	392740	46
15	9435	0407	8295	300724	45
16	.1782298	.9839889	.1811299	209005	44
17	5160	9370	4303	117579	43
18	8022	8850	7308	026446	42
19	.1790884	8329	.1820313	5.4935604	41
20	3746	7808	3318	845052	40
21	6607	7286	6324	754788	39
22	9469	6763	9330	664812	38
23	.1802330	6239	.1832336	575121	37
24	5191	5714	5343	485715	36
25	8052	5189	8350	396592	35
26	.1810913	4663	.1841357	307750	34
27	3774	4136	4365	219188	33
28	6635	3608	7373	130906	32
29	9495	3079	.1850381	042901	31
30	.1822355	2549	3390	5.3955172	30
	Coſine	Sine	Cotang.	Tangent	Min.
	100	Degrees		79	

Min.	10	Degrees		169	
	Sine	Coſine	Tangent	Cotangent	
30	.1822355	.9832549	.1853390	5.3955172	30
31	5215	2018	6399	867718	29
32	8075	1487	9408	780538	28
33	.1830935	0955	.1862418	693630	27
34	3795	0422	5428	606993	26
35	6654	.9829888	8438	520626	25
36	9513	9353	.1871449	434527	24
37	.1842373	8817	4460	348696	23
38	5232	8281	7471	263131	22
39	8091	7744	.1880483	177830	21
40	.1850949	7206	3495	092793	20
41	3808	6667	6507	008018	19
42	6666	6127	9520	5.2923505	18
43	9524	5587	.1892533	839251	17
44	.1862382	5046	5546	755255	16
45	5240	4504	8559	671517	15
46	8098	3961	.1901573	588035	14
47	.1870956	3417	4587	504809	13
48	3813	2872	7602	421836	12
49	6670	2327	.1910617	339116	11
50	9527	1781	3632	256647	10
51	.1882384	1234	6648	174428	9
52	5241	0686	9664	092459	8
53	8098	0137	.1922680	010738	7
54	.1890954	.9819587	5696	5.1929264	6
55	3811	9036	8713	848035	5
56	6667	8485	.1931730	767051	4
57	9523	7933	4748	686311	3
58	.1902379	7380	7766	605813	2
59	5234	6826	.1940784	525557	1
60	8090	6271	3803	445540	0
	Coſine	Sine	Cotang.	Tangent	Min.
	100	Degrees		79	

Min.	11	Degrees		168	
	Sine	Coſine	Tangent	Cotangent	
0	.1908090	.9816271	.1943803	5.1445540	60
1	.1910945	5716	6822	365763	59
2	3800	5160	9841	286224	58
3	6655	4603	.1952861	206921	57
4	9510	4045	5881	127855	56
5	.1922365	3486	8901	049024	55
6	5220	2926	.1961922	5.0970426	54
7	8074	2366	4943	892061	53
8	1.930928	1805	7964	813928	52
9	3782	1243	.1970986	736025	51
10	6636	0680	4008	658352	50
11	9490	0116	7030	580907	49
12	.1942344	.9809551	.1980053	503690	48
13	5197	8986	3076	426700	47
14	8050	8420	6100	349935	46
15	.1950903	7853	9124	273395	45
16	3756	7285	.1992148	197078	44
17	6609	6716	5172	120984	43
18	9461	6146	8197	045111	42
19	.1962314	5576	.2001222	4.9969459	41
20	5166	5005	4248	894027	40
21	8018	4433	7274	818813	39
22	.1970870	3860	.2010300	743817	38
23	3722	3286	3327	669037	37
24	6573	2711	6354	594474	36
25	9425	2136	9381	520125	35
26	.1982276	1560	.2022409	445990	34
27	5127	0893	5437	372068	33
28	7978	0405	8465	298358	32
29	.1990829	.9799826	.2031494	224859	31
30	3679	9247	4523	151570	30
	Coſine	Sine	Cotang.	Tangent	Min.
	101	Degrees		78	

Min.	11	Degrees		168	
	Sine	Coſine	Tangent	Cotangent	
30	.1993679	.9799247	.2034523	4.9151570	30
31	6530	8667	7552	078491	29
32	9380	8086	.2040582	005620	28
33	.2002230	7504	3612	4.8932956	27
34	5080	6921	6643	860499	26
35	7930	6337	9674	788248	25
36	.2010779	5752	.2052705	716201	24
37	3629	5167	5737	644359	23
38	6478	4581	8769	572719	22
39	9327	3994	.2061801	501282	21
40	.2022176	3406	4834	430045	20
41	5024	2817	7867	359010	19
42	7873	2228	.2070900	288174	18
43	.2030721	1638	3934	217536	17
44	3569	1047	6968	147096	16
45	6417	0455	.2080003	076854	15
46	9265	.9789862	3038	006808	14
47	.2042113	9268	6073	4.7936957	13
48	4961	8674	9109	867300	12
49	7808	8079	.2092145	797837	11
50	.2050655	7483	5181	728567	10
51	3502	6886	8218	659490	9
52	6349	6288	.2101245	590603	8
53	9195	5689	4293	521907	7
54	.2062042	5090	7331	453401	6
55	4888	4490	.2110369	385083	5
56	7734	3889	3407	316954	4
57	.2070580	3287	6446	249012	3
58	3426	2684	9485	181256	2
59	6271	2080	.2122526	113686	1
60	9117	1476	5565	046301	0
	Coſine	Sine	Cotang.	Tangent	Min.
	101	Degrees		78	

Min.	12	Degrees		167	
	Sine	Coſine	Tangent	Cotangent	
0	.2079117	.9781476	.2125565	4.7046301	60
1	.2081962	0871	8606	4.6979100	59
2	4807	0265	.2131647	12083	58
3	7652	.9779658	4688	4.6845248	57
4	.2090497	9050	7730	4.6778595	56
5	3341	8441	.2140772	12124	55
6	6186	7832	3814	4.6645832	54
7	9030	7222	6857	4.6579721	53
8	.2101874	6611	9900	13788	52
9	4718	5999	.2152944	4.6448034	51
10	7561	5386	5988	4.6382457	50
11	.2110405	4773	9032	17056	49
12	3248	4159	.2162077	4.6251832	48
13	6091	3544	5122	4.6186783	47
14	8934	2928	8167	21908	46
15	.2121777	2311	.2171213	4.6057207	45
16	4619	1693	4259	4.5992680	44
17	7462	1075	7306	28325	43
18	.2130304	0456	.2180353	4.5864141	42
19	3146	.9769836	3400	00129	41
20	5988	9215	6448	4.5736287	40
21	8829	8593	9496	4.5672614	39
22	.2141671	7970	.2192544	09111	38
23	4512	7347	5593	4.5545776	37
24	7353	6723	8642	4.5482608	36
25	.2150194	6098	.2201692	19608	35
26	3035	5472	4742	4.5356773	34
27	5876	4845	7793	4.5294105	33
28	8716	4217	.2210844	31601	32
29	.2161556	3589	3895	4.5169261	31
30	4396	2960	6947	07085	30
	Coſine	Sine	Cotang.	Tangent	Min.
	102	Degrees		77	

Min.	12	Degrees		167	
	Sine	Coſine	Tangent	Cotangent	
30	.2164396	.9762960	.2216947	4.5107085	30
31	7236	2330	9999	4.5045072	29
32	.2170076	1699	.2223051	4.4983221	28
33	2915	1067	6104	21532	27
34	5754	0435	9157	4.4860004	26
35	8593	.9759802	.2232211	4.4798636	25
36	.2181432	9168	5265	37428	24
37	4271	8533	8319	4.4676379	23
38	7110	7897	.2241374	15489	22
39	9948	7260	4429	4.4554756	21
40	.2192786	6623	7485	4.4494181	20
41	5624	5985	.2250541	33762	19
42	8462	5346	3597	4.4373499	18
43	.2201300	4706	6654	13392	17
44	4137	4065	9711	4.4253439	16
45	6974	3423	.2262769	4.4193641	15
46	9811	2781	5827	33996	14
47	.2212648	2138	8885	4.4074504	13
48	5485	1494	.2271944	15164	12
49	8321	0849	5003	4.3955976	11
50	.2221158	0203	8063	4.3896940	10
51	3994	.9749556	.2281123	38054	9
52	6830	8909	4183	4.3779317	8
53	9666	8261	7244	20731	7
54	.2232501	7612	.2290305	4.3662293	6
55	5337	6962	3367	04003	5
56	8172	6311	6429	4.3545861	4
57	.2241007	5660	9492	4.3487866	3
58	3841	5008	.2302555	30018	2
59	6676	4355	5618	4.3372316	1
60	9511	3701	8682	14759	0
	Coſine	Sine	Cotang.	Tangent	Min.
	102	Degrees		77	

Min.	13	Degrees		166	
	Sine	Coſine	Tangent	Cotangent	
0	.2249511	.9743701	.2308682	4.3314759	60
1	.2252345	3046	.2311746	4.3257347	59
2	5179	2390	4811	00079	58
3	8013	1734	7876	4.3142955	57
4	.2260846	1077	.2320941	4.3085974	56
5	3680	0419	4007	29136	55
6	6513	.9739760	7073	4.2972440	54
7	9346	9100	.2330140	15885	53
8	.2272179	8439	3207	4.2859472	52
9	5012	7778	6274	03199	51
10	7844	7116	9342	4.2747066	50
11	.2280677	6453	.2342410	4.2691072	49
12	3509	5789	5479	35218	48
13	6341	5124	8548	4.2579501	47
14	9172	4458	.2351617	23923	46
15	.2292004	3792	4687	4.2468482	45
16	4835	3125	7758	13177	44
17	7666	2457	.2360829	4.2358009	43
18	.2300497	1788	3900	02977	42
19	3328	1118	6972	4.2248080	41
20	6159	0448	.2370044	4.2193318	40
21	8989	.9729777	3116	38690	39
22	.2311819	9105	6189	4.2084196	38
23	4649	8432	9262	29835	37
24	7479	7758	.2382336	4.1975606	36
25	.2320309	7084	5410	21510	35
26	3138	6409	8485	4.1867546	34
27	5967	5733	.2391560	13713	33
28	8796	5056	4635	4.1760011	32
29	.2331625	4378	7711	06440	31
30	4454	3699	.2400787	4.1652998	30
	Coſine	Sine	Cotang.	Tangent	Min.
	103	Degrees		76	

Min.	13	Degrees		166	
	Sine	Coſine	Tangent	Cotangent	
30	.2334454	.9723699	.2400787	4.1652998	30
31	7282	3019	3864	4.1599685	29
32	.2340110	2339	6941	46501	28
33	2938	1658	.2410019	4.1493446	27
34	5766	0976	3097	40519	26
35	8594	0293	6176	4.1387719	25
36	.2351421	.9719609	9255	35046	24
37	4248	8925	.2422334	4.1282499	23
38	7075	8240	5414	30079	22
39	9902	7554	8494	4.1177784	21
40	.2362729	6867	.2431575	25614	20
41	5555	6179	4656	4.1073569	19
42	8381	5491	7737	21649	18
43	.2371207	4802	.2440819	4.0969852	17
44	4033	4112	3901	18178	16
45	6859	3421	6984	4.0866627	15
46	9684	2729	.2450067	15199	14
47	.2382510	2036	3151	4.0763892	13
48	5335	1343	6235	12707	12
49	8159	0649	9320	4.0661643	11
50	.2390984	.9709954	.2462405	10700	10
51	3808	9258	5491	4.0559877	9
52	6633	8561	8577	09174	8
53	9457	7863	.2471663	4.0458590	7
54	.2402280	7165	4750	08125	6
55	5104	6466	7837	4.0357779	5
56	7927	5766	.2480925	07550	4
57	.2410751	5065	4013	4.0257440	3
58	3574	4363	7102	07446	2
59	6396	3660	.2490191	4.0157570	1
60	9219	2957	3280	07809	0
	Coſine	Sine	Cotang.	Tangent	Min.
	103	Degrees		76	

Min.	14	*Degrees*		165	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.2419219	.9702957	.2493280	4.0107809	60
1	.2422041	2253	6370	4.0058165	59
2	4863	1548	9460	08636	58
3	7685	0842	.2502551	3.9959223	57
4	.2430507	0135	5642	09924	56
5	3329	.9699428	8734	3.9860739	55
6	6150	8720	.2511826	11669	54
7	8971	8011	4919	3.9762712	53
8	.2441792	7301	8012	13868	52
9	4613	6590	.2521106	3.9665137	51
10	7433	5879	4200	16518	50
11	.2450254	5167	7294	3.9568011	49
12	3074	4454	.2530389	19615	48
13	5894	3740	3484	3.9471331	47
14	8713	3025	6580	23157	46
15	.2461533	2309	9676	3.9375094	45
16	4352	1592	.2542773	27141	44
17	7171	0875	5870	3.9279297	43
18	9990	0157	8968	31563	42
19	.2472809	.9689438	.2552066	3.9183937	41
20	5627	8718	5165	36420	40
21	8445	7998	8264	3.9089011	39
22	.2481263	7277	.2561363	41710	38
23	4081	6555	4463	3.8994516	37
24	6899	5832	7563	47429	36
25	9716	5108	.2570664	00448	35
26	.2492533	4383	3766	3.8853574	34
27	5350	3657	6868	06805	33
28	8167	2931	9970	3.8760142	32
29	.2500984	2204	.2583073	13584	31
30	3800	1476	6176	3.8667131	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	104	*Degrees*		75	

Min.	14	Degrees		165	
	Sine	Coſine	Tangent	Cotangent	
30	.2503800	.9681476	.2586176	3.8667131	30
31	6616	0747	9280	20782	29
32	9432	0018	.2592384	3.8574537	28
33	.2512248	.9679288	5488	28396	27
34	5063	8557	8593	3.8482358	26
35	7879	7825	.2601699	36424	25
36	.2520694	7092	4805	3.8390591	24
37	3508	6358	7911	44861	23
38	6323	5623	.2611018	3.8299233	22
39	9137	4888	4126	53707	21
40	.2531952	4152	7234	08281	20
41	4766	3415	.2620342	3.8162957	19
42	7579	2677	3451	17733	18
43	.2540393	1938	6560	3.8072608	17
44	3206	1199	9670	27585	16
45	6019	0459	.2632780	3.7982661	15
46	8832	.9669718	5891	37835	14
47	.2551645	8976	9002	3.7893109	13
48	4458	8233	.2642114	48481	12
49	7270	7490	5226	03951	11
50	.2560082	6746	8339	3.7759519	10
51	2894	6001	.2651452	15185	9
52	5705	5255	4566	3.7670947	8
53	8517	4508	7680	26807	7
54	.2571328	3760	.2660794	3.7582763	6
55	4139	3012	3909	38815	5
56	6950	2263	7025	3.7494963	4
57	9760	1513	.2670141	51207	3
58	.2582570	0762	3257	07546	2
59	5381	0010	6374	3.7363980	1
60	8190	.9659258	9492	20508	0
	Coſine	Sine	Cotang.	Tangent	Min.
	104	Degrees		75	

Min.	15	Degrees		164	
	Sine	Coſine	Tangent	Cotangent	
0	.2588190	.9659258	.2679492	3.7320508	60
1	.2591000	8505	.2682610	3.7277131	59
2	3810	7751	5728	33847	58
3	6619	6996	8847	3.7190658	57
4	9428	6240	.2691967	47561	56
5	.2602237	5483	5087	04558	55
6	5045	4726	8207	3.7061648	54
7	7853	3968	.2701328	18830	53
8	.2610661	3209	4449	3.6976103	52
9	3469	2449	7571	33469	51
10	6277	1688	.2710693	3.6890927	50
11	9085	0927	3816	48475	49
12	.2621892	0165	6940	06115	48
13	4699	.9649402	.2720064	3.6763845	47
14	7506	8638	3188	21665	46
15	.2630312	7873	6313	3.6679575	45
16	3118	7107	9438	37575	44
17	5924	6341	.2732564	3.6595665	43
18	8730	5574	5690	53844	42
19	.2641536	4806	8817	12111	41
20	4342	4037	.2741944	3.6470467	40
21	7147	3267	5072	28911	39
22	9952	2497	8201	3.6387444	38
23	.2652757	1726	.2751330	46064	37
24	5561	0954	4459	04771	36
25	8365	0181	7589	3.6263566	35
26	.2661169	.9639407	.2760719	22447	34
27	3973	8633	3850	3.6181415	33
28	6777	7858	6981	40469	32
29	9581	7082	.2770113	3.6099609	31
30	.2672384	6305	3245	58835	30
	Coſine	Sine	Cotang.	Tangent	Min.
	105	Degrees		74	

Min.	15		Degrees	164	
	Sine	Coſine	Tangent	Cotangent	
30	.2672384	.9636305	.2773245	3.6058835	30
31	5187	5527	6378	18146	29
32	7989	4748	9512	3.5977543	28
33	.2680792	3969	.2782646	37024	27
34	3594	3189	5780	3.5896590	26
35	6396	2408	8915	56241	25
36	9198	1626	.2792050	15975	24
37	.2692000	0843	5186	3.5775794	23
38	4801	0059	8322	35696	22
39	7602	.9629275	.2801459	3.5695681	21
40	.2700403	8490	4597	55749	20
41	3204	7704	7735	15900	19
42	6004	6917	.2810873	3.5576133	18
43	8805	6130	4012	36449	17
44	.2711605	5342	7152	3.5496846	16
45	4404	4552	.2820292	57325	15
46	7204	3762	3432	17886	14
47	.2720003	2972	6573	3.5378528	13
48	2802	2180	9715	39251	12
49	5601	1387	.2832857	00054	11
50	8400	0594	5999	3.5260938	10
51	.2731198	.9619800	9142	21902	9
52	3996	9005	.2842286	3.5182946	8
53	6794	8209	5430	44070	7
54	9592	7413	8575	05273	6
55	.2742390	6616	.2851720	3.5066555	5
56	5187	5818	4866	27916	4
57	7984	5019	8012	3.4989356	3
58	.2750781	4219	.2861159	50874	2
59	3578	3418	4306	12470	1
60	6374	2617	7454	3.4874144	0
	Coſine	Sine	Cotang.	Tangent	Min.
	105		Degrees	74	

Min.	16	*Degrees*		163	
	Sine	*Cofine*	*Tangent*	*Cotangent*	
0	.2756374	.9612617	.2867454	3.4874144	60
1	9170	1815	.2870602	35896	59
2	.2761965	1012	3751	3.4797726	58
3	4761	0208	6900	59632	57
4	7556	.9609403	.2880050	21616	56
5	.2770352	8598	3201	3.4683676	55
6	3147	7792	6352	45813	54
7	5941	6984	9503	08026	53
8	8736	6177	.2892655	3.4570315	52
9	.2781530	5368	5808	32679	51
10	4324	4558	8961	3.4495120	50
11	7118	3748	.2902114	57635	49
12	9911	2937	5268	20226	48
13	.2792704	2125	8423	3.4382891	47
14	5497	1312	.2911578	45631	46
15	8290	0499	4734	08446	45
16	.2801083	.9599684	7890	3.4271334	44
17	3875	8869	.2921047	34297	43
18	6667	8053	4205	3.4197333	42
19	9459	7236	7363	60443	41
20	.2812251	6418	.2930521	23626	40
21	5042	5600	3680	3.4086882	39
22	7833	4781	6839	50210	38
23	.2820624	3961	9999	13612	37
24	3415	3140	.2943160	3.3977085	36
25	6205	2318	6321	·40631	35
26	8995	1496	9483	04249	34
27	.2831785	0672	.2952645	3.3867938	33
28	4575	.9589848	5808	31699	32
29	7364	9023	8971	3.3795531	31
30	.2840153	8197	.2962135	59434	30
	Cofine	*Sine*	*Cotang.*	*Tangent*	Min.
	106	*Degrees*		73	

Min.	16	Degrees		163	
	Sine	Coſine	Tangent	Cotangent	
30	.2840153	.9588197	.2962135	3.3759434	30
31	2942	7371	5299	23408	29
32	5731	6543	8464	3.3687453	28
33	8520	5715	.2971630	51568	27
34	.2851308	4886	4796	15753	26
35	4096	4056	7962	3.3580008	25
36	6884	3226	.2981129	44333	24
37	9672	2394	4297	08728	23
38	.2862458	1562	7465	3.3473191	22
39	5245	0729	.2990634	37724	21
40	8032	.9579895	3803	02326	20
41	.2870819	9060	6973	3.3366997	19
42	3605	8225	.3000144	31736	18
43	6391	7389	3315	3.3296543	17
44	9177	6552	6486	61419	16
45	.2881963	5714	9658	26362	15
46	4748	4875	.3012831	3.3191373	14
47	7533	4035	6004	56452	13
48	.2890318	3195	9178	21598	12
49	3103	2354	.3022352	3.3086811	11
50	5887	1512	5527	52091	10
51	8671	0669	8703	17438	9
52	.2901455	.9569825	.3031879	3.2982851	8
53	4239	8981	5055	48330	7
54	7022	8136	8232	13876	6
55	9805	7290	.3041410	3.2879487	5
56	.2912588	6443	4588	45164	4
57	5371	5595	7767	10907	3
58	8153	4747	.3050946	3.2776715	2
59	.2920935	3898	4126	42588	1
60	3717	3048	7307	08526	0
	Coſine	Sine	Cotang.	Tangent	Min.
	106	Degrees		73	

Min.	17	*Degrees*		162	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.2923717	.9563048	.3057307	3.2708526	60
1	6499	2197	.3060488	3.2674529	59
2	9280	1345	3669	40596	58
3	.2932061	0492	6851	06728	57
4	4842	.9559639	.3070034	3.2572924	56
5	7623	8785	3218	39184	55
6	.2940403	7930	6402	05508	54
7	3183	7074	9586	3.2471895	53
8	5963	6217	.3082771	38346	52
9	8743	5360	5957	04860	51
10	.2951522	4502	9143	3.2371438	50
11	4302	3643	.3092330	38078	49
12	7080	2784	5517	04780	48
13	9859	1923	8705	3.2271546	47
14	.2962638	1061	.3101893	38373	46
15	5416	0199	5082	05263	45
16	8194	.9549336	8272	3.2172215	44
17	.2970971	8473	.3111462	39228	43
18	3749	7608	4653	06304	42
19	6526	6742	7844	3.2073440	41
20	9303	5876	.3121036	40638	40
21	.2982079	5009	4229	07897	39
22	4856	4141	7422	3.1975217	38
23	7632	3273	.3130616	42598	37
24	.2990408	2403	3810	10039	36
25	3184	1533	7005	3.1877540	35
26	5959	0662	.3140200	45102	34
27	8734	.9539790	3396	12724	33
28	.3001509	8917	6593	3.1780406	32
29	4284	8044	9790	48147	31
30	7058	7169	.3152988	15948	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	107	*Degrees*		72	

Min.	17	Degrees		162	
	Sine	Coſine	Tangent	Cotangent	
30	.3007058	.9537169	.3152988	3.1715948	30
31	9832	6294	6186	3.1683808	29
32	.3012606	5418	9385	51728	28
33	5380	4542	.3162585	19706	27
34	8153	3664	5785	3.1587704	26
35	.3020926	2786	8986	55840	25
36	3699	1907	.3172187	23994	24
37	6471	1027	5389	3.1492207	23
38	9244	0146	8591	60478	22
39	.3032016	.9529264	.3181794	28807	21
40	4788	8382	4998	3.1397194	20
41	7559	7499	8202	65639	19
42	.3040331	6615	.3191407	34141	18
43	3102	5730	4613	02701	17
44	5872	4844	7819	3.1271317	16
45	8643	3958	.3201025	39991	15
46	.3051413	3071	4232	08722	14
47	4183	2183	7440	3.1177509	13
48	6953	1294	.3210649	46353	12
49	9723	0404	3858	15254	11
50	.3062492	.9519514	7067	3.1084210	10
51	5261	8623	.3220277	53223	9
52	8029	7731	3488	22291	8
53	.3070798	6838	6700	3.0991416	7
54	3566	5944	9912	60596	6
55	6334	5050	.3233125	29831	5
56	9102	4154	6338	3.0899122	4
57	.3081869	3258	9552	68468	3
58	4636	2361	.3242766	37869	2
59	7403	1464	5981	07325	1
60	.3090170	0565	9197	5.0776835	0
	Coſine	Sine	Cotang.	Tangent	Min.
	107	Degrees		72	

NATURAL SINES

Min.	18	*Degrees*		161	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.3090170	.9510565	.3249197	3.0776835	60
1	2936	.9509666	.3252413	46400	59
2	5702	8766	5630	16020	58
3	8468	7865	8848	3.0685693	57
4	.3101234	6963	.3262066	55421	56
5	3999	6060	5285	25203	55
6	6764	5157	8504	3.0595038	54
7	9529	4253	.3271724	64928	53
8	.3112294	3348	4944	34870	52
9	5058	2442	8165	04866	51
10	7822	1536	.3281387	3.0474915	50
11	.3120586	0629	4610	45018	49
12	3349	.9499721	7833	15173	48
13	6112	8812	.3291056	3.0385381	47
14	8875	7902	4280	55641	46
15	.3131638	6991	7505	25954	45
16	4400	6080	.3300731	3.0296320	44
17	7163	5168	3957	66737	43
18	9925	4255	7184	37207	42
19	.3142686	3341	.3310411	07728	41
20	5448	2426	3639	3.0178301	40
21	8209	1511	6868	48926	39
22	.3150969	0595	.3320097	19602	38
23	3730	.9489678	3327	3.0090330	37
24	6490	8760	6557	61109	36
25	9250	7841	9788	31939	35
26	.3162010	6922	.3333020	02820	34
27	4770	6002	6252	2.9973751	33
28	7529	5081	9485	44734	32
29	.3170288	4159	.3342719	15766	31
30	3047	3237	5953	2.9886850	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	108	*Degrees*		71	

Min.	18	Degrees		161	
	Sine	Coſine	Tangent	Cotangent	
30	·3173047	·9483236	·3345953	2.9886850	30
31	5805	2313	9188	57983	29
32	8563	1389	·3352424	29166	28
33	·3181321	0464	5660	00400	27
34	4079	·9479538	8897	2.9771683	26
35	6836	8611	·3362134	43016	25
36	9593	7684	5372	14399	24
37	·3192350	6756	8611	2.9685831	23
38	5106	5827	·3371850	57312	22
39	7863	4897	5090	28842	21
40	·3200619	3966	8330	00422	20
41	3374	3035	·3381571	2.9572050	19
42	6130	2103	4813	43727	18
43	8885	1170	8056	15453	17
44	·3211640	0236	·3391299	2.9487227	16
45	4395	·9469301	4543	59050	15
46	7149	8366	7787	30921	14
47	9903	7430	·3401032	02840	13
48	·3222657	6493	4278	2.9374807	12
49	5410	5555	7524	46822	11
50	8164	4616	·3410771	18885	10
51	·3230917	3676	4019	2.9290995	9
52	3670	2736	7267	63152	8
53	6422	1795	·3420516	35358	7
54	9174	0853	3765	07610	6
55	·3241926	·9459910	7015	2.9179909	5
56	4678	8967	·3430266	52256	4
57	7429	8023	3158	24649	3
58	·3250180	7078	6770	2.9097089	2
59	2931	6132	·3440023	69576	1
60	5682	5185	3276	42109	0
	Coſine	Sine	Cotang.	Tangent	Min.
	108	Degrees		71	

NATURAL SINES

Min.	19	Degrees		160	
	Sine	Cosine	Tangent	Cotangent	
0	.3255682	.9455186	.3443276	2.9042109	60
1	8432	4238	6530	14688	59
2	.3261182	3290	9785	2.8987314	58
3	3931	2341	.3453040	59986	57
4	6681	1391	6296	32704	56
5	9430	0440	9553	05467	55
6	.3272179	.9449489	.3462810	2.8878277	54
7	4928	8537	6068	51132	53
8	7676	7584	9327	24033	52
9	.3280424	6630	.3472586	2.8796979	51
10	3172	5675	5846	69970	50
11	5919	4720	9107	43007	49
12	8666	3764	.3482368	16088	48
13	.3291413	2807	5630	2.8689215	47
14	4160	1849	8893	62386	46
15	6906	0890	.3492156	35602	45
16	9652	.9439931	5420	08863	44
17	.3302398	8971	8685	2.8582168	43
18	5144	8010	.3501950	55517	42
19	7889	7048	5216	28911	41
20	.3310634	6085	8483	02349	40
21	3379	5122	.3511750	2.8475831	39
22	6123	4157	5018	49356	38
23	8867	3192	8287	22926	37
24	.3321611	2227	.3521556	2.8396539	36
25	4355	1260	4826	70196	35
26	7098	0293	8097	43896	34
27	9841	.9429325	.3531368	17639	33
28	.3332584	8356	4640	2.8291426	32
29	5327	7386	7913	65256	31
30	8069	6415	.3541186	39129	30
	Cosine	Sine	Cotang.	Tangent	Min.
	109	Degrees		70	

Min.	19	Degrees		160	
	Sine	*Cofine*	*Tangent*	*Cotangent*	
30	.3338069	.9426415	.3541186	2.8239129	30
31	.3340810	5443	4460	13045	29
32	3552	4471	7735	2.8187003	28
33	6293	3498	.3551010	61004	27
34	9034	2524	4286	35048	26
35	.3351775	1550	7563	09134	25
36	4516	0575	.3560840	2.8083263	24
37	7256	.9419598	4118	57433	23
38	9996	8621	7397	31646	22
39	.3362735	7644	.3570676	05901	21
40	5475	6665	3956	2.7980198	20
41	8214	5686	7237	54537	19
42	.3370953	4705	.3580518	28917	18
43	3691	3724	3800	03339	17
44	6429	2743	7083	2.7877802	16
45	9167	1760	.3590367	52307	15
46	.3381905	0777	3651	26853	14
47	4642	.9409793	6936	01440	13
48	7379	8808	.3600222	2.7776069	12
49	.3390116	7822	3508	50738	11
50	2853	6835	6795	25448	10
51	5589	5848	.3610083	00199	9
52	8325	4860	3371	2.7674990	8
53	.3401060	3871	6660	49822	7
54	3795	2881	9950	24695	6
55	6530	1891	.3623240	2.7599608	5
56	9265	0899	6531	74561	4
57	.3412000	.9399907	9823	49554	3
58	4734	8914	.3633115	24588	2
59	7468	7921	6408	2.7499661	1
60	.3420201	6926	9702	74774	0
	Cofine	*Sine*	*Cotang.*	*Tangent*	Min.
	109	Degrees		70	

Min.	20	Degrees		159	
	Sine	Coſine	Tangent	Cotangent	
0	.3420201	.9396926	.3639702	2.7474774	60
1	2935	5931	.3642997	49927	59
2	5668	4935	6292	25120	58
3	8400	3938	9588	00352	57
4	.3431133	2940	.3652885	2.7375623	56
5	3865	1942	6182	50934	55
6	6597	0943	9480	26284	54
7	9329	.9389942	.3662779	01674	53
8	.3442060	8942	6079	2.7277102	52
9	4792	7940	9379	52569	51
10	7521	6937	.3672680	28075	50
11	.3450252	5934	5982	03620	49
12	2982	4930	9284	2.7179204	48
13	5712	3925	.3682587	54826	47
14	8441	2919	5891	30487	46
15	.3461171	1913	9195	06186	45
16	3900	0906	.3692500	2.7081923	44
17	6628	.9379898	5806	57699	43
18	9357	8889	9113	33513	42
19	.3472085	7880	.3702420	09364	41
20	4812	6869	5728	2.6985254	40
21	7540	5858	9037	61181	39
22	.3480267	4846	.3712346	37147	38
23	2994	3833	5656	13149	37
24	5720	2820	8967	2.6889190	36
25	8447	1806	.3722278	65267	35
26	.3491173	0790	5590	41383	34
27	3898	.9369774	8903	17535	33
28	6624	8758	.3732217	2.6793725	32
29	9349	7740	5532	69951	31
30	.3502074	6722	8847	46215	30
	Coſine	Sine	Cotang.	Tangent	Min.
	110	Degrees		69	

Min.	20	Degrees		159	
	Sine	Coſine	Tangent	Cotangent	
30	·3502074	·9366722	·3738847	2.6746215	30
31	4799	5703	·3742163	22516	29
32	7523	4683	5479	2.6698853	28
33	·3510246	3662	8797	75227	27
34	2970	2641	·3752115	51638	26
35	5693	1618	5434	28085	25
36	8416	0595	8753	04569	24
37	·3521139	·9359571	·3762073	2.6581089	23
38	3862	8547	5394	57645	22
39	6584	7521	8716	34238	21
40	9306	6495	·3772038	10867	20
41	·3532027	5468	5361	2.6487531	19
42	4748	4440	8685	64232	18
43	7469	3412	·3782010	40969	17
44	·3540190	2382	5335	17741	16
45	2910	1352	8661	2.6394549	15
46	5630	0321	·3791988	71392	14
47	8350	·9349289	5316	48271	13
48	·3551070	8257	8644	25186	12
49	3789	7223	·3801973	02136	11
50	6508	6189	5303	2.6279121	10
51	9226	5154	8633	56141	9
52	·3561944	4118	·3811964	33196	8
53	4662	3082	5296	10286	7
54	7380	2045	8629	2.6187411	6
55	·3570097	1007	·3821962	64571	5
56	2814	·9339968	5296	41766	4
57	5531	8928	8631	18995	3
58	8248	7888	·3831967	2.6096259	2
59	·3580964	6846	5303	73558	1
60	3679	5804	8640	50891	0
	Coſine	Sine	Cotang.	Tangent	Min.
	110	Degrees		69	

Min.	21	Degrees		158	
	Sine	Coſine	Tangent	Cotangent	
0	.3583679	.9335804	.3838640	2.6050891	60
1	6395	4761	.3841978	28258	59
2	9110	3718	5317	05659	58
3	.3591825	2673	8656	2.5983095	57
4	4540	1628	.3851996	60564	56
5	7254	0582	5337	38068	55
6	9968	.9329535	8679	15606	54
7	.3602682	8488	.3862021	2.5893177	53
8	5395	7439	5364	70782	52
9	8108	6390	8708	48421	51
10	.3610821	5340	.3872053	26094	50
11	3534	4290	5398	03800	49
12	6246	3238	8744	2.5781539	48
13	8958	2186	.3882091	59312	47
14	.3621669	1133	5439	37118	46
15	4380	0079	8787	14957	45
16	7091	.9319024	.3892136	2.5692830	44
17	9802	7969	5486	70735	43
18	.3632512	6912	8837	48674	42
19	5222	5855	.3902189	26645	41
20	7932	4797	5541	04649	40
21	.3640641	3739	8894	2.5582686	39
22	3351	2679	.3912248	60756	38
23	6059	1619	5602	38858	37
24	8768	0558	8957	16992	36
25	.3651476	.9309496	.3922313	2.5495160	35
26	4184	8433	5670	73359	34
27	6891	7370	9028	51591	33
28	9599	6306	.3932386	29855	32
29	.3662306	5241	5745	08151	31
30	5012	4176	9105	2.5386479	30
	Coſine	Sine	Cotang.	Tangent	Min.
	111	Degrees		68	

Min.	21	Degrees		158	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	.3665012	.9304176	.3939105	2.5386479	30
31	7719	3109	.3942466	64839	29
32	.3670425	2042	5827	43231	28
33	3130	0974	9189	21655	27
34	5836	.9299905	.3952552	00111	26
35	8541	8835	5916	2.5278598	25
36	.3681246	7765	9280	57117	24
37	3950	6694	.3962645	35667	23
38	6654	5622	6011	14249	22
39	9358	4549	9378	2.5192863	21
40	.3692061	3475	.3972746	71507	20
41	4765	2401	6114	50183	19
42	7468	1326	9483	28890	18
43	.3700170	0250	.3982853	07629	17
44	2872	.9289173	6224	2.5086398	16
45	5574	8095	9596	65198	15
46	8276	7017	.3992968	44029	14
47	.3710977	5938	6341	22891	13
48	3678	4858	9715	01784	12
49	6379	3778	.4003089	2.4980707	11
50	9079	2696	6465	59661	10
51	.3721780	1614	9841	38645	9
52	4479	0531	.4013218	17660	8
53	7179	.9279447	6596	2.4896706	7
54	9878	8362	9975	75781	6
55	.3732577	7277	.4023354	54887	5
56	5275	6191	6734	34023	4
57	7973	5104	.4030115	13190	3
58	.3740671	4016	3497	2.4792386	2
59	3369	2928	6879	71612	1
60	6066	1839	.4040262	50869	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	111	Degrees		68	

Min.	22	Degrees		157	
	Sine	Coſine	Tangent	Cotangent	
0	.3746066	.9271839	.4040262	2.4750869	60
1	8763	0748	3646	30155	59
2	.3751459	.9269658	7031	09470	58
3	4156	8566	.4050417	2.4688816	57
4	6852	7473	3804	68191	56
5	9547	6380	7191	47596	55
6	.3762243	5286	.4060579	27030	54
7	4938	4191	3968	06494	53
8	7632	3096	7358	2.4585987	52
9	.3770327	2000	.4070748	65509	51
10	3021	0902	4139	45061	50
11	5714	.9259805	7531	24642	49
12	8408	8706	.4080924	04252	48
13	.3781101	7606	4318	2.4483891	47
14	3794	6506	7713	63559	46
15	6486	5405	.4091108	43256	45
16	9178	4303	4504	22982	44
17	.3791870	3201	7901	02736	43
18	4562	2097	.4101299	2.4382519	42
19	7253	0993	4697	62331	41
20	9944	.9249888	8097	42172	40
21	.3802634	8782	.4111497	22041	39
22	5324	7676	4898	01938	38
23	8014	6568	8300	2.4281864	37
24	.3810704	5460	.4121703	61819	36
25	3393	4351	5106	41801	35
26	6082	3242	8510	21812	34
27	8770	2131	.4131915	01851	33
28	.3821459	1020	5321	2.4181918	32
29	4147	.9239908	8728	62013	31
30	6834	8795	.4142136	42136	30
	Coſine	Sine	Cotang.	Tangent	Min.
	112	Degrees		67	

Min.	22	Degrees		157	
	Sine	Coſine	Tangent	Cotangent	
30	.3826834	.9238795	.4142136	2.4142136	30
31	9522	7682	5544	22286	29
32	.3832209	6567	8953	02465	28
33	4895	5452	.4152363	2.4082672	27
34	7582	4336	5774	62906	26
35	.3840268	3220	9186	43168	25
36	2953	2102	.4162599	23457	24
37	5639	0984	6012	03774	23
38	8324	.9229865	9426	2.3984118	22
39	.3851008	8745	.4172841	64490	21
40	3693	7624	6257	44889	20
41	6377	6503	9674	25316	19
42	9060	5381	.4183091	05769	18
43	.3861744	4258	6509	2.3886250	17
44	4427	3134	9928	66758	16
45	7110	2010	.4193348	47293	15
46	9792	0884	6769	27855	14
47	.3872474	.9219758	.4200191	08444	13
48	5156	8631	3613	2.3789060	12
49	7837	7504	7036	69703	11
50	.3880518	6375	.4210460	50372	10
51	3199	5246	3885	31068	9
52	5880	4116	7311	11791	8
53	8560	2986	.4220738	2.3692540	7
54	.3891239	1854	4166	73316	6
55	3919	0722	7594	54118	5
56	6598	.9209589	.4231023	34946	4
57	9277	8455	4453	15801	3
58	.3901955	7320	7884	2.3596683	2
59	4633	6185	.4241316	77590	1
60	7311	5049	4749	58524	0
	Coſine	Sine	Cotang.	Tangent	Min.
	112	Degrees		67	

NATURAL SINES

23 *Degrees* 156

Min.	*Sine*	*Coſine*	*Tangent*	*Cotangent*	
0	.3907311	.9205049	.4244749	2.3558524	60
1	9989	3912	8182	39483	59
2	.3912666	2774	.4251616	20469	58
3	5343	1635	5051	01481	57
4	8019	0496	8487	2.3482519	56
5	.3920695	.9199356	.4261924	63582	55
6	3371	8215	5362	44672	54
7	6047	7073	8800	25787	53
8	8722	5931	.4272239	06928	52
9	.3931397	4788	5679	2.3388095	51
10	4071	3644	9120	69287	50
11	6745	2499	.4282562	50505	49
12	9419	1353	6005	31748	48
13	.3942093	0207	9449	13017	47
14	4766	.9189060	.4292894	2.3294311	46
15	7439	7912	6339	75630	45
16	.3950111	6763	9785	56975	44
17	2783	5614	.4303232	38345	43
18	5455	4464	6680	19740	42
19	8127	3313	.4310129	01160	41
20	.3960798	2161	3579	2.3182606	40
21	3468	1009	7030	64076	39
22	6139	.9179855	.4320481	45571	38
23	8809	8701	3933	27091	37
24	.3971479	7546	7386	08636	36
25	4148	6391	.4330840	2.3090206	35
26	6817	5234	4295	71801	34
27	9486	4077	7751	53420	33
28	.3982155	2919	.4341208	35064	32
29	4823	1760	4666	16732	31
30	7491	0601	8123	2.2998425	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*

113 *Degrees* 66

Min.	23	Degrees		156	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	.3987491	.9170601	.4348123	2.2998425	30
31	.3990158	.9169440	.4351583	80143	29
32	2825	8279	5043	61885	28
33	5492	7118	8504	43651	27
34	8158	5955	,4361966	25442	26
35	.4000825	4791	5429	07257	25
36	3490	3627	8893	2.2889096	24
37	6156	2462	.4372358	70959	23
38	8821	1296	5823	52846	22
39	.4011486	0130	9289	34758	21
40	4150	.8158963	.4382756	16693	20
41	6814	7795	6224	2.2798653	19
42	9478	6626	9693	80636	18
43	.4022141	5456	.4393163	62643	17
44	4804	4286	6634	44674	16
45	7467	3115	.4400106	26729	15
46	.4030129	1943	3578	08807	14
47	2791	0770	7051	2.2690909	13
48	5453	.9149597	.4410525	73035	12
49	8114	8422	4000	55184	11
50	.4040775	7247	7476	37357	10
51	3436	6072	.4420953	19553	9
52	6096	4895	4331	01773	8
53	8756	3718	7910	2.2584016	7
54	.4051416	2540	.4431390	66283	6
55	4075	1361	4871	48572	5
56	6734	0181	8353	30885	4
57	9393	.9139001	.4441835	13221	3
58	.4062051	7819	5318	2.2495580	2
59	4709	6637	8802	77962	1
60	7366	5455	.4452287	60368	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	113	Degrees		66	

NATURAL SINES

Min.	24	Degrees		155	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.4067366	.9135455	.4452287	2.2460368	60
1	.4070023	4271	5773	42796	59
2	2681	3087	9260	25247	58
3	5337	1902	.4462748	07721	57
4	7993	0716	6237	2.2390218	56
5	.4080649	.9129529	9727	72738	55
6	3305	8342	.4473217	55280	54
7	5960	7154	6708	37845	53
8	8615	5965	.4480200	20433	52
9	.4091269	4775	3693	03043	51
10	3923	3584	7187	2.2285676	50
11	6577	2393	.4490682	68331	49
12	9230	1201	4178	51009	48
13	.4101883	0008	7675	33709	47
14	4536	.9118814	.4501173	16432	46
15	7189	7620	4672	2.2199177	45
16	9841	6425	8172	81944	44
17	.4112492	5229	.4511673	64733	43
18	5144	4032	5174	47545	42
19	7795	2835	8676	30379	41
20	.4120445	1637	.4522179	13234	40
21	3096	0438	5683	2.2096112	39
22	5745	.9109238	9188	79012	38
23	8395	8038	.4532694	61934	37
24	.4131044	6837	6201	44878	36
25	3693	5635	9709	27843	35
26	6342	4432	.4543218	10831	34
27	8990	3228	6728	2.1993840	33
28	.4141638	2024	.4550239	76871	32
29	4285	0819	3751	59923	31
30	6932	.9099613	7264	42997	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	114	Degrees		65	

Min.	24	Degrees		155	
	Sine	Cosine	Tangent	Cotangent	
30	.4146932	.9099613	.4557264	2.1942997	30
31	9579	8406	.4560777	26093	29
32	.4152226	7199	4291	09210	28
33	4872	5990	7806	2.1892349	27
34	7517	4781	.4571322	75510	26
35	.4160163	3571	4839	58691	25
36	2808	2361	8357	41894	24
37	5453	1150	.4581876	25119	23
38	8097	.9089938	5396	08364	22
39	.4170741	8725	8917	2.1791631	21
40	3385	7511	.4592439	74920	20
41	6028	6297	5962	58229	19
42	8671	5082	9486	41559	18
43	.4181313	3866	.4603011	24911	17
44	3955	2649	6537	08283	16
45	6597	1432	.4610064	2.1691677	15
46	9239	0214	3591	75091	14
47	.4191880	.9078995	7119	58527	13
48	4521	7775	.4620648	41983	12
49	7161	6554	4178	25460	11
50	9801	5333	7709	08958	10
51	.4202441	4111	.4631242	2.1592476	9
52	5080	2888	4776	76015	8
53	7719	1664	8311	59575	7
54	.4210358	0440	.4641846	43156	6
55	2996	.9069215	5382	26757	5
56	5634	7989	8919	10378	4
57	8272	6762	.4652457	2.1494020	3
58	.4220909	5535	5996	77683	2
59	3546	4307	9536	61366	1
60	6183	3078	4663077	45069	0
	Cosine	Sine	Cotang.	Tangent	Min.
	141	Degrees		65	

Min.	25	*Degrees*		154	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.4226183	.9063078	.4663077	2.1445069	60
1	8819	1848	6619	28793	59
2	.4231455	0618	.4670162	12537	58
3	4090	.9059386	3706	2.1396301	57
4	6725	8154	7251	80085	56
5	9360	6921	.4680797	63889	55
6	.4241994	5688	4343	47714	54
7	4628	4454	7890	31559	53
8	7262	3219	.4691438	15423	52
9	9895	1983	4988	2.1299308	51
10	.4252528	0746	8539	83213	50
11	5161	.9049509	.4702090	67137	49
12	7793	8271	5643	51082	48
13	.4260425	7032	9196	35046	47
14	3056	5792	.4712751	19030	46
15	5687	4551	6306	03034	45
16	8318	3310	9863	2.1187057	44
17	.4270949	2068	.4723420	71101	43
18	3579	0825	6978	55164	42
19	6209	.9039582	.4730538	39246	41
20	8838	8338	4098	23348	40
21	.4281467	7093	7659	07470	39
22	4095	5847	.4741222	2.1091611	38
23	6723	4600	4785	75771	37
24	9351	3353	8349	59951	36
25	.4291979	2105	.4751914	44150	35
26	4607	0856	5481	28369	34
27	7233	.9029606	9048	12607	33
28	9859	8356	.4762616	2.0996864	32
29	.4302485	7105	6185	81140	31
30	5111	5853	9755	65436	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	115	*Degrees*		64	

Min.	25	Degrees		154	
	Sine	Coſine	Tangent	Cotangent	
30	.4305111	.9025853	.4769755	2.0965436	30
31	7736	4600	.4773326	49751	29
32	.4310361	3347	6899	34084	28
33	2986	2092	.4780472	18437	27
34	5610	0838	4046	02809	26
35	8234	.9019582	7621	2.0887200	25
36	.4320857	8325	.4791197	71610	24
37	3481	7068	4774	56039	23
38	6103	5810	8352	40486	22
39	8726	4551	.4801932	24953	21
40	.4331348	3292	5512	09438	20
41	3970	2031	9093	2.0793942	19
42	6591	0770	.4812675	78465	18
43	9212	.9009508	6258	63007	17
44	.4341832	8245	9842	47567	16
45	4453	6982	.4823427	32146	15
46	7072	5718	7014	16743	14
47	9692	4453	.4830601	01359	13
48	.4352311	3187	4189	2.0685993	12
49	4930	1921	7778	70646	11
50	7548	0654	.4841368	55318	10
51	.4360166	.8999386	4959	40009	9
52	2784	8117	8552	24716	8
53	5401	6848	.4852145	09442	7
54	8018	5578	5739	2.0594187	6
55	.4370634	4307	9334	78950	5
56	3251	3035	.4862931	63732	4
57	5866	1763	6528	48531	3
58	8482	0489	.4870126	33349	2
59	.4381097	.8989215	3726	18184	1
60	3711	7940	7326	03038	0
	Coſine	Sine	Cotang.	Tangent	Min.
	115	Degrees		64	

	26	*Degrees*		153	
Min.	*Sine*	*Coſine*	*Tangent*	*Cotangent*	
0	.4383711	.8987940	.4877326	2.0503038	60
1	6326	6665	.4880927	2.0487910	59
2	8940	5389	4530	72800	58
3	.4391553	4112	8133	57708	57
4	4166	2834	.4891737	42634	56
5	6779	1515	5343	27578	55
6	9392	0276	8949	12540	54
7	.4402004	.8978996	.4902557	2.0397519	53
8	4615	7715	6166	82517	52
9	7227	6433	9775	67532	51
10	9838	5151	.4913386	52565	50
11	.4412448	3868	6997	37615	49
12	5058	2584	.4920610	22683	48
13	7668	1299	4224	07769	47
14	.4420278	0014	7838	2.0292873	46
15	2887	.8968727	.4931454	77994	45
16	5496	7440	5071	63133	44
17	8104	6152	8689	48289	43
18	.4430712	4864	.4942308	33462	42
19	3319	3575	5928	18653	41
20	5927	2285	9549	03862	40
21	8534	0994	.4953171	2.0189088	39
22	.4441140	.8959703	6794	74331	38
23	3746	8411	.4960418	59592	37
24	6352	7118	4043	44869	36
25	8957	5824	7669	30164	35
26	.4451562	4529	.4971297	15477	34
27	4167	3234	4925	00806	33
28	6771	1938	8554	2.0086153	32
29	9375	0641	.4982185	71516	31
30	.4461978	.8949344	5816	56897	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	116	*Degrees*		63	

Min.	26	Degrees		153	
	Sine	Coſine	Tangent	Cotangent	
30	.4461978	.8949344	.4985816	2.0056897	30
31	4581	8045	9449	42295	29
32	7184	6746	.4993082	27710	28
33	9786	5446	6717	13142	27
34	.4472388	4146	.5000352	1.9998590	26
35	4990	2844	3989	84056	25
36	7591	1542	7627	69539	24
37	.4480192	0239	.5011266	55038	23
38	2792	.8938936	4906	40554	22
39	5392	7632	8547	26087	21
40	7992	6326	.5022189	11637	20
41	.4490591	5021	5832	1.9897204	19
42	3190	3714	9476	82787	18
43	5789	2406	.5033121	68387	17
44	8387	1098	6767	54003	16
45	.4500985	.8929789	.5040415	39636	15
46	3582	8480	4063	25286	14
47	6179	7169	7713	10952	13
48	8776	5858	.5051363	1.9796635	12
49	.4511372	4546	5015	82334	11
50	3968	3234	8668	68050	10
51	6563	1920	.5062322	53782	9
52	9158	0606	5977	39531	8
53	.4521753	.8919291	9633	25296	7
54	4347	7975	.5073290	11077	6
55	6941	6659	6948	1.9696874	5
56	9535	5342	.5080607	82688	4
57	.4532128	4024	4267	68518	3
58	4721	2705	7928	54364	2
59	7313	1385	.5091591	40227	1
60	9905	0065	5254	26105	0
	Coſine	Sine	Cotang.	Tangent	Min.
	116	Degrees		63	

Min.	27	Degrees		152	
	Sine	Coſine	Tangent	Cotangent	
0	.4539905	.8910065	.5095254	1.9626105	60
1	.4542497	.8908744	8919	12000	59
2	5088	7422	.5102585	1.9597910	58
3	7679	6100	6252	83837	57
4	.4550269	4777	9919	69780	56
5	2859	3453	.5113588	55739	55
6	5449	2128	7259	41713	54
7	8038	0802	.5120930	27704	53
8	.4560627	.8899476	4602	13711	52
9	3216	8149	8275	1.9499733	51
10	5804	6821	.5131950	85771	50
11	8392	5493	5625	71826	49
12	.4570979	4164	9302	57896	48
13	3566	2834	.5142980	43981	47
14	6153	1503	6658	30083	46
15	8739	0171	.5150338	16200	45
16	.4581325	.8888839	4019	02333	44
17	3910	7506	7702	1.9388481	43
18	6496	6172	.5161385	74645	42
19	9080	4837	5069	60825	41
20	.4591665	3502	8755	47020	40
21	4248	2166	.5172441	33231	39
22	6832	0830	6129	19457	38
23	9415	.8879492	9818	05698	37
24	.4601998	8154	.5183508	1.9291956	36
25	4580	6815	7199	78228	35
26	7162	5475	.5190891	64516	34
27	9744	4134	4584	50819	33
28	.4612325	2792	8278	37138	32
29	4906	1451	.5201974	23472	31
30	7486	0108	5670	09821	30
	Coſine	Sine	Cotang.	Tangent	Min.
	117	Degrees		62	

Min.	27	Degrees		152	
	Sine	Coſine	Tangent	Cotangent	
30	.4617486	.8870108	.5205670	1.9209821	30
31	.4620066	.8868764	9368	1.9196186	29
32	2646	7420	.5213067	82565	28
33	5225	6075	6767	68960	27
34	7804	4730	.5220468	55370	26
35	.4630382	3383	4170	41795	25
36	2960	2036	7874	28236	24
37	5538	0688	.5231578	14691	23
38	8115	.8859339	5284	01162	22
39	.4640692	7989	8990	1.9087647	21
40	3269	6639	.5242698	74147	20
41	5845	5288	6407	60663	19
42	3420	3936	.5250117	47193	18
43	.4650996	2583	3829	33738	17
44	3571	1230	7541	20299	16
45	6145	.6849876	.5261254	06874	15
46	8719	8522	4969	1.8993464	14
47	.4661293	7166	8685	80068	13
48	3866	5810	.5272402	66688	12
49	6439	4453	6120	53322	11
50	9012	3095	9839	39971	10
51	.4671584	1736	.5283559	26634	9
52	4156	0377	7281	13313	8
53	6727	.8839017	.5291004	00006	7
54	9298	7656	4727	1.8886713	6
55	.4681869	6295	8452	73436	5
56	4439	4932	.5302178	60172	4
57	7009	3569	5906	46924	3
58	9578	2206	9634	33690	2
59	.4692147	0841	.5313364	20470	1
60	4716	.8829476	7094	07265	0
	Coſine	Sine	Cotang.	Tangent	Min.
	117	Degrees		62	

Min.	28	Degrees		151	
	Sine	Coſine	Tangent	Cotangent	
0	.4694716	.8829476	.5317094	1.8807265	60
1	7284	8110	.5320826	1.8794074	59
2	9852	6743	4559	80898	58
3	.4702419	5376	8293	67736	57
4	4986	4007	.5332029	54588	56
5	7553	2638	5765	41455	55
6	.4710119	1269	9503	28336	54
7	2685	.8819898	.5343242	15231	53
8	5250	8527	6982	02141	52
9	7815	7155	.5350723	1.8689064	51
10	.4720380	5782	4465	76003	50
11	2944	4409	8208	62955	49
12	5508	3035	.5361953	49921	48
13	8071	1660	5699	36902	47
14	.4730634	0284	9446	23896	46
15	3197	.8808907	.5373194	10905	45
16	5759	7530	6943	1.8597928	44
17	8321	6152	.5380694	84965	43
18	.4740882	4773	4445	72015	42
19	3443	3394	8198	59080	41
20	6004	2014	.5391952	46159	40
21	8564	0633	5707	33252	39
22	.4751124	.8799251	9464	20358	38
23	3683	7869	.5403221	07479	37
24	6242	6486	6980	1.8494613	36
25	8801	5102	.5410740	81761	35
26	.4761359	3717	4501	68923	34
27	3917	2332	8263	56099	33
28	6474	0946	.5422027	43289	32
29	9031	.8789559	5791	30492	31
30	.4771588	8171	9557	17709	30
	Coſine	Sine	Cotang.	Tangent	Min.
	118	Degrees		61	

Min.	28	Degrees		151	
	Sine	Coſine	Tangent	Cotangent	
30	.4771588	.8788171	.5429557	1.8417709	30
31	4144	6783	.5433324	04939	29
32	6700	5394	7092	1.8392184	28
33	9255	4004	.5440862	79442	27
34	.4781810	2613	4632	66713	26
35	4364	1222	8404	53999	25
36	6919	.8779830	.5452177	41297	24
37	9472	8437	5951	28610	23
38	.4792026	7043	9726	15936	22
39	4579	5649	.5463503	03275	21
40	7131	4254	7281	1.8290628	20
41	9683	2858	.5471060	77994	19
42	.4802235	1462	4840	65374	18
43	4786	0064	8621	52767	17
44	7337	.8768666	.5482404	40173	16
45	9888	7268	6188	27593	15
46	.4812438	5868	9973	15026	14
47	4987	4468	.5493759	02473	13
48	7537	3067	7546	1.8189932	12
49	.4820086	1665	.5501335	77405	11
50	2634	0262	5125	64892	10
51	5182	.8758859	8916	52391	9
52	7730	7455	.5512708	39904	8
53	.4330277	6051	6502	27430	7
54	2824	4645	.5520297	14969	6
55	5370	3239	4093	02521	5
56	7916	1832	7890	1.8090086	4
57	.4840462	0424	.5531688	77664	3
58	3007	.8749016	5488	65256	2
59	5552	7607	9288	52860	1
60	8096	6197	.5543090	40478	0
	Coſine	Sine	Cotang.	Tangent	Min.
	118	Degrees		61	

Min.	29	Degrees		150	
	Sine	Coſine	Tangent	Cotangent	
0	.4848096	.8746197	.5543090	1.8040478	60
1	.4850640	4786	6894	28108	59
2	3184	3375	.5550698	15751	58
3	5727	1963	4504	03408	57
4	8270	0550	8311	1.7991077	56
5	.4860812	.8739137	.5562119	78759	55
6	3354	7722	5929	66454	54
7	5895	6307	9739	54162	53
8	8436	4891	.5573551	41883	52
9	.4870977	3475	7364	29616	51
10	3517	2058	.5581179	17362	50
11	6057	0640	4994	05121	49
12	8597	.8729221	8811	1.7892893	48
13	.4881136	7801	.5592629	80678	47
14	3674	6381	6448	68475	46
15	6212	4960	.5600269	56285	45
16	8750	3538	4091	44107	44
17	.4891288	2116	7914	31943	43
18	3824	0693	.5611738	19790	42
19	6361	.8719269	5564	07651	41
20	8897	7844	9391	1.7795524	40
21	.4901433	6419	.5623219	83409	39
22	3968	4993	7048	71307	38
23	6503	3566	.5630879	59218	37
24	9037	2138	4710	47141	36
25	.4911572	0713	8543	35076	35
26	4105	.8709281	.5642378	23024	34
27	6638	7851	6213	10985	33
28	9171	6420	.5650050	1.7698958	32
29	.4921704	4989	3888	86943	31
30	4236	3557	7728	74940	30
	Coſine	Sine	Cotang.	Tangent	Min.
	119	Degrees		60	

Min.	29	*Degrees*		150	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	.4924236	.8703557	.5657728	1.7674940	30
31	6767	2124	.5661568	62950	29
32	9298	0691	5410	50972	28
33	.4931829	.8699256	9253	39007	27
34	4359	7821	.5673098	27053	26
35	6889	6386	6944	15112	25
36	9419	4949	.5680791	03183	24
37	.4941948	3512	4639	1.7591267	23
38	4476	2074	8488	79362	22
39	7005	0635	.5692339	67470	21
40	9532	.8689196	6191	55590	20
41	.4952060	7756	.5700045	43722	19
42	4587	6315	3899	31866	18
43	7113	4873	7755	20023	17
44	9639	3431	.5711612	08191	16
45	.4962165	1988	5471	1.7496371	15
46	4690	0544	9331	84564	14
47	7215	.8679100	.5723192	72768	13
48	9740	7655	7054	60984	12
49	.4972264	6209	.5730918	49213	11
50	4787	4762	4783	37453	10
51	7310	3314	8649	25705	9
52	9833	1866	.5742516	13969	8
53	.4982355	0417	6385	02245	7
54	4877	.8668967	.5750255	1.7390533	6
55	7399	7517	4126	78833	5
56	9920	6066	7999	67144	4
57	.4992441	4614	.5761873	55468	3
58	4961	3161	5748	43803	2
59	7481	1708	9625	32149	1
60	.5000000	0254	.5773503	20508	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	119	*Degrees*		60	

Min.	30	*Degrees*		149	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.5000000	.8660254	.5773503	1.7320508	60
1	2519	.8658799	7382	08878	59
2	5037	7344	.7581262	1.7297260	58
3	7556	5887	5144	85654	57
4	.5010073	4430	9027	74060	56
5	2591	2973	.5792911	62477	55
6	5107	1514	6797	50905	54
7	7624	0055	.5800684	39345	53
8	.5020140	.8648595	4573	27797	52
9	2655	7134	8462	16261	51
10	5170	5673	.5812353	04736	50
11	7685	4211	6245	1.7193222	49
12	.5030199	2748	.5820139	81720	48
13	2713	1284	4034	70230	47
14	5227	.8639820	7930	58751	46
15	7740	8355	.5831828	47283	45
16	.5040252	6889	5727	35827	44
17	2765	5423	9627	24382	43
18	5276	3956	.5843528	12949	42
19	7788	2488	7431	01527	41
20	.5050298	1019	.5851335	1.7090116	40
21	2809	.8629549	5241	78717	39
22	5319	8079	9148	67329	38
23	7828	6608	.5863056	55953	37
24	.5060338	5137	6965	44587	36
25	2846	3664	.5870876	33233	35
26	5355	2191	4788	21890	34
27	7863	0717	8702	10559	33
28	.5070370	.8619243	.5882617	1.6999238	32
29	2877	7768	6533	87929	31
30	5384	6292	.5890450	76631	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	120	*Degrees*		59	

Min.	30	Degrees		149	
	Sine	*Cosine*	*Tangent*	*Cotangent*	
30	.5075384	.8616292	.5890450	1.6976631	30
31	7890	4815	4369	65344	29
32	.5080396	3337	8289	54069	28
33	2901	1859	.5902211	42804	27
34	5406	0380	6134	31550	26
35	7910	.8608900	.5910058	20308	25
36	.5090414	7420	3983	09077	24
37	2918	5939	7910	1.6897856	23
38	5421	4457	.5921839	86647	22
39	7924	2975	5768	75449	21
40	.5100426	1491	9699	64261	20
41	2928	0007	.5933632	53085	19
42	5429	.8598523	7566	41919	18
43	7930	7037	.5941501	30765	17
44	.5110431	5551	5437	19621	16
45	2931	4064	9375	08489	15
46	5431	2576	.5953314	1.6797367	14
47	7930	1088	7254	86256	13
48	.5120429	.8589599	.5961196	75156	12
49	2927	8109	5140	64067	11
50	5425	6618	9084	52988	10
51	7922	5127	.5973030	41921	9
52	.5130420	3635	6978	30864	8
53	2916	2142	.5980927	19818	7
54	5412	0649	4877	08782	6
55	7908	.8579155	8828	1.6697758	5
56	.5140404	7660	.5992781	86744	4
57	2899	6164	6735	75741	3
58	5393	4668	.6000691	64748	2
59	7887	3171	4648	53766	1
60	.5150381	1673	8606	42795	0
	Cosine	*Sine*	*Cotang.*	*Tangent*	Min.
	120	Degrees		59	

Min.	31		Degrees		148
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.5150381	.8571673	.6008606	1.6642795	60
1	2874	0174	.6012566	31834	59
2	5367	.8568675	6527	20884	58
3	7859	7175	.6020490	09945	57
4	.5160351	5674	4454	1.6599016	56
5	2842	4173	8419	88097	55
6	5333	2671	.6032386	77189	54
7	7824	1168	6354	66292	53
8	.5170314	.8559664	.6040323	55405	52
9	2804	8160	4294	44529	51
10	5293	6655	8266	33663	50
11	7782	5149	.6052240	22808	49
12	.5180270	3643	6215	11963	48
13	2758	2135	.6060192	01128	47
14	5246	0627	4170	1.6490304	46
15	7733	.8549119	8149	79490	45
16	.5190219	7609	.6072130	68686	44
17	2705	6099	6112	57893	43
18	5191	4588	.6080095	47111	42
19	7676	3077	4080	36338	41
20	.5200161	1564	8067	25576	40
21	2646	0051	.6092054	14824	39
22	5130	.8538538	6043	04082	38
23	7613	7023	.6100034	1.6393351	37
24	.5210096	5508	4026	82630	36
25	2579	3992	8019	71919	35
26	5061	2475	.6112014	61218	34
27	7543	0958	6011	50528	33
28	.5220024	.8529440	.6120008	39847	32
29	2505	7921	4007	29177	31
30	4986	6402	8008	18517	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	121		Degrees		58

Min.	31	Degrees		148	
	Sine	Coſine	Tangent	Cotangent	
30	.5224986	.8526402	.6128008	1.6318517	30
31	7466	4881	.6132010	07867	29
32	9945	3360	6013	1.6297227	28
33	.5232424	1839	.6140018	86597	27
34	4903	0316	4024	75977	26
35	7381	.8518793	8032	65368	25
36	9859	7269	.6152041	54768	24
37	.5242336	5745	6052	44178	23
38	4813	4219	.6160064	33599	22
39	7290	2693	4077	23029	21
40	9766	1167	8092	12469	20
41	.5252241	.8509639	.6172108	01920	19
42	4716	8111	6126	1.6191380	18
43	7191	6582	.6180145	80850	17
44	9665	5053	4166	70330	16
45	.5262139	3522	8188	59820	15
46	4613	1991	.6192211	49320	14
47	7085	0459	6236	38829	13
48	9558	.8498927	.6200263	28349	12
49	.5272030	7394	4291	17878	11
50	4502	5860	8320	07417	10
51	6973	4325	.6212351	1.6096966	9
52	9443	2790	6383	86525	8
53	.5281914	1254	.6220417	76094	7
54	4383	.8489717	4452	65672	6
55	6853	8179	8488	55260	5
56	9322	6641	.6232526	44858	4
57	.5291790	5102	6566	34465	3
58	4258	3562	.6240607	24082	2
59	6726	2022	4650	13709	1
60	9193	0481	8694	03345	0
	Coſine	Sine	Cotang.	Tangent	Min.
	121	Degrees		58	

Min.	32	Degrees		147	
	Sine	Coſine	Tangent	Cotangent	
0	.5299193	.8480481	.6248694	1.6003345	60
1	.5301659	.8478939	.6252739	1.5992991	59
2	4125	7396	6786	82647	58
3	6591	5853	.6260834	72312	57
4	9056	4309	4884	61987	56
5	.5311521	2765	8935	51672	55
6	3986	1219	.6272988	41366	54
7	6450	.8469673	7042	31070	53
8	8913	8126	.6281098	20783	52
9	.5321376	6579	5156	10505	51
10	3839	5030	9215	00238	50
11	6301	3481	.6293275	1.5889979	49
12	8763	1932	7336	79730	48
13	.5331224	0381	.6301399	69491	47
14	3685	.8458830	5464	59261	46
15	6145	7278	9530	49041	45
16	8605	5726	.6313598	38830	44
17	.5341064	4172	7667	28628	43
18	3523	2619	.6321738	18436	42
19	5982	1063	5810	08253	41
20	8440	.8449508	9883	1.5798079	40
21	.5350898	7952	.6333958	87915	39
22	3355	6395	8035	77760	38
23	5812	4838	.6342113	67615	37
24	8268	3279	6193	57479	36
25	.5360724	1720	.6350274	47352	35
26	3179	0160	4357	37234	34
27	5634	.8438600	8441	27126	33
28	8088	7039	.6362527	17026	32
29	.5370542	5477	6614	06936	31
30	2996	3914	.6370703	1.5696856	30
	Coſine	Sine	Cotang.	Tangent	Min.
	122	Degrees		57	

Min.	32	Degrees		147	
	Sine	Coſine	Tangent	Cotangent	
30	.5372996	.8433914	.6370703	1.5696856	30
31	5449	2351	4793	86784	29
32	7902	0787	8885	76722	28
33	.5380354	.8429222	.6382978	66669	27
34	2806	7657	7073	56625	26
35	5257	6091	.6391169	46590	25
36	7708	4524	5267	36564	24
37	.5390158	2956	9366	26548	23
38	2608	1388	.6403467	16540	22
39	5058	.8419819	7569	06542	21
40	7507	8249	.6411673	1.5596552	20
41	9955	6679	5779	86572	19
42	.5402403	5108	9886	76601	18
43	4851	3536	.6423995	66639	17
44	7298	1963	8105	56685	16
45	9745	0390	.6432216	46741	15
46	.5412191	.8408816	6329	36806	14
47	4637	7241	.6440444	26880	13
48	7082	5666	4560	16963	12
49	9527	4090	8678	07054	11
50	.5421971	2513	.6452797	1.5497155	10
51	4415	0935	6918	87264	9
52	6859	.8399357	.6461041	77383	8
53	9302	7778	5165	67510	7
54	.5431744	6199	9290	57646	6
55	4187	4618	.6473417	47792	5
56	6628	3037	7546	37946	4
57	9069	1455	.6481676	28108	3
58	.5441510	.8389873	5808	18280	2
59	3950	8290	9941	08460	1
60	6390	6706	.6494076	1.5398650	0
	Coſine	Sine	Cotang.	Tangent	Min.
	122	Degrees		57	

Min.	33	*Degrees*		146	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.5446390	.8386706	.6494076	1.5398650	60
1	8830	5121	8212	88848	59
2	.5451269	3536	.6502350	79055	58
3	3707	1950	6490	69270	57
4	6145	0363	.6510631	59494	56
5	8583	.8378775	4774	49727	55
6	.5461020	7187	8918	39969	54
7	3456	5598	.6523064	30220	53
8	5892	4009	7211	20479	52
9	8328	2418	.6531360	10747	51
10	.5470763	0827	5511	01023	50
11	3198	.8369236	9663	1.5291308	49
12	5632	7643	.6543817	81602	48
13	8066	6050	7972	71904	47
14	.5480499	4456	.6552129	62215	46
15	2932	2862	6287	52535	45
16	5365	1266	.6560447	42863	44
17	7797	.8359670	4609	33200	43
18	.5490228	8074	8772	23545	42
19	2659	6476	.6572937	13899	41
20	5090	4878	7103	04261	40
21	7520	3279	.6581271	1.5194632	39
22	9950	1680	5441	85012	38
23	.5502379	0080	9612	75400	37
24	4807	.8348479	.6593785	65796	36
25	7236	6877	7959	56201	35
26	9663	5275	.6602135	46614	34
27	.5512091	3672	6313	37036	33
28	4518	2068	.6610492	27466	32
29	6944	0463	4673	17905	31
30	9370	.8338858	8856	08352	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	123	*Degrees*		56	

Min.	33	Degrees		146	
	Sine	Coſine	Tangent	Cotangent	
30	.5519370	.8338858	.6618856	1.5108352	30
31	.5521795	7252	.6623040	1.5098807	29
32	4220	5646	7226	89271	28
33	6645	4038	.6631413	79743	27
34	9069	2430	5602	70224	26
35	.5531492	0822	9792	60713	25
36	3915	.8329212	.6643984	51210	24
37	6338	7602	8178	41716	23
38	8760	5991	.6652373	32230	22
39	.5541182	4380	6570	22752	21
40	3603	2768	.6660769	13282	20
41	6024	1155	4969	03820	19
42	8444	.8319541	9171	1.4994367	18
43	.5550864	7927	.6673375	84922	17
44	3283	6312	7580	75486	16
45	5702	4696	.6681787	66058	15
46	8121	3080	5995	56638	14
47	.5560539	1462	.6690205	47226	13
48	2956	.8309845	4417	37822	12
49	5373	8226	8630	28426	11
50	7790	6607	.6702845	19038	10
51	.5570206	4987	7062	09659	9
52	2621	3366	.6711280	00288	8
53	5036	1745	5500	1.4890925	7
54	7451	0123	9721	81570	6
55	9865	.8298500	.6723944	72223	5
56	.5582279	6877	8169	62884	4
57	4692	5252	.6732396	53553	3
58	7105	3628	6624	44230	2
59	9517	2002	.6740854	34916	1
60	.5591929	0376	5085	25610	0
	Coſine	Sine	Cotang.	Tangent	Min.
	123	Degrees		56	

Min.	34	Degrees		145	
	Sine	Coſine	Tangent	Cotangent	
0	.5591929	.8290376	.6745085	1.4825610	60
1	4340	.8288749	9318	16311	59
2	6751	7121	.6753553	07021	58
3	9162	5493	7790	1.4797738	57
4	.5601571	3864	.6762028	88463	56
5	3981	2234	6268	79197	55
6	6390	0603	.6770509	69938	54
7	8798	.8278972	4752	60688	53
8	.5611206	7340	8997	51445	52
9	3614	5707	.6783244	42210	51
10	6021	4074	7492	32983	50
11	8428	2440	.6791742	23764	49
12	.5620834	0806	5993	14553	48
13	3239	.8269170	.6800246	05350	47
14	5644	7534	4501	1.4696155	46
15	8049	5897	8758	86967	45
16	.5630453	4260	.6813016	77787	44
17	2857	2622	7276	68616	43
18	5260	0983	.6821538	59452	42
19	7663	.8259343	5801	50296	41
20	.5640066	7703	.6830066	41147	40
21	2467	6062	4333	32007	39
22	4869	4420	8601	22874	38
23	7270	2778	.6842871	13749	37
24	9670	1135	7143	04632	36
25	.5652070	.8249491	.6851417	1.4595522	35
26	4469	7847	5692	86420	34
27	6868	6202	9969	77326	33
28	9267	4556	.6864247	68240	32
29	.5661665	2909	8527	59161	31
30	4062	1262	.6872810	50090	30
	Coſine	Sine	Cotang.	Tangent	Min.
	124	Degrees		55	

Min.	34	Degrees		145	
	Sine	Coſine	Tangent	Cotangent	
30	.5664062	.8241262	.6872810	1.4550090	30
31	6459	.8239614	7094	41027	29
32	8856	7965	.6881379	31971	28
33	.5671252	6316	5666	22923	27
34	3648	4666	9955	13883	26
35	6043	3015	.6894246	04850	25
36	8437	1364	8538	1.4495825	24
37	.5680832	.8229712	.6902832	56808	23
38	3225	8059	7128	77798	22
39	5618	6405	.6911425	68796	21
40	8011	4751	5724	59801	20
41	.5690403	3096	.6920025	50814	19
42	2795	1440	4328	41834	18
43	5186	.8219784	8633	32862	17
44	7577	8127	.6932939	23897	16
45	9968	6469	7247	14940	15
46	.5702357	4811	.6941557	05991	14
47	4747	3152	5868	1.4397049	13
48	7136	1492	.6950181	88114	12
49	9524	.8209832	4496	79187	11
50	.5711912	8170	8813	70268	10
51	4299	6508	.6963131	61356	9
52	6686	4846	7451	52451	8
53	9073	3183	.6971773	43554	7
54	.5721459	1519	6097	34664	6
55	3844	.8199854	.6980422	25781	5
56	6229	8189	4749	16906	4
57	8614	6523	9078	08039	3
58	.5730998	4856	.6993409	1.4299178	2
59	3381	3189	7741	90326	1
60	5764	1520	.7002075	81480	0
	Coſine	Sine	Cotang.	Tangent	Min.
	124	Degrees		55	

Min.	35	Degrees		144	
	Sine	Coſine	Tangent	Cotangent	
0	.5735764	.8191520	.7002075	1.4281480	60
1	8147	.8189852	6411	72642	59
2	.5740529	8182	.7010749	63811	58
3	2911	6512	5089	54987	57
4	5292	4841	9430	46171	56
5	7672	3169	.7023773	37362	55
6	.5750052	1497	8118	28561	54
7	2432	.8179824	.7032465	19766	53
8	4811	8150	6813	10979	52
9	7190	6476	.7041163	02200	51
10	9568	4801	5515	1.4193427	50
11	.5761946	3125	9869	84662	49
12	4323	1449	.7054224	75904	48
13	6700	.8169772	8581	67153	47
14	9076	8094	.7062940	58409	46
15	.5771452	6416	7301	49673	45
16	3827	4736	.7071664	40943	44
17	6202	3056	6029	32221	43
18	8576	1376	.7080395	23506	42
19	.5780950	.8159695	4763	14799	41
20	3323	8013	9133	06098	40
21	5696	6330	.7093505	1.4097405	39
22	8068	4647	7878	88718	38
23	.5790440	2963	.7102253	80039	37
24	2812	1278	6630	71367	36
25	5183	.8149593	.7111009	62702	35
26	7553	7906	5390	54044	34
27	9923	6220	9773	45393	33
28	.5802292	4532	.7124157	36749	32
29	4661	2844	8543	28113	31
30	7030	1155	.7132931	19483	30
	Coſine	Sine	Cotang.	Tangent	Min.
	125	Degrees		54	

Min.	35	Degrees		144	
	Sine	Coſine	Tangent	Cotangent	
30	.5807030	.8141155	.7132931	1.4019483	30
31	9397	.8139465	7321	10860	29
32	.5811765	7775	.7141713	1.3902245	28
33	4132	6084	6106	93636	27
34	6498	4393	.7150501	85034	26
35	8864	2701	4898	76440	25
36	.5821230	1008	9297	67852	24
37	3595	.8129314	.7163698	59272	23
38	5959	7620	8101	50698	22
39	8323	5925	.7172505	42131	21
40	.5830687	4229	6911	33571	20
41	3050	2532	.7181319	25018	19
42	5412	0835	5729	16473	18
43	7774	.8119137	.7190141	07934	17
44	.5840136	7439	4555	1.3899401	16
45	2497	5740	8970	90876	15
46	4857	4040	.7203387	82358	14
47	7217	2339	7806	73846	13
48	9577	0638	.7212227	65342	12
49	.5851936	.8108936	6650	56844	11
50	4294	7234	.7221075	48353	10
51	6652	5530	5502	39869	9
52	9010	3826	9931	31392	8
53	.5861367	2122	.7234361	22922	7
54	3724	0416	8793	14458	6
55	6080	.8098710	.7243227	06001	5
56	8435	7004	7663	1.3797551	4
57	.5870790	5296	.7252101	89108	3
58	3145	3588	6541	80672	2
59	5499	1879	.7260983	72242	1
60	7853	0170	5426	63819	0
	Coſine	Sine	Cotang.	Tangent	Min.
	125	Degrees		54	

Min.	36	*Degrees*		143	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.5877853	.8090170	.7265426	1.3763819	60
1	.5880206	.8088460	9871	55403	59
2	2558	6749	.7274318	46994	58
3	4910	5037	8767	38591	57
4	7262	3325	.7283218	30195	56
5	9613	1612	7671	21805	55
6	.5891964	.8079898	.7292126	13423	54
7	4314	8185	6582	05047	53
8	6663	6470	.7301040	1.3696678	52
9	9012	4754	5501	88315	51
10	.5901361	3038	9963	79959	50
11	3709	1321	.7314427	71610	49
12	6057	.8069603	8894	63267	48
13	8404	7885	.7323362	54931	47
14	.5910750	6166	7831	46602	46
15	3096	4446	.7332303	38279	45
16	5442	2726	6777	29963	44
17	7787	1005	.7341253	21653	43
18	.5920132	.8059283	5730	13350	42
19	2476	7560	.7350210	05054	41
20	4819	5837	4691	1.3596764	40
21	7163	4113	9174	88481	39
22	9505	2389	.7363660	80204	38
23	.5931847	0664	8147	71934	37
24	4189	.8048938	.7372636	63670	36
25	6530	7211	7127	55413	35
26	8871	5484	.7381620	47162	34
27	.5941211	3756	6115	38918	33
28	3550	2028	.7390611	30680	32
29	5889	0299	5110	22449	31
30	8228	.8038569	9611	14224	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	126	*Degrees*		53	

Min.	36	Degrees		143	
	Sine	Coſine	Tangent	Cotangent	
30	.5948228	.8038569	.7399611	1.3514224	30
31	.5950566	6838	.7404114	06006	29
32	2903	5107	8618	1.3497794	28
33	5240	3375	.7413124	89589	27
34	7577	1642	7633	81390	26
35	9913	.8029909	.7422143	73197	25
36	.5962249	8175	6655	65011	24
37	4584	6440	.7431170	56832	23
38	6918	4705	5686	48658	22
39	9252	2969	.7440204	40492	21
40	.5971586	1232	4724	32331	20
41	3919	.8019495	9246	24177	19
42	6251	7756	.7453770	16029	18
43	8583	6018	8296	07888	17
44	.5980915	4278	.7462824	1.3399753	16
45	3246	2538	7354	91624	15
46	5576	0797	.7471886	83502	14
47	7906	.8009056	6420	75386	13
48	.5990236	7314	.7480956	67276	12
49	2565	5571	5494	59172	11
50	4893	3827	.7490033	51075	10
51	7221	2083	4575	42984	9
52	9549	0338	9119	34900	8
53	.6001876	.7998593	.7503665	26822	7
54	4202	6847	8212	18749	6
55	6528	5100	.7512762	10684	5
56	8853	3352	7314	02624	4
57	.6011179	1604	.7521867	1.3294571	3
58	3503	.7989855	6423	86524	2
59	5827	8105	.7530981	78483	1
60	8150	6355	5540	70448	0
	Coſine	Sine	Cotang.	Tangent	Min.
	126	Degrees		53	

NATURAL SINES

Min.	37	*Degrees*		142	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	.6018150	·7986355	·7535540	1.3270448	60
1	.6020473	4604	·7540102	62420	59
2	2795	2853	4666	54397	58
3	5117	1100	9232	46381	57
4	7439	·7979347	·7553799	38371	56
5	9760	7594	8369	30368	55
6	.6032080	5839	·7562941	22370	54
7	4400	4084	7514	14379	53
8	6719	2329	·7572090	06393	52
9	9038	0572	6668	1.3198414	51
10	.6041356	7·968815	·7581248	90441	50
11	3674	7058	5829	82474	49
12	5991	5299	·7590413	74513	48
13	8308	3540	4999	66559	47
14	.6050624	1780	9587	58610	46
15	2940	0020	·7604177	50668	45
16	5255	·7958259	8769	42731	44
17	7570	6497	·7613363	34801	43
18	9884	4735	7959	26876	42
19	.6062198	2972	·7622557	18958	41
20	4511	1208	7157	11046	40
21	6823	·7949444	·7631759	03140	39
22	9136	7678	6363	1.3095239	38
23	.6071447	5913	·7640969	87345	37
24	3758	4146	5577	79457	36
25	6069	2379	·7650188	71575	35
26	8379	0611	4800	63699	34
27	.6080689	·7938843	9414	55828	33
28	2998	7074	·7664031	47964	32
29	5306	5304	8649	40106	31
30	7614	3533	·7673270	32254	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	127	*Degrees*		52	

Min.	37	Degrees		142	
	Sine	Coſine	Tangent	Cotangent	
30	.6087614	.7933533	.7673270	1.3032254	30
31	9922	1762	7893	24407	29
32	.6092229	.7929990	.7682517	16567	28
33	4535	8218	7144	08732	27
34	6841	6445	.7691773	00904	26
35	9147	4671	6404	1.2993081	25
36	.6101452	2896	.7701037	85265	24
37	3756	1121	5672	77454	23
38	6060	.7919345	.7710309	69649	22
39	8363	7569	4948	61850	21
40	.6110666	5792	9589	54057	20
41	2969	4014	.7724233	46269	19
42	5270	2235	8879	38488	18
43	7572	0456	.7733526	30712	17
44	9873	.7908676	8175	22943	16
45	.6122173	6896	.7742827	15179	15
46	4473	5115	7481	07421	14
47	6772	3333	.7752137	1.2899669	13
48	9071	1550	6795	91922	12
49	.6131369	.7899767	.7761455	84182	11
50	3666	7983	6117	76447	10
51	5964	6198	.7770782	68718	9
52	8260	4413	5448	60995	8
53	.6140556	2627	.7780117	53277	7
54	2852	0841	4788	45566	6
55	5147	.7889054	9460	37860	5
56	7442	7266	.7794135	30160	4
57	9736	5477	8812	22466	3
58	.6152029	3688	.7803492	14776	2
59	4322	1898	8173	07093	1
60	6615	0108	.7812856	1.2799416	0
	Coſine	Sine	Cotang.	Tangent	Min.
	127	Degrees		52	

Min.	38	Degrees		141	
	Sine	Coſine	Tangent	Cotangent	
0	.6156615	.7880108	.7812856	1.2799416	60
1	8907	.7878316	7542	91745	59
2	.6161198	6524	.7822229	84079	58
3	3489	4732	6919	76419	57
4	5779	2939	.7831611	68764	56
5	8069	1145	6305	61116	55
6	.6170359	.7869350	.7841002	53473	54
7	2648	7555	5700	45836	53
8	4936	5759	.7850400	38204	52
9	7224	3963	5103	30578	51
10	9511	2165	9808	22957	50
11	.6181798	0367	.7864515	15342	49
12	4084	.7858569	9224	07733	48
13	6370	6770	.7873935	00130	47
14	8655	4970	8649	1.2692532	46
15	.6190939	3169	.7883364	84939	45
16	3224	1368	8082	77353	44
17	5507	.7849566	.7892802	69772	43
18	7790	7764	7524	62196	42
19	.6200073	5961	.7902248	54626	41
20	2355	4157	6975	47062	40
21	4636	2352	.7911703	39503	39
22	6917	0547	6434	31950	38
23	9198	.7838741	.7921167	24402	37
24	.6211478	6935	5902	16860	36
25	3757	5127	.7930640	09323	35
26	6036	3320	5379	01792	34
27	8314	1511	.7940121	1.2594267	33
28	.6220592	.7829702	4865	86747	32
29	2870	7892	9611	79232	31
30	5146	6082	.7954359	71723	30
	Coſine	Sine	Cotang.	Tangent	
	128	Degrees		51	Min.

Min.	38	*Degrees*		141	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	.6225146	.7826082	.7954359	1.2571723	30
31	7423	4270	9110	64219	29
32	9698	2459	.7963862	56721	28
33	.6231974	0646	8617	49229	27
34	4248	.7818833	.7973374	41742	26
35	6522	7019	8134	34260	25
36	8796	5205	.7982895	26784	24
37	.6241069	3390	7659	19313	23
38	3342	1574	.7992425	11848	22
39	5614	.7809757	7193	04388	21
40	7885	7940	.8001963	1.2496933	20
41	.6250156	6122	6736	89484	19
42	2427	4304	.8011511	82040	18
43	4696	2485	6288	74602	17
44	6966	0665	.8021067	67169	16
45	9235	.7798845	5848	59742	15
46	.6261503	7024	.8030632	52320	14
47	3771	5202	5418	44903	13
48	6038	3380	.8040206	37492	12
49	8305	1557	4997	30086	11
50	.6270571	.7789733	9790	22685	10
51	2837	7909	.8054585	15290	9
52	5102	6083	9382	07900	8
53	7366	4258	.8064181	00515	7
54	9631	2431	8983	1.2393136	6
55	.6281894	0604	.8073787	85762	5
56	4157	.7778777	8593	78393	4
57	6420	6949	.8083401	71030	3
58	8682	5120	8212	63672	2
59	.6290943	3290	.8093025	56319	1
60	3204	1460	7840	48972	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	128	*Degrees*		51	

Min.	39	Degrees		140	
	Sine	Coſine	Tangent	Cotangent	
0	.6293204	.7771460	.8097840	1.2348972	60
1	5464	.7769629	.8102658	41629	59
2	7724	7797	7478	34292	58
3	9983	5965	.8112300	26961	57
4	.6302242	4132	7124	19634	56
5	4500	2298	.8121951	12313	55
6	6758	0464	6780	04997	54
7	9015	.7758629	.8131611	1.2297687	53
8	.6311272	6794	6444	90381	52
9	3528	4957	.8141280	83081	51
10	5784	3121	6118	75786	50
11	8039	1283	.8150958	68496	49
12	.6320293	.7749445	5801	61211	48
13	2547	7606	.8160646	53932	47
14	4800	5767	5493	46658	46
15	7053	3926	.8170343	39389	45
16	9306	2086	5195	32125	44
17	.6331557	0244	.8180049	24866	43
18	3809	.7738402	4905	17613	42
19	6059	6559	9764	10364	41
20	8310	4716	.8194625	03121	40
21	.6340559	2872	9488	1.2195883	39
22	2808	1027	.8204354	88650	38
23	5057	.7729182	9222	81422	37
24	7305	7336	.8214093	74199	36
25	9553	5489	8965	66982	35
26	.6351800	3642	.8223840	59769	34
27	4046	1794	8718	52562	33
28	6292	.7719945	.8233597	45359	32
29	8537	8096	8479	38162	31
30	.6360782	6246	.8243364	30970	30
	Coſine	Sine	Cotang.	Tangent	Min.
	129	Degrees		50	

Min.	39	Degrees		140	
	Sine	Coſine	Tangent	Cotangent	
30	.6360782	.7716246	.8243364	1.2130970	30
31	3026	4395	8251	23783	29
32	5270	2544	.8253140	16601	28
33	7513	0692	8031	09424	27
34	9756	.7708839	.8262925	02252	26
35	.6371998	6986	7821	1.2095085	25
36	4240	5132	.8272719	87923	24
37	6481	3278	7620	80767	23
38	8721	1423	.8282523	73615	22
39	.6380961	.7699567	7429	66468	21
40	3201	7710	.8292337	59327	20
41	5440	5853	7247	52190	19
42	7678	3996	.8302160	45058	18
43	9916	2137	7075	37931	17
44	.6392153	0278	.8311992	30810	16
45	4390	.7688418	6912	23693	15
46	6626	6558	.8321834	16581	14
47	8862	4697	6759	09475	13
48	.6401097	2835	.8331686	02373	12
49	3332	0973	6615	1.1995276	11
50	5566	.7679110	.8341547	88184	10
51	7799	7246	6481	81097	9
52	.6410032	5382	.8351418	74015	8
53	2264	3517	6357	66938	7
54	4496	1651	.8361298	59866	6
55	6728	.7669785	6242	52799	5
56	8958	7918	.8371188	45736	4
57	.6421189	6051	6136	38679	3
58	3418	4183	.8381087	31626	2
59	5647	2314	6040	24579	1
60	7876	0444	.8390996	17536	0
	Coſine	Sine	Cotang.	Tangent	Min.
	129	Degrees		50	

NATURAL SINES

Min.	40	Degrees		139	
	Sine	Cosine	Tangent	Cotangent	
0	.6427876	.7660444	.8390996	1.1917536	60
1	.6430104	.7658574	5954	10498	59
2	2332	6704	.8400915	03465	58
3	4559	4832	5878	1.1896437	57
4	6785	2960	.8410844	89414	56
5	9011	1087	5812	82395	55
6	.6441236	.7649214	.8420782	75382	54
7	3461	7340	5755	68373	53
8	5685	5465	.8430730	61369	52
9	7909	3590	5708	54370	51
10	.6450132	1714	.8440688	47376	50
11	2355	.7639837	5670	40387	49
12	4577	7960	.8450655	33402	48
13	6798	6082	5643	26422	47
14	9019	4204	.8460633	19447	46
15	.6461240	2325	5625	12477	45
16	3460	0445	.8470620	05512	44
17	5679	.7628564	5617	1.1798551	43
18	7898	6683	.8480617	91595	42
19	.6470116	4802	5619	84644	41
20	2334	2919	.8490624	77698	40
21	4551	1036	5631	70756	39
22	6767	.7619152	.8500640	63820	38
23	8983	7268	5652	56888	37
24	.6481199	5383	.8510667	49960	36
25	3414	3497	5684	43038	35
26	5628	1611	.8520704	36120	34
27	7842	.7609724	5726	29207	33
28	.6490055	7837	.8530750	22298	32
29	2268	5949	5777	15395	31
30	4480	4060	.8540807	08496	30
	Cosine	Sine	Cotang.	Tangent	Min.
	130	Degrees		49	

Min.	40	Degrees		139	
	Sine	Cosine	Tangent	Cotangent	
30	.6494480	.7604060	.8540807	1.1708496	30
31	6692	2170	5839	01601	29
32	8903	0280	.8550873	1.1694712	28
33	.6501114	.7598389	5910	87827	27
34	3324	6498	.8560950	80947	26
35	5533	4606	5992	74071	25
36	7742	2713	.8571037	67200	24
37	9950	0820	6084	60334	23
38	.6512158	.7588926	.8581133	53472	22
39	4366	7031	6185	46615	21
40	6572	5136	.8591240	39763	20
41	8778	3240	6297	32916	19
42	.6520984	1343	.8601357	26073	18
43	3189	.7579446	6419	19234	17
44	5394	7548	.8611484	12400	16
45	7598	5650	6551	05571	15
46	9801	3751	.8621621	1.1598747	14
47	.6532004	1851	6693	91927	13
48	4206	.7569951	.8631768	85111	12
49	6408	8049	6846	78301	11
50	8609	6148	.8641926	71495	10
51	.6540810	4245	7009	64693	9
52	3010	2343	.8652094	57896	8
53	5209	0439	7181	51104	7
54	7408	.7558535	.8662271	44316	6
55	9607	6630	7364	37532	5
56	.6551804	4724	.8672460	30754	4
57	4002	2818	7558	23979	3
58	6198	0911	.8682659	17210	2
59	8395	.7549004	7762	10445	1
60	.6560590	7096	.8692868	03684	0
	Cosine	Sine	Cotang.	Tangent	Min.
	130	Degrees		49	

NATURAL SINES

Min.	41	Degrees		138	
	Sine	Coſine	Tangent	Cotangent	
0	.6560590	.7547096	.8692868	1.1503684	60
1	2785	5187	7976	1.1496928	59
2	4980	3278	.8703087	90176	58
3	7174	1368	8200	83429	57
4	9367	.7539457	.8713316	76687	56
5	.6571560	7546	8445	69949	55
6	3752	5634	.8723556	63215	54
7	5944	3721	8680	56486	53
8	8135	1808	.8733806	49762	52
9	.6580326	.7529894	8935	43041	51
10	2516	7980	.8744067	36326	50
11	4706	6065	9201	29615	49
12	6895	4149	.8754338	22908	48
13	9083	2233	9478	16206	47
14	.6591271	0316	.8764620	09508	46
15	3458	.7518398	9765	02815	45
16	5645	6480	.8774912	1.1396126	44
17	7831	4561	.8780062	89441	43
18	.6600017	2641	5215	82761	42
19	2202	0721	.8790370	76085	41
20	4386	.7508800	5528	69414	40
21	6570	6879	.8800689	62747	39
22	8754	4957	5852	56085	38
23	.6610936	3034	.8811018	49427	37
24	3119	1111	6186	42773	36
25	5300	.7499187	.8821357	36124	35
26	7481	7262	6531	29479	34
27	9662	5337	.8831707	22839	33
28	.6621842	3411	6886	16203	32
29	4022	1484	.8842068	09571	31
30	6200	.7489557	7253	02944	30
	Coſine	Sine	Cotang.	Tangent	Min.
	131	Degrees		48	

Min.	41	*Degrees*		138	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	.6626200	.7489557	.8847253	1.1302944	30
31	8379	7629	.8852440	1.1296321	29
32	.6630557	5701	7630	89702	28
33	2734	3772	.8862822	83088	27
34	4910	1842	8017	76478	26
35	7087	.7479912	.8873215	69872	25
36	9262	7981	8416	63271	24
37	.6641437	6049	.8883620	56674	23
38	3611	4117	8826	50081	22
39	5785	2184	.8894034	43493	21
40	7959	0251	9245	36909	20
41	.6650131	.7468317	.8904459	30329	19
42	2304	6382	9675	23754	18
43	4475	4446	.8914894	17183	17
44	6646	2510	.8920116	10616	16
45	8817	0574	5341	04053	15
46	.6660987	.7458636	.8930569	1.1197495	14
47	3156	6699	5799	90941	13
48	5325	4760	.8941032	84391	12
49	7493	2821	6268	77846	11
50	9661	0881	.8951506	71305	10
51	.6671828	.7448941	6747	64768	9
52	3994	6999	.8961991	58235	8
53	6160	5058	7238	51706	7
54	8326	3115	.8972487	45182	6
55	.6680490	1172	7739	38662	5
56	2655	.7439229	.8982994	32146	4
57	4818	7285	8252	25635	3
58	6981	5340	.8993512	19127	2
59	9144	3394	8775	12624	1
60	.6691306	1448	.9004041	06125	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	131	*Degrees*		48	

Min.	42	Degrees		137	
	Sine	Coſine	Tangent	Cotangent	
0	.6691306	.7431448	.9004041	1.1106125	60
1	3467	.7429501	9309	1.1099630	59
2	5628	7554	.9014580	93140	58
3	7789	5606	9854	86653	57
4	9948	3657	.9025131	80171	56
5	.6702108	1708	.9030411	73693	55
6	4266	.7419758	5694	67219	54
7	6424	7808	.9040979	60750	53
8	8582	5857	6267	54284	52
9	.6710739	3905	.9051558	47823	51
10	2895	1953	6851	41365	50
11	5051	0000	.9062147	34912	49
12	7206	.7408046	7446	28463	48
13	9361	6092	.9072748	22019	47
14	.6721515	4137	8053	15578	46
15	3668	2181	.9083360	09141	45
16	5821	0225	8671	02709	44
17	7973	.7398268	.9093984	1.0996281	43
18	.6730125	6311	9300	89856	42
19	2276	4353	.9104619	83436	41
20	4427	2394	9941	77020	40
21	6577	0435	.9115265	70608	39
22	8727	.7388475	.9120592	64201	38
23	.6740876	6515	5922	57797	37
24	3024	4553	.9131255	51397	36
25	5172	2592	6591	45002	35
26	7319	0629	9141929	38610	34
27	9466	.7378666	7270	32223	33
28	.6751612	6703	.9152615	25840	32
29	3757	4738	7962	19460	31
30	5902	2773	.9163312	13085	30
	Coſine	Sine	Cotang.	Tangent	Min.
	132	Degrees		47	

Min.	42	Degrees		137	
	Sine	Coſine	Tangent	Cotangent	
30	.6755902	.7372773	.9163312	1.0913085	30
31	8046	0808	8665	06714	29
32	.6760190	.7368842	.9174020	00347	28
33	2333	6875	9379	1.0893983	27
34	4476	4907	.9184740	87624	26
35	6618	2939	.9190104	81269	25
36	8760	0971	5471	74918	24
37	.6770901	.7359002	.9200841	68571	23
38	3041	7032	6214	62228	22
39	5181	5061	.9211590	55889	21
40	7320	3090	6968	49554	20
41	9459	1118	.9222350	43223	19
42	.6781597	.7349146	7734	36896	18
43	3734	7173	.9233122	30573	17
44	5871	5199	8512	24254	16
45	8007	3225	.9243905	17939	15
46	.6790143	1250	9301	11628	14
47	2278	.7339275	.9254700	05321	13
48	4413	7299	.9260101	1.0799018	12
49	6547	5322	5506	92718	11
50	8681	3345	.9270914	86423	10
51	.6800813	1367	6324	80132	9
52	2946	.7329388	.9281738	73844	8
53	5078	7409	7154	67561	7
54	7209	5429	.9292573	61282	6
55	9339	3449	7996	55006	5
56	.6811469	1467	.9303421	48734	4
57	3599	.7319486	8849	42467	3
58	5728	7503	.9314280	36203	2
59	7856	5521	9714	29943	1
60	9984	3537	.9325151	23687	0
	Coſine	Sine	Cotang.	Tangent	Min.
	132	Degrees		47	

43 *Degrees* 136

Min.	*Sine*	*Coſine*	*Tangent*	*Cotangent*	
0	.6819984	.7313537	.9325151	1.0723687	60
1	.6822111	1553	.9330591	17435	59
2	4237	.7309568	6034	11187	58
3	6363	7583	.9341479	04943	57
4	8489	5597	6928	1.0698702	56
5	.6830613	3610	.9352380	92466	55
6	2738	1623	7834	86233	54
7	4861	.7299635	.9363292	80004	53
8	6984	7646	8753	73779	52
9	9107	5657	.9374216	67558	51
10	.6841229	3668	9683	61341	50
11	3350	1677	.9385152	55128	49
12	5471	.7289686	.9390625	48918	48
13	7591	7695	6101	42713	47
14	9711	5702	.9401579	36511	46
15	.6851830	3710	7061	30313	45
16	3948	1716	.9412545	24119	44
17	6066	.7279722	8033	17929	43
18	8184	7728	.9423523	11742	42
19	.6860300	5732	9017	05560	41
20	2416	3736	.9434513	1.0599381	40
21	4532	1740	.9440013	93206	39
22	6647	.7269743	5516	87034	38
23	8761	7745	.9451021	80867	37
24	.6870875	5747	6530	74703	36
25	2988	3748	.9462042	68544	35
26	5101	1748	7556	62388	34
27	7213	.7259748	.9473074	56235	33
28	9325	7747	8595	50087	32
29	.6881435	5746	.9484119	43942	31
30	3546	3744	9646	37801	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*

133 *Degrees* 46

Min.	43	Degrees		136	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	.6883545	.7253744	.9489646	1.0537801	30
31	5655	1741	.9495176	31664	29
32	7765	.7249738	.9500709	25531	28
33	9873	7734	6245	19401	27
34	.6891981	5729	.9511784	13275	26
35	4089	3724	7326	07153	25
36	6195	1718	.9522871	01034	24
37	8302	.7239712	8420	1.0494920	23
38	.6900407	7705	.9533971	88809	22
39	2512	5698	9526	82702	21
40	4617	3690	.9545083	76598	20
41	6721	1681	.9550644	70498	19
42	8824	.7229671	6208	64402	18
43	.6910927	7661	.9561774	58310	17
44	3029	5651	7344	52221	16
45	5131	3640	.9572917	46136	15
46	7232	1628	8494	40055	14
47	9332	.7219615	.9584073	33977	13
48	.6921432	7602	9655	27904	12
49	3531	5589	.9595241	21833	11
50	5630	3574	.9600829	15767	10
51	7728	1559	6421	09704	9
52	9825	.7209544	.9612016	03645	8
53	6.931922	7528	7614	1.0397589	7
54	4018	5511	.9623215	91537	6
55	6114	3494	8819	85489	5
56	8209	1476	.9634427	79445	4
57	.6940304	.7199457	.9640037	73404	3
58	2398	7438	5651	67367	2
59	4491	5418	.9651268	61333	1
60	6584	3398	6888	55303	0
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	133	Degrees		46	

Min.	44	Degrees		135	
	Sine	Coſine	Tangent	Cotangent	
0	.6946584	.7193398	.9656888	1.0355303	60
1	8676	1377	.9662511	49277	59
2	.6950767	.7189355	8137	43254	58
3	2858	7333	.9673767	37235	57
4	4949	5310	9400	31220	56
5	7039	3287	.9685035	25208	55
6	9128	1263	.9690674	19199	54
7	.6961217	.7179238	6316	13195	53
8	3305	7213	.9701962	07194	52
9	5392	5187	7610	01196	51
10	7479	3161	.9713262	1.0295203	50
11	9565	1134	8917	89212	49
12	.6971651	.7169106	.9724575	83226	48
13	3736	7078	.9730236	77243	47
14	5821	5049	5901	71263	46
15	7905	3019	.9741569	65287	45
16	9988	0989	7240	59315	44
17	.6982071	.7158959	.9752914	53346	43
18	4153	6927	8591	47381	42
19	6234	4895	.9764272	41419	41
20	8315	2863	9956	35461	40
21	.6990396	0830	.9775643	29506	39
22	2476	.7148796	.9781333	23555	38
23	4555	6762	7027	17608	37
24	6633	4727	.9792724	11664	36
25	8711	2691	8424	05723	35
26	.7000789	0655	.9804127	1.0199786	34
27	2866	.7138618	9833	93853	33
28	4942	6581	.9815543	87923	32
29	7018	4543	.9821256	81997	31
30	9093	2504	6973	76074	30
	Coſine	Sine	Cotang.	Tangent	Min.
	134	Degrees		45	

Min.	44		Degrees	135	
	Sine	Coſine	Tangent	Cotangent	
30	.7009093	.7132504	.9826973	1.0176074	30
31	.7011167	0465	.9832692	70155	29
32	3241	.7128426	8415	64239	28
33	5314	6385	.9844141	58326	27
34	7387	4344	9871	52417	26
35	9459	2303	.9855603	46512	25
36	.7021530	0260	.9861339	1.0140610	24
37	3601	.7118218	7079	34712	23
38	5672	6174	.9872821	28817	22
39	7741	4130	8567	22925	21
40	9811	2086	.9884316	17037	20
41	.7031879	0041	.9890069	11153	19
42	3947	.7107995	5825	05272	18
43	6014	5948	9901584	1.0099394	17
44	8081	3901	7346	93520	16
45	.7040147	1854	9913112	87649	15
46	2213	.7099806	8881	81782	14
47	4278	7757	.9924654	75918	13
48	6342	5707	.9930429	70058	12
49	8406	3657	6208	64201	11
50	.7050469	1607	.9941991	58347	10
51	2532	.7089556	7777	1.0052497	9
52	4594	7504	.9953566	46651	8
53	6655	5451	9358	40807	7
54	8716	3398	.9965154	34968	6
55	.7060776	1345	.9970953	29131	5
56	2835	.7079291	6756	23298	4
57	4894	7236	.9982562	17469	3
58	6953	5180	8371	11642	2
59	9011	3124	.9994184	05819	1
60	.7071068	1068	1.0000000	1.0000000	0
	Coſine	Sine	Cotang.	Tangent	Min.
	134		Degrees	45	

A

TABLE

OF

Artificial or Logarithmic

Sines and Tangents,

TO EVERY

Degree and Minute of the Quadrant;

The Radius being 10.0000000;

From whence the Verſed Sines and Secants are eaſily found.

For,

Note 1. *Verſed Sine* = 2 × *Sine of* $\frac{1}{2}$ *Arch* — 9.6989700

2. *Secant* = 20 — *Coſine.*

When half the Arch comes to half an odd Minute, take the Sum of the next greater and leſſer Sines, inſtead of twice the Sine: but this holds not exactly in ſmall Arches.

Min.	0	Degrees		179	
	Sine	Coſine	Tangent	Cotangent	
0	0.0000000	10.0000000	0.0000000	Infinite	60
1	6.4637261	9.9999999	6.4637261	13.5362739	59
2	6.7647561	9	6.7647562	13.2352438	58
3	6.9408473	8	6.9408475	3.0591525	57
4	7.0657860	7	7.0657863	12.9342137	56
5	7.1626960	5	7.1626964	12.8373036	55
6	7.2418771	9.9999993	7.2418778	12.7581222	54
7	7.3088239	9	7.3088247	12.6911752	53
8	7.3668157	88	7.3668169	12.6331831	52
9	7.4179681	85	7.4179696	12.5820304	51
10	7.4637255	82	7.4637273	12.5362727	50
11	7.5051181	78	7.5051203	12.4948797	49
12	7.5429065	74	7.5429091	12.4570909	48
13	7.5776684	69	7.5776715	12.4223285	47
14	7.6098530	64	7.6098566	12.3901434	46
15	7.6398160	59	7.6398201	601799	45
16	7.6678445	9.9999953	7.6678492	321508	44
17	7.6941733	47	7.6941786	058214	43
18	7.7189966	40	7.7190026	12.2809974	42
19	7.7424775	34	7.7424841	575159	41
20	7.7647537	27	7.7647610	352390	40
21	7.7859427	9.9999919	7.7859508	140492	39
22	7.8061458	911	7.8061547	12.1938453	38
23	7.8254507	903	7.8254604	745396	37
24	7.8439338	894	7.8439444	560556	36
25	7.8616623	885	7.8616738	383262	35
26	7.8786953	9.9999876	7.8787077	212923	34
27	7.8950854	866	7.8950988	049012	33
28	7.9108793	856	7.9108938	12.0891062	32
29	7.9261190	845	7.9261344	738656	31
30	7.9408419	835	7.9408584	591416	30
	Coſine	Sine	Cotang.	Tangent	Min.
	90	Degrees		89	

Min.	0		Degrees	179	
	Sine	Cosine	Tangent	Cotangent	
30	7.9408419	9.9999835	7.9408584	12.0591416	30
31	550819	23	550996	449004	29
32	688698	12	688886	311114	28
33	822334	00	822534	177466	27
34	951980	9.9999788	952192	047808	26
35	8.0077867	75	8.0078092	11.9921908	25
36	200207	62	200445	799555	24
37	319195	48	319446	680554	23
38	435009	35	435274	564726	22
39	547814	21	548094	451906	21
40	657763	06	658057	341943	20
41	764997	9.9999691	8.0765306	11.9234694	19
42	869646	76	869970	130030	18
43	971832	60	972172	027828	17
44	8.1071669	44	8.1072025	11.8927975	16
45	169262	28	169634	830366	15
46	264710	11	265099	734901	14
47	358104	9.9999594	358510	641490	13
48	449532	77	449956	550044	12
49	539075	59	539516	460484	11
50	626808	41	627267	372733	10
51	8.1712804	22	8.1713282	11.8286718	9
52	797129	03	797626	202374	8
53	879848	9.9999484	880364	119636	7
54	961020	64	961556	038444	6
55	8.2040703	44	8.2041259	11.7958741	5
56	118949	24	119526	880474	4
57	195811	03	196408	803592	3
58	271335	9.9999382	271953	728047	2
59	345568	60	346208	653792	1
60	418553	38	419215	580785	0
	Cosine	Sine	Cotangent	Tangent	Min.
	90		Degrees	89	

Min.	1	*Degrees*		178	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	8.2418553	9.9999338	8.2419215	11.7580785	60
1	490332	316	491015	508985	59
2	560943	294	561649	438351	58
3	630424	271	631153	368847	57
4	698810	247	699563	300437	56
5	766136	224	766912	233088	55
6	8.2832434	9.9999200	8.2833234	11.7166766	54
7	897734	175	898559	101441	53
8	962067	150	962917	037083	52
9	8.3025460	125	026335	11.6973665	51
10	087941	100	088842	911158	50
11	149536	9.9999074	150462	849538	49
12	210269	047	211221	788779	48
13	270163	021	271143	728857	47
14	329243	9.9998994	330249	669751	46
15	387529	966	388563	611437	45
16	8.3445043	939	8.3446105	11.6553895	44
17	501805	911	502895	497105	43
18	557835	882	558953	441047	42
19	613150	853	614297	385703	41
20	667769	824	668945	331055	40
21	721710	9.9998794	8.3722915	11.6277085	39
22	774988	764	776222	223777	38
23	827620	734	828886	171114	37
24	879622	703	880918	119082	36
25	931008	672	932336	067664	35
26	981793	9.9998641	983152	016848	34
27	8.4031990	609	8.4033381	11.5966619	33
28	081614	577	083037	916963	32
29	130676	544	132132	867868	31
30	179190	512	180679	819321	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	Min.
	91	*Degrees*		88	

Min.	1	*Degrees*		178	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	8.4179190	9.9998512	8.4180679	11.5819321	30
31	227168	478	228690	771310	29
32	274621	445	276176	723824	28
33	321561	411	323150	676850	27
34	367999	376	369622	630378	26
35	413944	342	415603	584397	25
36	8.4459409	9.9998306	8.4461103	11.5538897	24
37	504402	271	506131	493869	23
38	548934	235	550699	449301	22
39	593013	199	594814	405186	21
40	636649	162	638486	361514	20
41	679850	9.9998125	8.4681725	11.5318275	19
42	722626	088	724538	275462	18
43	764984	050	766933	233067	17
44	806932	012	808920	191080	16
45	848479	9.9997974	850505	149495	15
46	8.4889632	935	8.4891696	11.5108304	14
47	930398	896	932502	067498	13
48	970784	856	972928	027072	12
49	8.5010798	817	8.5012982	11.4987018	11
50	050447	776	052671	947329	10
51	089736	9.9997736	092001	907999	9
52	128673	695	130978	869022	8
53	167264	653	169610	830390	7
54	205514	612	207902	792098	6
55	243430	570	245860	754140	5
56	8.5281017	9.9997527	8.5283490	11.4716510	4
57	318281	484	320797	679203	3
58	355228	441	357787	642213	2
59	391863	398	394466	605534	1
60	428192	354	430838	569162	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	*Min.*
	91	*Degrees*		88	

Min.	2	Degrees		177	
	Sine	Coſine	Tangent	Cotangent	
0	8.5428192	9·9997354	8.5430838	11.4569162	60
1	464218	309	466909	533091	59
2	499948	265	502683	497317	58
3	535386	220	538166	461834	57
4	570536	174	573362	426638	56
5	605404	128	608276	391724	55
6	8.5639994	082	8.5642912	11.4357088	54
7	674310	036	677275	322725	53
8	708357	9·9996989	711368	288632	52
9	742139	942	745197	254803	51
10	775660	894	778766	221234	50
11	8.5808923	846	8.5812077	11.4187923	49
12	841933	798	845136	154864	48
13	874694	749	877945	122055	47
14	907209	700	910509	089491	46
15	939483	650	942832	057168	45
16	971517	9·9996601	974917	025083	44
17	8.6003317	550	8.6006767	11.3993233	43
18	034886	500	038386	961614	42
19	066226	449	069777	930223	41
20	097341	398	100943	899057	40
21	8.6128235	9·9996346	8.6131889	11.3868111	39
22	158910	294	162616	837384	38
23	189369	242	193127	806873	37
24	219616	189	223427	776573	36
25	249653	136	253518	746482	35
26	8.6279484	082	8.6283402	11.3716598	34
27	309111	028	313083	686917	33
28	338537	9·9995974	342563	657437	32
29	367764	919	371845	628155	31
30	396796	865	400931	599069	30
	Coſine	Sine	Cotangent	Tangent	Min.
	92	Degrees		87	

	2	Degrees		177	
Min.	Sine	Coſine	Tangent	Cotangent	
30	8.6396796	9.9995865	8.6400931	11.3599069	30
31	425634	809	429825	570175	29
32	454282	753	458528	541472	28
33	482742	697	487044	512956	27
34	511016	641	515375	484625	26
35	539107	584	543522	456478	25
36	8.6567017	9.9995527	8.6571490	11.3428510	24
37	594748	469	599279	400721	23
38	622303	411	626891	373109	22
39	649684	353	654331	345669	21
40	676893	295	681598	318402	20
41	8.6703932	9.9995236	8.6708697	11.3291303	19
42	730804	176	735628	264372	18
43	757510	116	762393	237607	17
44	784052	056	788996	211004	16
45	810433	9.9994996	815437	184563	15
46	8.6836654	935	8.6841719	11.3158281	14
47	862718	874	867844	132156	13
48	888625	812	893813	106187	12
49	914379	750	919629	080371	11
50	939980	688	945292	054708	10
51	965431	9.9994625	970806	029194	9
52	990734	562	996173	003828	8
53	8.7015889	498	8.7021390	11.2978610	7
54	040899	435	046465	953535	6
55	065766	370	071395	928605	5
56	090490	9.9994306	8.7096185	11.2903815	4
57	115075	241	120834	879166	3
58	139520	176	145345	854655	2
59	163829	110	169719	830281	1
60	188002	044	193958	806042	0
	Coſine	Sine	Cotangent	Tangent	Min.
	92	Degrees		87	

Min.	3	Degrees		176	
	Sine	Coſine	Tangent	Cotangent.	
0	8.7188002	9.9994044	8.7193958	11.2806042	60
1	8.7212040	9.9993978	8.7218063	11.2781937	59
2	35946	911	42035	57965	58
3	59721	844	65877	34123	57
4	83366	776	89589	10411	56
5	8.7306882	708	8.7313174	11.2686826	55
6	30272	9.9993640	36631	63369	54
7	53535	572	59964	40036	53
8	76675	503	83172	16828	52
9	99691	433	8.7406258	11.2593742	51
10	8.7422586	364	29222	70778	50
11	45360	9.9993293	52067	47933	49
12	68015	223	74792	25208	48
13	90553	152	97400	02600	47
14	8.7512973	081	8.7519892	11.2480108	46
15	35278	009	42269	57731	45
16	57469	9.9992938	64531	35469	44
17	79546	865	86681	13319	43
18	8.7601512	793	8.7608719	11.2391281	42
19	23366	720	30647	69353	41
20	45111	646	52465	47535	40
21	66747	9.9992572	74175	25825	39
22	88275	498	95777	04223	38
23	8.7709697	424	8.7717274	11.2282726	37
24	31014	349	38665	61335	36
25	52226	274	59952	40048	35
26	73334	9.9992198	81136	18864	34
27	94340	122	8.7802218	11.2197782	33
28	8.7815244	046	23199	76801	32
29	36048	9.9991969	44079	55921	31
30	56753	892	64861	35139	30
	Coſine	Sine	Cotangent	Tangent	Min.
	93	Degrees		86	

Min.	3	Degrees		176	
	Sine	Coſine	Tangent	Cotangent	
30	8.7856753	9.9991892	8.7864861	11.2135139	30
31	77359	815	85544	14456	29
32	97867	737	8.7906130	11.2093870	28
33	8.7918278	659	26620	73380	27
34	38594	580	47014	52986	26
35	58814	501	67313	32687	25
36	78941	9.9991422	87519	12481	24
37	98974	342	8.8007632	11.1992368	23
38	8.8018915	262	27653	72347	22
39	38764	182	47583	52417	21
40	58523	101	67422	32578	20
41	78192	020	87172	12828	19
42	97772	9.9990938	8.8106834	11.1893166	18
43	8.8117264	856	26407	73593	17
44	36668	774	45894	54106	16
45	55985	691	65294	34706	15
46	75217	9.9990608	84608	15392	14
47	94363	525	8.8203838	11.1796162	13
48	8.8213425	441	22984	77016	12
49	32404	357	42046	57954	11
50	51299	273	61026	38974	10
51	70112	9.9990188	79924	20076	9
52	88844	103	98741	01259	8
53	8.8307495	017	8.8317478	11.1682522	7
54	26066	9.9989931	36134	63866	6
55	44557	845	54712	45288	5
56	62969	758	73211	26789	4
57	81304	671	91633	08367	3
58	99561	584	8.8409977	11.1590023	2
59	8.8417741	496	28245	71755	1
60	35845	408	46437	53563	0
	Coſine	Sine	Cotangent	Tangent	Min.
	93	Degrees		86	

Min.	4	Degrees		175	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	8.8435845	9.9989408	8.8446437	11.1553563	60
1	53874	319	64554	35446	59
2	71827	230	82597	17403	58
3	89707	141	8.8500566	11.1499434	57
4	8.8507512	052	18461	81539	56
5	25245	9.9988962	36283	63717	55
6	42905	871	54034	45966	54
7	60493	780	71713	28287	53
8	78010	689	89321	10679	52
9	95457	598	8.8606859	11.1393141	51
10	8.8612833	506	24327	75673	50
11	30139	9.9988414	41725	58275	49
12	47376	321	59055	40945	48
13	64545	228	76317	23683	47
14	81646	135	93511	06489	46
15	98680	041	8.8710638	11.1289362	45
16	8.8715646	9.9987947	27699	72301	44
17	32546	853	44694	55306	43
18	49381	758	61623	38377	42
19	66150	663	78487	21513	41
20	82854	567	95286	04714	40
21	99493	9.9987471	8.8812022	11.1187978	39
22	8.8816069	375	28694	71306	38
23	32581	278	45303	54697	37
24	49031	181	61850	38150	36
25	65418	084	78334	21666	35
26	81743	9.9986986	94757	05243	34
27	98007	888	8.8911119	11.1088881	33
28	8.8914209	790	27420	72580	32
29	30351	691	43660	56340	31
30	46433	591	59842	40158	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	94	Degrees		85	

Min.	4	Degrees		175	
	Sine	Coſine	Tangent	Cotangent	
30	8.8946433	9.9986591	8.8959842	11.1040158	30
31	62455	492	75963	24037	29
32	78418	392	92026	07974	28
33	94322	292	8.9008030	11.0991970	27
34	8.9010168	191	23977	76023	26
35	25955	090	39866	60134	25
36	41685	9.9985988	55697	44303	24
37	57358	886	71472	28528	23
38	72975	784	87190	12810	22
39	88535	682	8.9102853	11.0897147	21
40	8.9104039	579	18460	81540	20
41	19487	9.9985475	34012	65988	19
42	34881	372	49509	50491	18
43	50219	268	64952	35048	17
44	65504	163	80340	19660	16
45	80734	058	95675	04325	15
46	95911	9.9984953	8.9210957	11.0789043	14
47	8.9211034	848	26186	73814	13
48	26105	742	41363	58637	12
49	41123	636	56487	43513	11
50	56089	529	71560	28440	10
51	71003	9.9984422	86581	13419	9
52	85866	315	8.9301552	11.0698448	8
53	8.9300678	207	16471	83529	7
54	15439	099	31340	68660	6
55	30150	9.9983990	46160	53840	5
56	44811	881	60929	39071	4
57	59422	772	75650	24350	3
58	73983	663	90321	09670	2
59	88496	553	8.9404944	11.0595056	1
60	8.9402960	442	19518	80482	0
	Coſine	Sine	Cotangent	Tangent	Min.
	94	Degrees		85	

Min.	5	*Degrees*		174	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	8.9402960	9.9983442	8.9419518	11.0580482	60
1	17376	331	34044	65956	59
2	31743	220	48523	51477	58
3	46063	109	62954	37046	57
4	60335	9.9982997	77338	22662	56
5	74561	885	91676	08324	55
6	88739	772	8.9505967	11.0494033	54
7	8.9502871	660	20211	79789	53
8	16957	546	34410	65590	52
9	30996	433	48564	51436	51
10	44991	318	62672	37328	50
11	58940	204	76735	23265	49
12	72843	089	90754	09246	48
13	86703	9.9981974	8.9604728	11.0395272	47
14	8.9600517	859	18659	81341	46
15	14288	743	32545	67455	45
16	28014	626	46388	53612	44
17	41697	510	60188	39812	43
18	55337	393	73944	26056	42
19	68934	275	87658	12342	41
20	82487	158	8.9701330	11.0298670	40
21	95999	040	14959	85041	39
22	8.9709468	9.9980921	28547	71453	38
23	22895	802	42092	57908	37
24	36280	683	55597	44403	36
25	49624	563	69060	30940	35
26	62926	443	82483	17517	34
27	76188	323	95865	04135	33
28	89408	202	8.9809206	11.0190794	32
29	8.9802589	081	22507	77493	31
30	15729	9.9979960	35769	64231	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	*Min.*
	95	*Degrees*		84	

Min.	5		Degrees	174	
	Sine	Coſine	Tangent	Cotangent	
30	8.9815729	9.9979960	8.9835769	11.0164231	30
31	28829	838	48991	51009	29
32	41889	716	62173	37827	28
33	54910	593	75317	24683	27
34	67891	470	88421	11579	26
35	80834	347	8.9901487	11.0098513	25
36	93737	223	14514	85486	24
37	8.9906602	099	27503	72497	23
38	19429	9.9978975	40454	59546	22
39	32217	850	53367	46633	21
40	44968	725	66243	33757	20
41	57681	9.9978599	79081	20919	19
42	70356	473	91883	08117	18
43	82994	347	9.0004647	10.9995353	17
44	95595	220	17375	82625	16
45	9.0008160	093	30066	69934	15
46	20687	9.9977966	42721	57279	14
47	33179	838	55340	44660	13
48	45634	710	67924	32076	12
49	58053	582	80471	19529	11
50	70436	453	92984	07016	10
51	82784	9.9977323	9.0105461	10.9894539	9
52	95096	194	17903	82097	8
53	9.0107374	064	30310	69690	7
54	19616	9.9976933	42682	57318	6
55	31823	803	55021	44979	5
56	43996	672	9.0167325	10.9832675	4
57	56135	540	79594	20406	3
58	68239	408	91831	08169	2
59	80309	276	9.0204033	10.9795967	1
60	92346	143	16202	83798	0
	Coſine	Sine	Cotangent	Tangent	Min.
	95		Degrees	84	

Min.	6	Degrees		173	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.0192346	9.9976143	9.0216202	10.9783798	60
1	9.0204348	011	28338	71662	59
2	16318	9.9975877	40441	59559	58
3	28254	743	52510	47490	57
4	40157	609	64548	35452	56
5	52027	475	76552	23448	55
6	9.0263865	340	88524	11476	54
7	75669	205	9.0300464	10.9699536	53
8	87442	069	12373	87627	52
9	99182	9.9974933	24249	75751	51
10	9.0310890	797	36093	63907	50
11	22567	660	47906	10.9652094	49
12	34212	523	59688	40312	48
13	45825	386	71439	28561	47
14	57407	248	83159	16841	46
15	68958	110	94848	05152	45
16	80477	9.9973971	9.0406506	10.9593494	44
17	91966	833	18134	81866	43
18	9.0403424	693	29731	70269	42
19	14852	554	41299	58701	41
20	26249	414	52836	47164	40
21	37617	273	9.0464343	10.9535657	39
22	48954	132	75821	24179	38
23	60261	9.9972991	87270	12730	37
24	71538	850	98689	01311	36
25	82786	708	9.0510078	10.9489922	35
26	94005	566	21439	78561	34
27	9.0505194	423	32771	67229	33
28	16354	280	44074	55926	32
29	27485	137	55349	44651	31
30	38588	9.9971993	66595	33405	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	96	Degrees		83	

Min.	6	Degrees		173	
	Sine	Coſine	Tangent	Cotangent	
30	.90538588	9.9971993	9.0566595	10.9433405	30
31	49661	849	77813	22187	29
32	60706	704	89002	10998	28
33	71723	559	9.0600164	10.9399836	27
34	82711	414	11297	88703	26
35	93671	268	22403	77597	25
36	9.0604604	122	33482	66518	24
37	15509	9.9970976	44533	55467	23
38	26386	829	55556	44444	22
39	37235	682	66553	33447	21
40	48057	535	77522	22478	20
41	9.0658852	387	9.0688465	11535	19
42	69619	239	99381	00619	18
43	80360	090	9.0710270	10.9289730	17
44	91074	9.9969941	21133	78867	16
45	9.0701761	792	31969	68031	15
46	12421	642	42779	57221	14
47	23055	492	53563	46437	13
48	33663	342	64321	35679	12
49	44244	191	75053	24947	11
50	54799	040	85760	14240	10
51	9.0765329	9.9968888	96441	03559	9
52	75832	736	9.0807096	10.9192904	8
53	86310	584	17726	82274	7
54	96762	431	28331	71669	6
55	9.0807189	278	38911	61089	5
56	17590	125	49466	50534	4
57	27966	9.9967971	59996	40004	3
58	38317	817	70501	29499	2
59	48643	662	80981	19019	1
60	58945	507	91438	08562	0
	Coſine	Sine	Cotangent	Tangent	Min.
	96	Degrees		83	

Min.	7	Degrees		172	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.0858945	9.9967507	9.0891438	10.9108562	60
1	69221	352	9.0901869	10.9098131	59
2	79473	196	12277	87723	58
3	89700	040	22660	77340	57
4	99903	9.9966884	33020	66980	56
5	9.0910082	727	43355	56645	55
6	20237	570	9.0953667	10.9046333	54
7	30367	412	63955	36045	53
8	40474	254	74219	25781	52
9	50556	096	84460	15540	51
10	60615	9.9965937	94678	05322	50
11	9.0970651	778	9.1004872	10.8995128	49
12	80662	619	15044	84956	48
13	90651	459	25192	74808	47
14	9.1000616	299	35317	64683	46
15	10558	138	45420	54580	45
16	20477	9.9964977	9.1055500	10.8944500	44
17	30373	816	65557	34443	43
18	40246	655	75591	24409	42
19	50096	493	85604	14396	41
20	59924	330	95594	04406	40
21	9.1069729	167	9.1105562	10.8894438	39
22	79512	004	15508	84492	38
23	89272	9.9963841	25431	74569	37
24	99010	677	35333	64667	36
25	9.1108726	513	45213	54787	35
26	18420	348	9.1155072	10.8844928	34
27	28092	183	64909	35091	33
28	37742	018	74724	25276	32
29	47370	9.9962852	84518	15482	31
30	56977	686	94291	05709	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	97	Degrees		82	

Min.	7	Degrees		172	
	Sine	Cosine	Tangent	Cotangent	
30	9.1156977	9.9962686	9.1194291	10.8805709	30
31	66562	519	9.1204043	10.8795957	29
32	76125	352	13773	86227	28
33	85667	185	23482	76518	27
34	95188	017	33171	66829	26
35	9.1204688	9.9961849	42839	57161	25
36	14167	681	9.1252486	10.8747514	24
37	23624	512	62112	37888	23
38	33061	343	71718	28282	22
39	42477	174	81303	18697	21
40	51872	004	90868	09132	20
41	9.1261246	9.9960834	9.1300413	10.8699587	19
42	70600	663	09937	90063	18
43	79934	492	19442	80558	17
44	89247	321	28926	71074	16
45	98539	149	38391	61609	15
46	9.1307812	9.9959977	9.1347835	10.8652165	14
47	17064	804	57260	42740	13
48	26297	631	66665	33335	12
49	35509	458	76051	23949	11
50	44702	284	85417	14583	10
51	9.1353875	111	94764	05236	9
52	63028	9.9958936	9.1404092	10.8595908	8
53	72161	761	13400	86600	7
54	81275	586	22689	77311	6
55	90370	411	31959	68041	5
56	99445	235	9.1441210	58790	4
57	9.1408501	059	50442	49558	3
58	17537	9.9957882	59655	40345	2
59	26555	705	68850	31150	1
60	35553	528	78025	21975	0
	Cosine	Sine	Cotangent	Tangent	Min.
	97	Degrees		82	

Min.	8	Degrees		171	
	Sine	Coſine	Tangent	Cotangent	
0	9.1435553	9.9957528	9.1478025	10.8521975	60
1	44532	350	87182	12818	59
2	53493	172	96321	03679	58
3	62435	9.9956993	9.1505441	10.8494559	57
4	71358	815	14543	85457	56
5	80262	635	23627	76373	55
6	9.1489148	9.9956456	32692	67308	54
7	98015	276	41739	58261	53
8	9.1506864	095	50769	49231	52
9	15694	9.9955915	59780	40220	51
10	24507	734	68773	31227	50
11	33301	552	9.1577748	10.8422252	49
12	42076	370	86706	13294	48
13	50834	188	95646	04354	47
14	59574	005	9.1604569	10.8395431	46
15	68296	9.9954822	13473	86527	45
16	9.1577000	639	22361	77639	44
17	85686	455	31231	68769	43
18	94354	271	40083	59917	42
19	9.1603005	087	48919	51081	41
20	11639	9.9953902	57737	42263	40
21	20254	717	9.1666538	10.8333462	39
22	28853	531	75322	24678	38
23	37434	345	84089	15911	37
24	45998	159	92839	07161	36
25	54544	9.9952972	9.1701572	10.8298428	35
26	9.1663074	785	10289	89711	34
27	71586	597	18989	81011	33
28	80081	409	27672	72328	32
29	88559	221	36338	63662	31
30	97021	033	44988	55012	30
	Coſine	Sine	Cotangent	Tangent	Min.
	98	Degrees		81	

Min.	8	Degrees		171	
	Sine	Coſine	Tangent	Cotangent	
30	9.1697021	9.9952033	9.1744988	10.8255012	30
31	9.1705465	9.9951844	53622	46378	29
32	13893	654	62239	37761	28
33	22305	464	70840	29160	27
34	30699	274	79425	20575	26
35	39077	084	87993	12007	25
36	9.1747439	9.9950893	96546	03454	24
37	55784	702	9.1805082	10.8194918	23
38	64112	510	13602	86398	22
39	72425	318	22107	77894	21
40	80721	126	30595	69405	20
41	9.1789001	9.9949933	9.1839068	10.8160932	19
42	97265	740	47525	52475	18
43	9.1805512	546	55966	44034	17
44	13744	352	64392	35608	16
45	21960	158	72802	27198	15
46	9.1830160	9.9948964	9.1881196	10.8118804	14
47	38344	769	89575	10425	13
48	46512	573	97939	02061	12
49	54665	377	9.1906287	10.8093713	11
50	62802	181	14621	85379	10
51	9.1870923	9.9947985	22939	77061	9
52	79029	788	31241	68759	8
53	87120	591	39529	60471	7
54	95195	393	47802	52198	6
55	9.1903254	195	56059	43941	5
56	11299	9.9946997	9.1964302	10.8035698	4
57	19328	798	72530	27470	3
58	27342	599	80743	19257	2
59	35341	399	88941	11059	1
60	43324	199	97125	02875	0
	Coſine	Sine	Cotangent	Tangent	Min.
	98	Degrees		81	

Min.	9	*Degrees*		170	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	0.1943324	9.9946199	9.1997125	10.8002875	60
1	51293	9.9945999	9.2005294	10.7994706	59
2	59247	798	13449	86551	58
3	67186	597	21588	78411	57
4	75110	396	29714	70286	56
5	83019	194	37825	62175	55
6	90913	9.9944992	9.2045922	10.7954078	54
7	98793	789	54004	45996	53
8	9.2006658	587	62072	37928	52
9	14509	383	70126	29874	51
10	22345	180	78165	21835	50
11	30167	9.9943975	9.2086191	10.7913809	49
12	37974	771	94203	05797	48
13	45766	566	9.2102200	10.7897800	47
14	53545	361	10184	89816	46
15	613 9	156	18153	81847	45
16	9.2069059	9.9942950	9.2126109	10.7873891	44
17	76795	743	34051	65949	43
18	84516	537	41980	58020	42
19	92224	330	49894	50106	41
20	99917	122	57795	42205	40
21	9.2107597	9.9941914	9.2165683	10.7834317	39
22	15263	706	73556	26444	38
23	22914	498	81417	18583	37
24	30552	289	89264	10736	36
25	38176	079	97097	02903	35
26	9.2145787	9.9940870	9.2204917	10.7795083	34
27	53384	659	12724	87276	33
28	60967	449	20518	79482	32
29	68536	238	28298	71702	31
30	76092	027	36065	63935	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	99	*Degrees*		80	

Min.	9	Degrees		170	
	Sine	Coſine	Tangent	Cotangent	
30	9.2176092	9.9940027	9.2236065	10.7763935	30
31	83635	9.9939815	43819	56181	29
32	91164	603	51561	48439	28
33	98680	391	59289	40711	27
34	9.2206182	178	67004	32996	26
35	13671	9.9938965	74706	25294	25
36	21147	752	9.2282395	10.7717605	24
37	28609	538	90071	09929	23
38	36059	324	97735	02265	22
39	43495	109	9.2305386	10.7694614	21
40	50918	9.9937894	13024	86976	20
41	9.2258328	679	20650	79350	19
42	65725	463	28262	71738	18
43	73110	247	35863	64137	17
44	80481	030	43451	56549	16
45	87839	9.9936813	51026	48974	15
46	95185	596	9.2358589	10.7641411	14
47	9.2302518	378	66139	33861	13
48	09838	160	73678	26322	12
49	17145	9.9935942	81203	18797	11
50	24440	723	88717	11283	10
51	9.2331722	504	96218	03782	9
52	38992	285	9.2403708	10.7596292	8
53	46249	065	11185	88815	7
54	53494	9.9934844	18650	81350	6
55	60726	624	26103	73897	5
56	9.2367946	403	9.2433543	66457	4
57	75153	181	40972	59028	3
58	82349	9.9933959	48389	51611	2
59	89532	737	55794	44206	1
60	96702	515	63188	36812	0
	Coſine	Sine	Cotangent	Tangent	Min.
	99	Degrees		80	

Min.	10	Degrees		169	
	Sine	Coſine	Tangent	Cotangent	
0	9.2396702	9.9933515	9.2463188	10.7536812	60
1	9.2403861	292	70569	29431	59
2	11007	068	77939	22061	58
3	18141	9.9932845	85297	14703	57
4	25264	621	92643	07357	56
5	32374	396	99978	00022	55
6	9.2439472	171	9.2507301	10.7492699	54
7	46558	9.9931946	14612	85388	53
8	53632	720	21912	78088	52
9	60695	494	29200	70800	51
10	67746	268	36477	63523	50
11	9.2474784	041	9.2543743	10.7456257	49
12	81811	9.9930814	50997	49003	48
13	88827	587	58240	41760	47
14	95830	359	65472	34528	46
15	9.2502822	131	72692	27308	45
16	09803	9.9929902	9.2579901	10.7420099	44
17	16772	673	87099	12901	43
18	23729	444	94285	05715	42
19	30675	214	9.2601461	10.7398539	41
20	37609	9.9928984	08625	91375	40
21	9.2544532	753	15779	84221	39
22	51444	522	22921	77079	38
23	58344	291	30053	69947	37
24	65233	059	37173	62827	36
25	72110	9.9927827	44283	55717	35
26	9.2578977	595	9.2651382	10.7348618	34
27	85832	362	58470	41530	33
28	92676	129	65547	34453	32
29	99509	9.9926895	72613	27387	31
30	9.2606330	661	79669	20331	30
	Coſine	Sine	Cotangent	Tangent	Min.
	100	Degrees		79	

Min.	10	Degrees		169	
	Sine	Coſine	Tangent	Cotangent	
30	9.2606330	9.9926661	9.2679669	10.7320331	30
31	13141	427	86714	13286	29
32	19941	192	93749	06251	28
33	26729	9.9925957	9.2700772	10.7299228	27
34	33507	722	07786	92214	26
35	40274	486	14788	85212	25
36	9.2647030	250	9.2721780	10.7278220	24
37	53775	013	28762	71238	23
38	60509	9.9924776	35733	64267	22
39	67232	539	42694	57306	21
40	73945	301	49644	50356	20
41	9.2680647	063	9.2756584	10.7243416	19
42	87338	9.9923824	63514	36486	18
43	94019	585	70434	29566	17
44	9.2700689	346	77343	22657	16
45	07348	106	84242	15758	15
46	13997	9.9922866	9.2791131	10.7208869	14
47	20635	626	98009	01991	13
48	27263	385	9.2804878	10.7195122	12
49	33880	144	11736	88264	11
50	40487	9.9921902	18585	81415	10
51	9.2747083	660	9.2825423	10.7174577	9
52	53669	418	32251	67749	8
53	60245	175	39070	60930	7
54	66811	9.9920932	45878	54122	6
55	73366	689	52677	47323	5
56	9.2779911	445	9.2859466	10.7140534	4
57	86445	201	66245	33755	3
58	92970	9.9919956	73014	26986	2
59	99484	711	79773	20227	1
60	9.2805988	466	86523	13477	0
	Coſine	Sine	Cotangent	Tangent	Min.
	100	Degrees		79	

Min.	11	*Degrees*		168	
	Sine	*Coſine*	*Tangent*	*Cotangent.*	
0	9.2805988	9.9919466	9.2886523	10.7113477	60
1	12483	220	93262	06737	59
2	18967	9.9918974	99993	00007	58
3	25441	727	9.2906713	10.7093287	57
4	31905	480	13424	86576	56
5	38359	233	20126	79874	55
6	9.2844803	9.9917986	26817	10.7073183	54
7	51237	737	33500	66500	53
8	57661	489	40172	59828	52
9	64076	240	46836	53164	51
10	70480	9.9916991	53489	46511	50
11	9.2876875	741	9.2960134	10.7039866	49
12	83260	492	66769	33231	48
13	89636	241	73395	26605	47
14	96001	9.9915990	80011	19989	46
15	9.2902357	739	86618	13382	45
16	08704	488	93216	10.7006784	44
17	15040	236	99804	00196	43
18	21367	9.9914984	9.3006383	10.6993617	42
19	27685	731	12954	87046	41
20	33993	478	19514	80486	40
21	9.2940291	225	9.3026066	10.6973934	39
22	46580	9.9913971	32609	67391	38
23	52859	717	39143	60857	37
24	59129	462	45667	54333	36
25	65390	207	52183	47817	35
26	9.2971641	9.9912952	9.3058689	10.6941311	34
27	77883	696	65187	34813	33
28	84116	440	71675	28325	32
29	90339	184	78155	21845	31
30	96553	9.9911927	84626	15374	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	*Min.*
	101	*Degrees*		78	

Min.	11	Degrees		168	
	Sine	Coſine	Tangent	Cotangent	
30	9.2996553	9.9911927	9.3084626	10.6915374	30
31	9.3002758	670	91088	08912	29
32	08953	412	97541	02459	28
33	15140	154	9.3103985	10.6896015	27
34	21317	9.9910896	10421	89579	26
35	27485	637	16848	83152	25
36	9.3033644	378	9.3123266	10.6876734	24
37	39794	119	29675	70325	23
38	45934	9.9909859	36076	63924	22
39	52066	598	42468	57532	21
40	58189	338	48851	51149	20
41	9.3064303	077	9.3155226	10.6844774	19
42	70407	9.9908815	61592	38408	18
43	76503	553	67950	32050	17
44	82590	291	74299	25701	16
45	88668	029	80640	19360	15
46	94737	9.9907766	9.3186972	10.6813028	14
47	9.3100798	502	93295	06705	13
48	06849	239	99611	00389	12
49	12892	9.9906974	9.3205918	10.6794082	11
50	18926	710	12216	87784	10
51	9.3124951	445	18506	81494	9
52	30968	180	24788	75212	8
53	36976	9.9905914	31061	68939	7
54	42975	648	37327	62673	6
55	48965	382	43584	56416	5
56	9.3154947	115	6.3249832	10.6750168	4
57	60921	9.9904848	56073	43927	3
58	66885	580	62305	37695	2
59	72841	312	68529	31471	1
60	78789	044	74745	25255	0
	Coſine	Sine	Cotangent	Tangent	Min.
	101	Degrees		78	

Min.	12	Degrees		167	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.3178789	9.9904044	9.3274745	10.6725255	60
1	84728	9.9903775	80953	19047	59
2	90659	506	87153	12847	58
3	96581	237	93345	06655	57
4	9.3202495	9.9902967	99528	00472	56
5	08400	697	9.3305704	10.6694296	55
6	14297	426	11872	88128	54
7	20186	155	18031	81969	53
8	26066	9.9901883	24183	75817	52
9	31938	612	30327	69673	51
10	37802	339	36463	63537	50
11	9.3243657	067	9.3342591	10.6657409	49
12	49505	9.9900794	48711	51289	48
13	55344	521	54823	45177	47
14	61174	247	60927	39073	46
15	66997	9.9899973	67024	32976	45
16	9.3272811	698	9.3373113	10.6626887	44
17	78617	423	79194	20806	43
18	84416	148	85267	14733	42
19	90206	9.9898873	91333	08667	41
20	95988	597	97391	02609	40
21	9.3301761	320	9.3403441	10.6596559	39
22	07527	043	09484	90516	38
23	13285	9.9897766	15519	84481	37
24	19035	489	21546	78454	36
25	24777	211	27566	72434	35
26	9.3330511	9.9896932	9.3433578	10.6566422	34
27	36237	654	39583	60417	33
28	41955	374	45580	54420	32
29	47665	095	51570	48430	31
30	53368	9.9895815	57552	42448	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	102	Degrees		77	

Min.	12	Degrees		167	
	Sine	Coſine	Tangent	Cotangent	
30	9.3353368	9.9895815	9.3457552	10.6542448	30
31	59062	535	63527	36473	29
32	64749	254	69494	30506	28
33	70428	9.9894973	75454	24546	27
34	76099	692	81407	18593	26
35	81762	410	87352	12648	25
36	9.3387418	128	9.3493290	06710	24
37	93065	9.9893845	99220	00780	23
38	98706	562	9.3505143	10.6494857	22
39	9.3404338	279	11059	88941	21
40	09963	9.9892995	16968	83032	20
41	15580	711	9.3522869	77131	19
42	21190	427	28763	71237	18
43	26792	142	34650	65350	17
44	32386	9.9891856	40530	59470	16
45	37973	571	46402	53598	15
46	9.3443552	285	9.3552267	10.6447733	14
47	49124	9.9890998	58126	41874	13
48	54688	711	63977	36023	12
49	60245	424	69821	30179	11
50	65794	137	75658	24342	10
51	9.3471336	9.9889849	9.3581487	10.6418513	9
52	76870	560	87310	12690	8
53	82397	271	93126	06874	7
54	87917	9.9888982	98935	01065	6
55	93429	693	9.3604736	10.6395264	5
56	98934	403	10531	89469	4
57	9.3504432	113	16319	83681	3
58	09922	9.9887822	22100	77900	2
59	15405	531	27874	72126	1
60	20880	239	33641	66359	0
	Coſine	Sine	Cotangent	Tangent	Min.
	102	Degrees		77	

Min.	13	Degrees		166	
	Sine	Coſine	Tangent	Cotangent	
0	9·3520880	9·9887239	9·3633641	10.6366359	60
1	26349	9·9886947	39401	60599	59
2	31810	655	45155	54845	58
3	37264	363	50901	49099	57
4	42710	070	56641	43359	56
5	48150	9·9885776	62374	37626	55
6	9·3553582	482	9·3668100	10.6331900	54
7	59007	188	73819	26181	53
8	64426	9·9884894	79532	20468	52
9	69836	599	85238	14762	51
10	75240	303	90937	09063	50
11	9·3580637	008	96629	03371	49
12	86027	6.9883712	9·3702315	10.6297685	48
13	91409	415	07994	92006	47
14	96785	118	13667	86333	46
15	9·3602154	9·9882821	19333	80667	45
16	07515	523	9·3724992	10.6275008	44
17	12870	225	30645	69355	43
18	18217	6.9881927	36291	63709	42
19	23558	628	41930	58070	41
20	28892	329	47563	52437	40
21	9·3634219	029	9·3753190	10.6246810	39
22	39539	9·9880729	58810	41190	38
23	44852	429	64423	35577	37
24	50158	128	70030	29970	36
25	55458	9·9879827	75631	24369	35
26	9·3660750	525	9·3781225	10.6218775	34
27	66036	223	86813	13187	33
28	71315	9·9878921	92394	07606	32
29	76587	618	97969	02031	31
30	81853	315	9·3803537	10.6196463	30
	Coſine	Sine	Cotangent	Tangent	Min.
	103	Degrees		76	

Min.	13	Degrees		166	
	Sine	Coſine	Tangent	Cotangent	
30	9.3681853	9.9878315	9.3803537	10.6196463	30
31	87111	012	09100	90900	29
32	92363	9.9877708	14655	85345	28
33	97608	404	20205	79795	27
34	9.3702847	099	25748	74252	26
35	08079	9.9876794	31285	68715	25
36	9.3713304	488	9.3836816	10.6163184	24
37	18523	183	42340	57660	23
38	23735	9.9875876	47858	52142	22
39	28940	570	53370	46630	21
40	34139	263	58876	41124	20
41	9.3739331	9.9874955	9.3864376	10.6135624	19
42	44517	648	69869	30131	18
43	49696	339	75356	24644	17
44	54868	031	80837	19163	16
45	60034	9.9873722	86312	13688	15
46	9.3765194	413	91781	08219	14
47	70347	103	97244	02756	13
48	75493	9.9872793	9.3902700	10.6097300	12
49	80633	482	08151	91849	11
50	85767	171	13595	86405	10
51	9.3790894	9.9871860	9.3919034	10.6080966	9
52	96015	549	24466	75534	8
53	9.3801129	236	29893	70107	7
54	06237	9.9870924	35313	64687	6
55	11339	611	40727	59272	5
56	9.3816434	298	9.3946136	10.6053864	4
57	21523	9.9869984	51538	48462	3
58	26605	670	56935	43065	2
59	31682	356	62326	37674	1
60	36752	041	67711	32289	0
	Coſine	Sine	Cotangent	Tangent	Min.
	103	Degrees		76	

Min.	14	Degrees		165	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.3836752	9.9869041	9.3967711	10.6032289	60
1	41815	9.9868726	73089	26911	59
2	46873	410	78463	21537	58
3	51924	094	83830	16170	57
4	56969	9.9867778	89191	10809	56
5	62008	461	94547	05453	55
6	9.3867040	144	99896	00104	54
7	72067	9.9866827	9.4005240	10.5994760	53
8	77087	509	10578	89422	52
9	82101	191	15910	84090	51
10	87109	9.9865872	21237	78763	50
11	92111	553	9.4026558	10.5973442	49
12	97106	233	31873	68127	48
13	9.3902096	9.9864913	37182	62818	47
14	07079	593	42486	57514	46
15	12057	272	47784	52216	45
16	9.3917028	9.9863951	9.4053076	10.5946924	44
17	21993	630	58363	41637	43
18	26952	308	63644	36356	42
19	31905	9.9862986	68919	31081	41
20	36852	663	74189	25810	40
21	9.3941794	340	9.4079453	10.5920547	39
22	46729	017	84712	15288	38
23	51658	9.9861693	89965	10035	37
24	56581	369	95212	04788	36
25	61499	045	9.4100454	10.5899546	35
26	9.3966410	9.9860720	05690	94310	34
27	71315	394	10921	89079	33
28	76215	069	16146	83854	32
29	81109	9.9859742	21366	78634	31
30	85996	416	26581	73419	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	104	Degrees		75	

Min.	15	Degrees		164	
	Sine	Coſine	Tangent	Cotangent	
30	9.4268988	9.9839105	9.4429883	10.5570117	30
31	73541	9.9838755	34786	65214	29
32	78089	404	39685	60315	28
33	82631	052	44579	55421	27
34	87169	9.9837701	49468	50532	26
35	91701	348	54352	45648	25
36	96228	9.9836996	9.4459232	10.5540768	24
37	9.4300750	643	64107	35893	23
38	05267	290	68978	31022	22
39	09779	9.9835936	73843	26157	21
40	14286	582	78704	21296	20
41	9.4318788	227	9.4483561	10.5516439	19
42	23285	9.9834872	88413	11587	18
43	27777	517	93260	06740	17
44	32264	161	98102	01898	16
45	36746	9.9833805	9.4502940	10.5497060	15
46	9.4341223	449	07774	92226	14
47	45694	092	12602	87398	13
48	50161	9.9832735	17427	82573	12
49	54623	377	22246	77754	11
50	59080	019	27061	72939	10
51	9.4363532	9.9831661	9.4531872	10.5468128	9
52	67980	302	36678	63322	8
53	72422	9.9830942	41479	58521	7
54	76859	583	46276	53724	6
55	81292	223	51069	48931	5
56	9.4385719	9.9829862	9.4555857	10.5444143	4
57	90142	501	60641	39359	3
58	94560	140	65420	34580	2
59	98973	9.9828778	70194	29806	1
60	03381	416	74964	25036	0
	Coſine	Sine	Cotangent	Tangent	Min.
	105	Degrees		74	

Min.	16	Degrees		163	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9 4403381	9.9828416	9.4574964	10.5425036	60
1	07784	8054	79730	20270	59
2	12182	7691	84491	15509	58
3	16576	7328	89248	10752	57
4	20965	6964	94001	05999	56
5	25349	6600	98749	01251	55
6	9.4429728	9.9826236	9.4603492	10.5396508	54
7	34103	5871	08232	91768	53
8	38472	5506	12967	87033	52
9	42837	5140	17697	82303	51
10	47197	4774	22423	77577	50
11	9.4451553	9.9824408	9.4627145	10.5372855	49
12	55904	4041	31863	68137	48
13	60250	3674	36576	63424	47
14	64591	3306	41285	58715	46
15	68927	2938	45990	54010	45
16	9.4473259	9.9822569	9.4650690	10.5349310	44
17	77586	2201	55386	44614	43
18	81909	1831	60078	39922	42
19	86227	1462	64765	35235	41
20	90540	1092	69448	30552	40
21	94849	0721	9.4674127	10.5325873	39
22	99153	0351	78802	21198	38
23	9.4503452	9.9819979	83473	16527	37
24	07747	9608	83139	11861	36
25	12037	9236	92801	07199	35
26	9.4516322	9.9818863	97459	02541	34
27	20603	8490	02112	10.5297888	33
28	24879	8117	06762	93238	32
29	29151	7744	11407	88593	31
30	33418	7370	16048	83952	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	106	Degrees		73	

Min.	16	Degrees		163	
	Sine	Coſine	Tangent	Cotangent	
30	9.4533418	9.9817370	9.4716048	10.5283952	30
31	37681	6995	20685	79315	29
32	41939	6620	25318	74682	28
33	46192	6245	29947	70053	27
34	50441	5870	34572	65428	26
35	54686	5494	39192	60808	25
36	9.4558926	9.9815117	9.4743808	10.5256192	24
37	63161	4740	48421	51579	23
38	67392	4363	53029	46971	22
39	71618	3986	57633	42367	21
40	75840	3608	62233	37767	20
41	9.4580058	9.9813229	9.4766829	10.5233171	19
42	84271	2850	71421	28579	18
43	88480	2471	76009	23991	17
44	92684	2091	80592	19408	16
45	96884	1711	85172	14828	15
46	9.4601079	9.9811331	9.4789748	10.5210252	14
47	05270	0950	94319	05681	13
48	09456	0569	98887	01113	12
49	13638	0187	9.4803451	10.5196549	11
50	17816	9.9809805	08011	91989	10
51	9.46219 9	9423	9.4812566	10.5187434	9
52	26158	9040	17118	82882	8
53	30323	8657	21666	78334	7
54	34483	8273	26210	73790	6
55	38639	7889	30750	69250	5
56	9.4642790	9.9807505	9.4835286	10.5164714	4
57	46938	7120	39818	60182	3
58	51081	6735	44346	55654	2
59	55219	6349	48870	51130	1
60	59353	5963	53390	46610	0
	Coſine	Sine	Cotangeut	Tangent	Min.
	106	Degrees		73	

Min.	17	Degrees		162	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.4659353	9.9805963	9.4853390	10.5146610	60
1	63483	5577	57907	42093	59
2	67609	5190	62419	37581	58
3	71730	4803	66928	33072	57
4	75848	4415	71433	28567	56
5	79960	4027	75933	24067	55
6	9.4684069	9.9803639	9.4880430	10.5119570	54
7	88173	3250	84924	15076	53
8	92273	2860	89413	10587	52
9	96369	2471	93898	06102	51
10	9.4700461	2081	98380	01620	50
11	04548	9.9801690	9.4902858	10.5097142	49
12	08631	1299	07332	92668	48
13	12710	0908	11802	88198	47
14	16785	0516	16269	83731	46
15	20856	0124	20731	79269	45
16	9.4724922	9.9799732	9.4925190	10.5074810	44
17	28985	9339	29646	70354	43
18	33043	8946	34097	65903	42
19	37097	8552	38545	61455	41
20	41146	8158	42988	57012	40
21	9.4745192	9.9797764	9.4947429	10.5052571	39
22	49234	7369	51865	48135	38
23	53271	6973	56298	43702	37
24	57304	6578	60727	39273	36
25	61334	6182	65152	34848	35
26	9.4765359	9.9795785	9.4969574	10.5030426	34
27	69380	5388	73991	26009	33
28	73396	4991	78406	21594	32
29	77409	4593	82816	17184	31
30	81418	4195	87223	12777	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	
	107	Degrees		72	Min.

Min.	17 Sine	Degrees Cosine	Tangent	162 Cotangent	
30	9.4781418	9.9794195	9.4987223	10.5012777	30
31	85423	3796	91626	08374	29
32	89423	3398	96026	03974	28
33	93420	2998	9.5000422	10.4999578	27
34	97412	2599	04814	95186	26
35	9.4801401	2198	09203	90797	25
36	05385	9.9791798	9.5013588	10.4986412	24
37	09366	1397	17969	82031	23
38	13342	0996	22347	77653	22
39	17315	0594	26721	73279	21
40	21283	0192	31092	68908	20
41	9.4825248	9.9789789	9.5035459	10.4964541	19
42	29208	9386	39822	60178	18
43	33165	8983	44182	55818	17
44	37117	8579	48538	51462	16
45	41066	8175	52891	47109	15
46	9.4845010	9.9787770	9.5057240	10.4942760	14
47	48951	7365	61586	38414	13
48	52888	6960	65928	34072	12
49	56820	6554	70267	29733	11
50	60749	6148	74602	25398	10
51	9.4864674	9.9785741	9.5078933	10.4921067	9
52	68595	5334	83261	16739	8
53	72512	4927	87586	12414	7
54	76426	4519	91907	08093	6
55	30335	4111	96224	03776	5
56	9.4884240	9.9783702	9.5100539	10.4899461	4
57	88142	3293	04849	95151	3
58	92040	2883	09156	90844	2
59	95934	2474	13460	86540	1
60	99824	2063	17760	82240	0
	Cosine	Sine	Cotangent	Tangent	Min.
	107	Degrees		72	

Min.	18	*Degrees*		161	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9·4899824	9·9782062	9·5117760	10.4882240	60
1	9·4903710	1653	22057	77943	59
2	07592	1241	26351	73649	58
3	11471	0830	30641	69359	57
4	15345	0418	34927	65073	56
5	19216	0006	39210	60790	55
6	9·4923083	9·9779593	9·5143490	10.4856510	54
7	26946	9180	47766	52234	53
8	30806	8766	52039	47961	52
9	34661	8353	56309	43691	51
10	38513	7938	60575	39425	50
11	9·4942361	9·9777523	9·5164838	10.4835162	49
12	46205	7108	69097	30903	48
13	50046	6693	73353	26647	47
14	53883	6277	77606	22394	46
15	57716	5860	81855	18145	45
16	9·4961545	9·9775444	9·5186101	10.4813899	44
17	65370	5026	90344	09656	43
18	69192	4609	94583	05417	42
19	73010	4191	98819	01181	41
20	76824	3772	9·5203052	10.4796948	40
21	9·4980635	9·9773354	07282	92718	39
22	84442	2934	11508	88492	38
23	38245	2515	15730	84270	37
24	92045	2095	19950	80050	36
25	95840	1674	24166	75834	35
26	99633	9·9771253	9·5228379	10.4771621	34
27	9·5003421	0832	32589	67411	33
28	07206	0410	36795	63205	32
29	10987	9·9769988	40999	59001	31
30	14764	9566	45199	54801	30
	Coſine	*Sine*	*Cotang.*	*Tangent*	*Min.*
	108	*Degrees*		71	

Min.	18	Degrees		161	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	9.5014764	9.9769566	9.5245199	10.4754801	30
31	18538	9143	49395	50605	29
32	22308	8720	53589	46411	28
33	26075	8296	57779	42221	27
34	29838	7872	61966	38034	26
35	33597	7447	66150	33850	25
36	9.5037353	9.9767022	6.5270331	10.4729669	24
37	41105	6597	74508	25492	23
38	44853	6171	78682	21318	22
39	48598	5745	82853	17147	21
40	52339	5318	87021	12979	20
41	9.5056077	9.9764891	9.5291186	10.4708814	19
42	59811	4464	95347	04653	18
43	63542	4036	99505	00495	17
44	67269	3608	9.5303661	10.4696339	16
45	70992	3179	07813	92187	15
46	9.5074712	9.9762750	9.5311961	88039	14
47	78428	2321	16107	83893	13
48	82141	1891	20250	79750	12
49	85850	1461	24389	75611	11
50	89556	1030	28526	71474	10
51	93258	9.9760599	9.5332659	10.4667341	9
52	96956	0167	36789	63211	8
53	9.5100651	9.9759736	40916	59084	7
54	04343	9303	45040	54960	6
55	08031	8870	49161	50839	5
56	9.5111716	9.9758437	9.5353278	10.4646722	4
57	15397	8004	57393	42607	3
58	19074	7570	61505	38495	2
59	22749	7135	65613	34387	1
60	26419	6701	69719	30281	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	108	Degrees		71	

Min.	19	Degrees		160	
	Sine	*Cofine*	*Tangent*	*Cotangent*	
0	9.5126419	9.9756701	9.5369719	10.4630281	60
1	30086	6265	73821	26179	59
2	33750	5830	77920	22080	58
3	37410	5394	82017	17983	57
4	41067	4957	86110	13890	56
5	44721	4521	90200	09800	55
6	9.5148371	9.9754083	94287	05713	54
7	52017	3646	98371	01629	53
8	55660	3208	9.5402453	10.4597547	52
9	59300	2769	06531	93469	51
10	62936	2330	10606	89394	50
11	9.5166569	9.9751891	9.5414678	10.4585322	49
12	70198	1451	18747	81253	48
13	73824	1011	22813	77187	47
14	77447	0570	26877	73123	46
15	81066	0129	30937	69063	45
16	9.5184682	9.9749688	9.5434994	10.4565006	44
17	88295	9246	39048	60952	43
18	91904	8804	43100	56900	42
19	95510	8361	47148	52852	41
20	99112	7918	51193	48807	40
21	9.5202711	9.9747475	9.5455236	10.4544764	39
22	06307	7031	59276	40724	38
23	09899	6587	63312	36688	37
24	13488	6142	67346	32654	36
25	17074	5697	71377	28623	35
26	9.5220656	9.9745251	9.5475405	10.4524595	34
27	24235	4806	79430	20570	33
28	27811	4359	83452	16548	32
29	31383	3913	87471	12529	31
30	34953	3466	91487	08513	30
	Cofine	*Sine*	*Cotangent*	*Tangent*	Min.
	109	Degrees		70	

Min.	19	Degrees		160	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	9.5234953	9.9743466	9.5491487	10.4508513	30
31	38518	3018	95500	04500	29
32	42081	2570	99511	00489	28
33	45640	2122	9.5503519	10.4496481	27
34	49196	1673	07523	92477	26
35	52749	1224	11525	88475	25
36	9.5256298	9.9740774	9.5515524	10.4484476	24
37	59844	0324	19521	80479	23
38	63387	9.9739873	23514	76486	22
39	66927	9422	27504	72496	21
40	70463	8971	31492	68508	20
41	9.5273997	9.9738519	9.5535477	10.4464523	19
42	77526	8067	39459	60541	18
43	81053	7615	43438	56562	17
44	84577	7162	47415	52585	16
45	88097	6709	51388	48612	15
46	9.5291614	9.9736255	9.5555359	10.4444641	14
47	95128	5801	59327	40673	13
48	98638	5346	63292	36708	12
49	9.5302146	4891	67255	32745	11
50	05650	4435	71214	28786	10
51	9.5309151	9.9733980	9.5575171	10.4424829	9
52	12649	3523	79125	20875	8
53	16143	3067	83077	16923	7
54	19635	2610	87025	12975	6
55	23123	2152	90971	09029	5
56	9.5326608	9.9731694	94914	10.4405086	4
57	30090	1236	98854	01146	3
58	33569	0777	9.5602792	10.4397208	2
59	37044	0318	06727	93273	1
60	40517	9.9729858	10659	89341	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	109	Degrees		70	

Min.	20	Degrees		159	
	Sine	Coſine	Tangent	Cotangent.	
0	9.5340517	9 9729858	9.5610659	10.4389341	60
1	43986	9398	14588	85412	59
2	47452	8939	18515	81485	58
3	50915	8477	22439	77561	57
4	54375	8016	26360	73640	56
5	57832	7554	30278	69722	55
6	9.5361286	9.9727092	9.5634194	10.4365806	54
7	64737	6629	38107	61893	53
8	68184	6166	42018	57982	52
9	71628	5703	45925	54075	51
10	75069	5239	49831	50169	50
11	9.5378508	9.9724775	9.5653733	10.4346267	49
12	81943	4310	57633	42367	48
13	85375	3845	61530	38470	47
14	88804	3380	65424	34576	46
15	92230	2914	69316	30684	45
16	9.5395653	9.9722448	9.5673205	10.4326795	44
17	99073	1981	77091	22909	43
18	9.5402489	1514	80975	19025	42
19	05903	1047	84856	15144	41
20	09314	0579	88735	11265	40
21	9.5412721	0110	92611	10.4307389	39
22	16126	9.9719642	96484	03516	38
23	19527	9172	9.5700355	10.4299645	37
24	22926	8703	04223	95777	36
25	26321	8233	08088	91912	35
26	9.5429713	9.9717762	9.5711951	10.4288049	34
27	33103	7291	15811	84189	33
28	36489	6820	19669	80331	32
29	39873	6348	23524	76476	31
30	43253	5876	27377	72623	30
	Coſine	Sine	Cotangent	Tangent	Min.
	110	Degrees		69	

Min.	20	Degrees		159	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	9.5443253	9.9715876	9.5727377	10.4272623	30
31	46630	5404	31227	68773	29
32	50005	4931	35074	64926	28
33	73376	4457	38919	61081	27
34	56745	3984	42761	57239	26
35	60110	3509	46601	53399	25
36	63472	9.9713035	9.5750438	10.4249562	24
37	66832	2560	54272	45728	23
38	70189	2084	58104	41896	22
39	73542	1608	61934	38066	21
40	76893	1132	65761	34239	20
41	9.5480240	0655	9.5769585	10.4230415	19
42	83585	0178	73407	26593	18
43	86927	9.9709701	77226	22774	17
44	90266	9223	81043	18957	16
45	93602	8744	84858	15142	15
46	96935	9.9708265	9.5788669	10.4211331	14
47	9.5500265	7786	92479	07521	13
48	03592	7306	96286	03714	12
49	06916	6826	9.5800090	10.4199910	11
50	10237	6346	03892	96108	10
51	9.5513556	9.9705865	9.5807691	92309	9
52	16871	5383	11488	88512	8
53	20184	4902	15282	84718	7
54	23494	4419	19074	80926	6
55	26801	3937	22864	77136	5
56	9.5530105	9.9703454	9.5826651	10.4173349	4
57	33406	2970	30435	69565	3
58	36704	2486	34217	65783	2
59	39999	2002	37997	62003	1
60	43292	1517	41774	58226	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	110	Degrees		69	

Min.	21	Degrees		158	
	Sine	Coſine	Tangent	Cotangent	
0	9.5543292	9.9701517	6.5841774	10.4158226	60
1	6581	1032	5549	4451	59
2	9868	0547	9321	0679	58
3	9.5553152	0061	9.5853091	10.4146909	57
4	6433	9.9699574	6859	3141	56
5	9711	9087	9.5860624	10.4139376	55
6	9.5562987	9.9698600	4386	5614	54
7	6259	8112	8147	1853	53
8	9529	7624	9.5871904	10.4128096	52
9	9.5572796	7136	6660	4340	51
10	6060	6647	9413	0587	50
11	9321	9.9696158	9.5883163	10.4116837	49
12	9.5582579	5668	6912	3088	48
13	5835	5177	9.5890657	10.4109343	47
14	9088	4687	4401	5599	46
15	9.5592338	4196	8142	1858	45
16	5585	9.9693704	9.5901881	10.4098119	44
17	8829	3212	5617	4383	43
18	9.5602071	2720	9351	0649	42
19	5310	2227	9.5913082	10.4086918	41
20	8546	1734	6812	3188	40
21	9.5611779	9.9691241	6.5920539	10.4079461	39
22	5010	0746	4263	5737	38
23	8237	0252	7985	2015	37
24	9.5621462	9.9689757	9.5931705	10.4068295	36
25	4685	9262	5423	4577	35
26	7904	9.9688766	9138	0862	34
27	9.5631121	8270	9.5942851	10.4057149	33
28	4335	7773	6561	3439	32
29	7546	7276	9.5950269	10.4049731	31
30	9.5640754	6779	3975	6025	30
	Coſine	Sine	Cotangent	Tangent	Min.
	111	Degrees		68	

Min.	21	*Degrees*		158	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	9.5640754	9.9686779	9.5953975	10.4046025	30
31	3960	6281	7679	2321	29
32	7163	5783	9.5961380	10.4038620	28
33	9.5650363	5284	5079	4921	27
34	3561	4785	8776	1224	26
35	6756	4286	9.5972470	10.4027530	25
36	9948	9.9683786	6162	3838	24
37	9.5663137	3285	9852	0148	23
38	6324	2784	9.5983540	10.4016460	22
39	9508	2283	7225	2775	21
40	9.5672689	1781	9.5990908	10.4009092	20
41	5868	9.9681279	4588	5411	19
42	9044	0777	8267	1733	18
43	9.5682217	0274	9.6001943	10.3998057	17
44	5387	9.9679771	5617	4383	16
45	8555	9267	9289	0711	15
46	9.5691721	9.9678763	9.6012958	10.3987042	14
47	4883	8258	6625	3375	13
48	8043	7753	9.6020290	10.3979710	12
49	9.5701200	7247	3953	6047	11
50	4355	6741	7613	2387	10
51	7506	9.9676235	9.6031271	10.3968729	9
52	9.5710656	5728	4927	5073	8
53	3802	5221	8581	1419	7
54	6946	4713	9.6042233	10.3957767	6
55	9.5720087	4205	5882	4118	5
56	3226	9.9673697	9529	0471	4
57	6362	3188	9.6053174	10.3946826	3
58	9495	2679	6817	3183	2
59	9.5732626	2169	9.6060457	10.3939543	1
60	5754	1659	4096	5904	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	111	*Degrees*		68	

Min.	22	Degrees		157	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.5735754	9.9671659	9.6064096	10.3935904	60
1	8880	1148	7732	2268	59
2	9.5742003	0637	9.6071366	10.3928634	58
3	5123	0125	4997	5003	57
4	8240	9.9669614	8627	1373	56
5	9.5751356	9101	9.6082254	10.3917746	55
6	4468	8588	5880	4120	54
7	7578	8075	9503	0497	53
8	9.5760685	7562	9.6093124	10.3906876	52
9	3790	7048	6742	3258	51
10	6892	6533	9.6100359	10.3899641	50
11	9991	9.9666018	3973	6027	49
12	9.5773088	5503	7586	2414	48
13	6183	4987	9.6111196	10.3888804	47
14	9275	4471	4804	5196	46
15	9.5782364	3954	8409	1591	45
16	5450	9.9663437	9.6122013	10.3877987	44
17	8535	2920	5615	4385	43
18	9.5791616	2402	9214	0786	42
19	4695	1884	9.6132812	10.3867188	41
20	7772	1365	6407	3593	40
21	9.5800845	0846	9.6140000	0000	39
22	3917	0326	3591	10.3856409	38
23	6986	9.9659806	7180	2820	37
24	9.5810052	9285	9.6150766	10.3849234	36
25	3116	8764	4351	5649	35
26	6117	9.9658243	7934	2066	34
27	9236	7721	9.6161514	10.3838486	33
28	9.5822292	7199	5093	4907	32
29	5345	6677	8669	1331	31
30	8397	6153	6.6172243	10.3827757	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	112	Degrees		67	

Min.	22	Degrees		175	
	Sine	Coſine	Tangent	Cotangent	
30	9.5828397	9.9656153	9.6172243	10.3827757	30
31	9.5831445	5630	5815	4185	29
32	4491	5106	9385	0615	28
33	7535	4582	9.6182953	10.3817047	27
34	9.5840576	4057	6519	3481	26
35	3615	3532	9.6190083	10.3809917	25
36	6651	9.9653006	3645	6355	24
37	9685	2480	7205	2795	23
38	9.5852716	1953	9.6200762	10.3799238	22
39	5745	1426	4318	5682	21
40	8771	0899	7872	2128	20
41	9.5861795	0371	9.6211423	10.3788577	19
42	4816	9.9649843	4973	5027	18
43	7835	9314	8520	1480	17
44	9.5870851	8785	9.6222066	10.3777934	16
45	3865	8256	5609	4391	15
46	6876	9.9647726	9150	0850	14
47	9885	7195	9.6232690	10.3767310	13
48	9.5882892	6665	6227	3773	12
49	5896	6133	9763	0237	11
50	8897	5602	9.6243296	10.3756704	10
51	9.5891897	9.9645069	6827	3173	9
52	4893	4537	9.6250356	10.3749644	8
53	7888	4004	3884	6116	7
54	9.5900880	3470	7409	2591	6
55	3869	2937	9.6260932	10.3739068	5
56	6856	9.9642402	4454	5546	4
57	9841	1868	7973	2027	3
58	9.5912823	1332	9.6271491	10.3728509	2
59	5803	0797	5006	4994	1
60	8780	0261	8519	1481	0
	Coſine	Sine	Cotangent	Tangent	Min.
	112	Degrees		67	

Min.	23	Degrees		156	
	Sine	Coſine	Tangent	Cotangent	
0	9.5918780	9.9640261	9.6278519	10.3721481	60
1	9.5921755	9.9639724	9.6282031	10.3717969	59
2	4728	9187	5540	4460	58
3	7698	8650	9048	0952	57
4	9.5930666	8112	9.6292553	10.3707447	56
5	3631	7574	6057	3943	55
6	6594	9.9637036	9558	0442	54
7	9555	6496	9.6303058	10.3696942	53
8	9.5942513	5957	6556	3444	52
9	5469	5417	9.6310052	10.3689948	51
10	8422	4877	3545	6455	50
11	9.5951373	9.9634336	7037	2963	49
12	4322	3795	9.6320527	10.3679473	48
13	7268	3253	4015	5985	47
14	9.5960212	2711	7501	2499	46
15	3154	2168	9.6330985	10.3669015	45
16	6093	9.9631625	4468	5532	44
17	9030	1082	7948	2052	43
18	9.5971965	0538	9.6341426	10.3658574	42
19	4897	9.9629994	4903	5097	41
20	7827	9449	8378	1622	40
21	9.5980754	9.9628904	9.6351850	10.3648150	39
22	3679	8358	5321	4679	38
23	6602	7812	8790	1210	37
24	9523	7266	9.6362257	10.3637743	36
25	9.5992441	6719	5722	4278	35
26	5357	9.9626172	9185	0815	34
27	8270	5624	9.6372646	10.3627354	33
28	9.6001181	5076	6106	3894	32
29	4090	4527	9563	0437	31
30	6997	3978	9.6383019	10.3616981	30
	Coſine	Sine	Cotangent	Tangent	Min.
	113	Degrees		66	

Min.	23	Degrees		156	
	Sine	Coſine	Tangent	Cotangent	
30	9.6006997	9.9623978	9.6383019	10.3616981	30
31	9901	3428	6473	3527	29
32	9.6012803	2878	6925	0075	28
33	5703	2328	9.6393375	10.3606625	27
34	8600	1777	6823	3177	26
35	9.6021495	1226	9.6400269	10.3599731	25
36	4388	9.9620674	3714	6286	24
37	7278	0122	7156	2844	23
38	9.6030166	9.9619569	9.6410597	10.3589403	22
39	3052	9016	4036	5964	21
40	5936	8463	7473	2527	20
41	8817	9.9617909	9.6420908	10.3579092	19
42	9.6041696	7355	4342	5658	18
43	4573	6800	7773	2227	17
44	7448	6245	9.6431203	10.3568797	16
45	9.6050320	5689	4631	5369	15
46	3190	9.9615133	8057	1943	14
47	6057	4576	9.6441481	10.3558519	13
48	8923	4020	4903	5097	12
49	9.6061786	3463	8324	1676	11
50	4647	2904	9.6451743	10.3548257	10
51	7506	9.9612346	5160	4840	9
52	9.6070362	1787	8575	1425	8
53	3216	1228	9.6461988	10.3538012	7
54	6068	0668	5400	4600	6
55	8918	0108	8810	1190	5
56	9.6081765	9.9609548	9.6472217	10.3527783	4
57	4611	8987	5624	4376	3
58	7454	8426	9028	0972	2
59	9.6090294	7864	9.6482431	10.3517569	1
60	3133	7302	5831	4169	0
	Coſine	Sine	Cotangent	Tangent	Min.
	113	Degrees		66	

Min.	24 Sine	Degrees Cosine	Tangent	155 Cotangent	
0	9.6093133	9.9607302	9.6485831	10.3514169	60
1	5969	6739	9230	0770	59
2	8803	6176	9.6492628	10.3507372	58
3	9.6101635	5612	6023	3977	57
4	4465	5048	9417	0583	56
5	7293	4484	9.6502809	10.3497191	55
6	9.6110118	9.9603919	6199	3801	54
7	2941	3354	9587	0413	53
8	5762	2788	9.6512974	10.3487026	52
9	8580	2222	6359	3641	51
10	9.6121397	1655	9742	0258	50
11	4211	9.9601088	9.6523123	10.3476877	49
12	7023	0520	6503	3497	48
13	9833	9.9599952	9881	0119	47
14	9.6132641	9384	9.6533257	10.3466743	46
15	5446	8815	6631	3369	45
16	8250	9.9598246	9.6540004	80.3459996	44
17	9.6141051	7676	3375	6625	43
18	3850	7106	6744	3256	42
19	6647	6535	9.6550112	10.3449888	41
20	9441	5964	3477	6523	40
21	9.6152234	9.9595393	6841	3159	39
22	5024	4821	9.6560204	10.3439796	38
23	7812	4248	3564	6436	37
24	9.6160598	3675	6923	3077	36
25	3382	3102	9.6570280	10.3429720	35
26	6164	9.9592528	3636	6364	34
27	8944	1954	6989	3011	33
28	9.6171721	1380	9.6580341	10.3419659	32
29	4496	0805	3692	6308	31
30	7270	0229	7041	2960	30
	Cosine	Sine	Cotangent	Tangent	Min.
	114	Degrees		65	

	24	Degrees		155	
Min.	Sine	Coſine	Tangent	Cotangent	
30	9.6177270	9.9590229	9.6587041	10.3412960	30
31	9.6180041	9.9589653	9.6590387	10.3409613	29
32	2809	9077	3733	6267	28
33	5576	8500	7076	2924	27
34	8341	7923	9.6600418	10.3399582	26
35	9.6191103	7345	3758	6242	25
36	3864	9.9586767	7097	2903	24
37	6622	6188	9.6610434	10.3389566	23
38	9378	5609	3769	6231	22
39	9.6202132	5030	7103	2897	21
40	4884	4450	9.6620434	10.3379566	20
41	7634	9.9583869	3765	6235	19
42	9.6210382	3288	7093	2907	18
43	3127	2707	9.6630420	10.3369580	17
44	5871	2125	3745	6255	16
45	8612	1543	7069	2931	15
46	9.6221351	9.9580961	9.6640391	10.3359609	14
47	4088	0378	3711	6289	13
48	6824	9.9579794	7030	2970	12
49	9557	9210	9.6650346	10.3349654	11
50	9.6232287	8626	3662	6338	10
51	5016	9.9578041	6975	3025	9
52	7743	7456	9.6660288	10.3339712	8
53	9.6240467	6870	3598	6402	7
54	3190	6284	6907	3093	6
55	5911	5697	9.6670214	10.3329786	5
56	8629	9.9575110	3519	6481	4
57	9.6251346	4522	6823	3177	3
58	4060	3934	9.6680126	10.3319874	2
59	6772	3346	3426	6574	1
60	9483	2757	6725	3275	0
	Coſine	Sine	Cotangent	Tangent	Min.
	114	Degrees		65	

Min.	25	Degrees		154	
	Sine	Coſine	Tangent	Cotangent	
0	9.6259483	9.9572757	9.6686725	10.3313275	60
1	9.6262191	9.9572168	9.6690023	10.3309977	59
2	4897	1578	3319	6681	58
3	7601	0988	6613	3387	57
4	9.6270303	0397	9906	0094	56
5	3003	9.9569806	9.6703197	10.3296803	55
6	5701	9215	6486	3514	54
7	8397	8623	9774	0226	53
8	9.6281090	8030	9.6713060	10.3286940	52
9	3782	7437	6345	3655	51
10	6472	6844	9628	0372	50
11	9160	9.9566250	9.6722910	10.3277090	49
12	9.6291845	5656	6190	3810	48
13	4529	5061	9468	0532	47
14	7211	4466	9.6732745	10.3267255	46
15	9890	3870	6020	3980	45
16	9.6302568	9.9563274	9294	0706	44
17	5243	2678	9.6742566	10.3257434	43
18	7917	2081	5836	4164	42
19	9.6310589	1483	9105	0895	41
20	3258	0886	9.6752372	10.3247628	40
21	5926	0287	5638	4362	39
22	8591	9.9559689	8903	1097	38
23	9.6321255	9089	9.6762165	10.3237835	37
24	3916	8490	5426	4574	36
25	6576	7890	8686	1314	35
26	9233	9.9557289	9.6771944	10.3228056	34
27	9.6331889	6688	5201	4799	33
28	4542	6087	8456	1544	32
29	7194	5485	9.6781709	10.3218291	31
30	9844	4882	4961	5039	30
	Coſine	Sine	Cotangent	Tangent	Min.
	115	Degrees		64	

Min.	25	Degrees		154	
	Sine	Coſine	Tangent	Cotangent	
30	9.6339844	9.9554882	9.6784961	10.3215039	30
31	9.6342491	4280	8211	1789	29
32	5137	3676	9.6791460	10.3208540	28
33	7780	3073	4708	5292	27
34	9.6350422	2469	7953	2047	26
35	3062	1864	9.6801198	10.3198802	25
36	5699	9.9551259	4440	5560	24
37	8335	0653	7682	2318	23
38	9.6360969	0047	9.6810921	10.3189079	22
39	3601	9.9549441	4160	5840	21
40	6231	8834	7396	2604	20
41	8859	9.9548227	9.6820632	10.3179368	19
42	9.6371484	7619	3865	6135	18
43	4108	7011	7098	2902	17
44	6731	6402	9.6830328	10.3169672	16
45	9351	5793	3557	6443	15
46	9.6381969	9.9545184	6785	3215	14
47	4585	4574	9.6840011	10.3159989	13
48	7199	3963	3236	6764	12
49	9812	3352	6459	3541	11
50	9.6392422	2741	9681	0319	10
51	5030	9.9542129	9.6852901	10.3147099	9
52	7637	1517	6120	3880	8
53	9.6400241	0904	9338	0662	7
54	2844	0291	9.6862553	10.3137447	6
55	5445	9.9539677	5768	4232	5
56	8044	9063	8981	1019	4
57	9.6410640	8448	9.6872192	10.3127808	3
58	3235	7833	5402	4598	2
59	5828	7218	8611	1389	1
60	8420	6602	9.6881818	10.3118182	0
	Coſine	Sine	Cotangent	Tangent	Min.
	115	Degrees		64	

Min.	26	Degrees		153	
	Sine	Coſine	Tangent	Cotangent	
0	9.6418420	9.9536602	9.6881818	10.3118182	60
1	9.6421009	5985	5023	4977	59
2	3596	5369	8227	1773	58
3	6182	4751	9.6891430	10.3108570	57
4	8765	4134	4631	5369	56
5	9.6431347	3515	7831	2169	55
6	3926	9.9532897	9.6901030	10.3098970	54
7	6504	2278	4226	5774	53
8	9080	1658	7422	2578	52
9	9.6441654	1038	9.6910616	10.3089384	51
10	4226	0418	3809	6191	50
11	6796	9.9529797	7000	3000	49
12	9365	9175	9.6920189	10.3079811	48
13	9.6451931	8553	3378	6622	47
14	4496	7931	6565	3435	46
15	7058	7308	9750	0250	45
16	9619	9.9526685	9.6932934	10.3067066	44
17	9.6462178	6061	6117	3883	43
18	4735	5437	9298	0702	42
19	7290	4813	9.6942478	10.3057522	41
20	9844	4188	5656	4344	40
21	9.6472395	9.9523562	8833	1167	39
22	4945	2936	9.6952009	10.3047991	38
23	7492	2310	5183	4817	37
24	9.6480038	1683	8355	1645	36
25	2582	1055	9.6961527	10.3038473	35
26	5124	0428	4697	5303	34
27	7665	9.9519799	7865	2135	33
28	9.6490203	9171	9.6971032	10.3028968	32
29	2740	8541	4198	5802	31
30	5274	7912	7363	2637	30
	Coſine	Sine	Cotangent	Tangent	Min.
	116	Degrees		63	

Min.	26	Degrees		153	
	Sine	Coſine	Tangent	Cotangent	
30	9.6495274	9.9517912	9.6977363	10.3022637	30
31	7807	7282	9.6980526	10.3019474	29
32	9.6500338	6651	3687	6313	28
33	2868	6020	6847	3153	27
34	5395	5389	9.6990006	10.3009994	26
35	7920	4757	3164	6836	25
36	9.6510444	9.9514124	6320	3680	24
37	2966	3492	9474	0526	23
38	5486	2858	9.7002628	10.2997372	22
39	8004	2224	5780	4220	21
40	9.6520521	1590	8930	1070	20
41	3035	9.9510956	9.7012080	10.2987920	19
42	5548	0320	5227	4773	18
43	8059	9.9509685	8374	1626	17
44	9.6530568	9049	9.7021519	10.2978481	16
45	3075	8412	4663	5337	15
46	5581	9.9507775	7805	2195	14
47	8084	7138	9.7030946	10.2969054	13
48	9.6540586	6500	4086	5914	12
49	3086	5861	7225	2775	11
50	5584	5223	9.7040362	10.2959638	10
51	8081	9.9504583	3497	6503	9
52	9.6550575	3944	6632	3368	8
53	3068	3303	9765	0235	7
54	5559	2663	9.7052897	10.2947103	6
55	8048	2022	6027	3973	5
56	9.6560536	9.9501380	9156	0844	4
57	3021	0738	9.7062284	10.2937716	3
58	5505	0095	5410	4590	2
59	7987	9.9499452	8535	1465	1
60	9.6570468	8809	9.7071659	10.2928341	0
	Coſine	Sine	Cotangent	Tangent	Min.
	116	Degrees		63	

Min.	27	Degrees		152	
	Sine	Coſine	Tangent	Cotangent	
0	9.6570468	9.9498809	9.7071659	10.2928341	60
1	2946	8165	4781	5219	59
2	5423	7521	7902	2098	58
3	7898	6876	9.7081022	10.2918978	57
4	9.6580371	6230	4141	5859	56
5	2842	5585	7258	2742	55
6	9.6585312	9.9494938	9.7090374	10.2909626	54
7	7780	4292	3488	6512	53
8	9.6590246	3645	6601	3399	52
9	2710	2997	9713	0287	51
10	5173	2349	9.7102824	10.2897176	50
11	7634	9.9491700	5933	4067	49
12	9.6600093	1051	9041	0959	48
13	2550	0402	9.7112148	10.2887852	47
14	5005	9.9489752	5254	4746	46
15	7459	9101	8358	1642	45
16	9911	9.9488450	9.7121461	10.2878539	44
17	9.6612361	7799	4562	5438	43
18	4810	7147	7662	2338	42
19	7257	6495	9.7130761	10.2869239	41
20	9702	5842	3859	6141	40
21	9.6622145	9.9485189	6956	3044	39
22	4586	4535	9.7140051	10.2859949	38
23	7026	3881	3145	6855	37
24	9464	3226	6237	3763	36
25	9.6631900	2572	9329	0671	35
26	4335	9.9481916	9.7152419	10.2847581	34
27	6768	1260	5508	4492	33
28	9199	0604	8595	1405	32
29	9.6641628	9.9479947	9.7161682	10.2838318	31
30	4056	9289	4767	5233	30
	Coſine	Sine	Cotangent	Tangent	Min.
	117	Degrees		62	

Min.	27	Degrees		152	
	Sine	Coſine	Tangent	Cotangent	
30	9.6644056	9.9479289	9.7164767	10.2835233	30
31	6482	8631	7851	2149	29
32	8906	7973	9.7170933	10.2829067	28
33	9.6651329	7314	4014	5986	27
34	3749	6655	7094	2906	26
35	6168	5995	9.7180173	10.2819827	25
36	8586	4.9475335	3251	6749	24
37	9.6661001	4674	6327	3673	23
38	3415	4013	9402	0598	22
39	5828	3352	9.7192476	10.2807524	21
40	8238	2689	5549	4451	20
41	9.6670647	9.9472027	8620	1380	19
42	3054	1364	9.7201690	10.2798309	18
43	5459	0700	4759	5241	17
44	7863	0036	7827	2173	16
45	9.6680265	9.9469372	9.7210893	10.2789107	15
46	2665	8707	3958	6042	14
47	5064	8042	7022	2978	13
48	7461	7376	9.7220085	10.2779915	12
49	9856	6710	3147	6853	11
50	9.6692250	6043	6207	3793	10
51	4642	9.9465376	9266	0734	9
52	7032	4708	9.7232324	10.2767676	8
53	9420	4040	5381	4619	7
54	9.6701807	3371	8436	1564	6
55	4192	2702	9.7241490	10.2758510	5
56	6576	9.9462032	4543	5457	4
57	8958	1362	7595	2405	3
58	9.6711338	0692	9.7250646	10.2749354	2
59	3716	0021	3695	6305	1
60	6093	9.9459349	6744	3256	0
	Coſine	Sine	Cotangent	Tangent	Min.
	117	Degrees		62	

Min.	28	Degrees		151	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.6716093	9·9459349	9·7256744	10.2743256	60
1	8468	8677	9791	0209	59
2	9.6720841	8005	9·7262837	10.2737163	58
3	3213	7332	5881	4119	57
4	5583	6659	8925	1075	56
5	7952	5985	9·7271967	10.2728033	55
6	9.6730319	9·9455310	5008	4992	54
7	2684	4636	8048	1952	53
8	5047	3960	9·7281087	10.2718913	52
9	7409	3285	4124	5876	51
10	9769	2609	7161	2839	50
11	9.6742128	9·9451932	9·7290196	10.2709804	49
12	4485	1255	3230	6770	48
13	6840	0577	6263	3737	47
14	9194	9·9449899	9295	0705	46
15	9.6751546	9220	9·7302325	10.2697675	45
16	3896	8541	5354	4646	44
17	6245	7862	8383	1617	43
18	8592	7182	9·7311410	10.2688590	42
19	9.6760937	6501	4436	5564	41
20	3281	5821	7460	2540	40
21	5623	9·9445139	9·7320484	10.2679516	39
22	7963	4457	3506	6494	38
23	9.6770302	3775	6527	3473	37
24	2640	3092	9547	0453	36
25	4975	2409	9·7332566	10.2667434	35
26	7309	9·9441725	5584	4416	34
27	9642	1041	8601	1399	33
28	9.6781972	0356	9·7341616	10.2658384	32
29	4301	9·9439671	4631	5369	31
30	6629	8985	7644	2356	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	118	Degrees		61	

Min.	28	Degrees		151	
	Sine	Coſine	Tangent	Cotangent	
30	9.6786629	9.9438985	9.7347644	10.2652356	30
31	8955	8299	9.7350656	10.2649344	29
32	9.6791279	7612	3667	6333	28
33	3602	6925	6677	3323	27
34	5923	6238	9685	0315	26
35	8243	5549	9.7362693	10.2637307	25
36	9.6800560	9.9434861	5699	4301	24
37	2877	4172	8705	1295	23
38	5191	3482	9.7371709	10.2628291	22
39	7504	2792	4712	5288	21
40	9816	2102	7714	2286	20
41	9.6812126	9.9431411	9.7380715	10.2619285	19
42	4434	0720	3714	6286	18
43	6741	0028	6713	3287	17
44	9046	9.9429335	9710	0290	16
45	9.6821349	8643	9.7392707	10.2607293	15
46	3651	7949	5702	4298	14
47	5951	7255	8696	1304	13
48	8250	6561	9.7401689	10.2598311	12
49	9.6830548	5866	4681	5319	11
50	2843	5171	7672	2328	10
51	5137	9.9424476	9.7410662	10.2589338	9
52	7430	3779	3650	6350	8
53	9720	3083	6638	3362	7
54	9.6842010	2386	9624	0376	6
55	4297	1688	9.7422609	10.2577391	5
56	6583	9 9420990	5594	4406	4
57	8868	0291	8577	1423	3
58	9.6851151	9.9419592	9.7431559	10.2568441	2
59	3432	8893	4540	5460	1
60	5712	8193	7520	2480	0
	Coſine	Sine	Cotangent	Tangent	Min.
	118	Degrees		61	

Min.	29	*Degrees*		150	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.6855712	9.9418193	9.7437520	10.2562480	60
1	7991	7492	9.7440499	10.2559501	59
2	9.6860267	6791	3476	6524	58
3	2542	6090	6453	3547	57
4	4816	5388	9428	0572	56
5	7088	4685	9.7452403	10.2547597	55
6	9359	9.9413982	5376	4624	54
7	9.6871628	3279	8349	1651	53
8	3895	2575	9.7461320	10.2538680	52
9	6161	1871	4290	5710	51
10	8425	1166	7259	2741	50
11	9.6880688	0461	9.7470227	10.2529773	49
12	2949	9.9409755	3194	6806	48
13	5209	9048	6160	3840	47
14	7467	8342	9125	0875	46
15	9723	7634	9.7482089	10.2517911	45
16	9.6891978	9.9406927	5052	4948	44
17	4232	6219	8013	1987	43
18	6484	5510	9.7490974	10.2509026	42
19	8734	4801	3934	6066	41
20	9.6900983	4091	6892	3108	40
21	3231	9.9403381	9850	0150	39
22	5476	2670	9.7502806	10.2497194	38
23	7721	1959	5762	4238	37
24	9964	1248	8716	1284	36
25	9.6912205	0535	9.7511669	10.2488331	35
26	4445	9.9399823	4622	5378	34
27	6683	9110	7573	2427	33
28	8919	8396	9.7520523	10.2479477	32
29	9.6921155	7682	3472	6528	31
30	3388	6968	6420	3580	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	*Min.*
	119	*Degrees*		60	

Min.	29	Degrees		150	
	Sine	Coſine	Tangent	Cotangent	
30	9.6923388	9.9396968	9.7526420	10.2473580	30
31	5620	6253	9368	0632	29
32	7851	5537	9.7532314	10.2467686	28
33	9.6930080	4821	5259	4741	27
34	2308	4105	8203	1797	26
35	4534	3388	9.7541146	10.2458854	25
36	9.6936758	9.9392671	4088	5912	24
37	8981	1953	7029	2971	23
38	9.6941203	1234	9969	0031	22
39	3423	0515	9.7552908	10.2447092	21
40	5642	9.9389796	5846	4154	20
41	7859	9076	8783	1217	19
42	9.6950074	8356	9.7561718	10.2438282	18
43	2288	7635	4653	5347	17
44	4501	6914	7587	2413	16
45	6712	6192	9.7570520	10.2429480	15
46	8922	9.9385470	3452	6548	14
47	9.6961130	4747	6383	3617	13
48	3336	4024	9313	0687	12
49	5541	3300	9.7582242	10.2417758	11
50	7745	2576	5170	4830	10
51	9947	9.9381851	8096	1904	9
52	9.6972148	1126	9.7591022	10.2408978	8
53	4347	0400	3947	6053	7
54	6545	9.9379674	6871	3129	6
55	8741	8947	9794	0206	5
56	9.6980936	9.9378220	9.7602716	10.2397284	4
57	3129	7492	5637	4363	3
58	5321	6764	8557	1443	2
59	7511	6035	9.7611476	10.2388524	1
60	9700	5806	4394	5606	0
	Coſine	Sine	Cotangent	Tangent	Min.
	119	Degrees		60	

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Min.	30	Degrees		149	
	Sine	Coſine	Tangent	Cotangent	
0	9.6989700	9.9375306	9.7614394	10.2385606	60
1	9.6991887	4577	7311	2689	59
2	4073	3847	9.7620227	10.2379773	58
3	6258	3116	3142	6858	57
4	8441	2385	6056	3944	56
5	9.7000622	1653	8969	1031	55
6	2802	9.9370921	9.7631881	10.2368119	54
7	4981	0189	4792	5208	53
8	7158	9.9369456	7702	2298	52
9	9334	8722	9.7640612	10.2359388	51
10	9.7011508	7988	3520	6480	50
11	3681	9.9367254	6427	3573	49
12	5852	6519	9334	0666	48
13	8022	5783	9.7652239	10.2347761	47
14	9.7020190	5047	5143	4857	46
15	2357	4311	8047	1953	45
16	4523	9.9363574	9.7660949	10.2339051	44
17	6687	2836	3851	6149	43
18	8849	2098	6751	3249	42
19	9.7031011	1360	9651	0349	41
20	3170	0621	9.7672550	10.2327450	40
21	5329	9.9359881	5448	4552	39
22	7486	9141	8344	1656	38
23	9641	8401	9.7681240	10.2318760	37
24	9.7041795	7660	4135	5865	36
25	3947	6918	7029	2971	35
26	6099	9.9356177	9922	0078	34
27	8248	5434	9.7692814	10.2307186	33
28	9,7050397	4691	5705	4295	32
29	2543	3948	8596	1404	31
30	4689	3204	9.7701485	10.2298515	30
	Coſine	Sine	Cotangent	Tangent	Min.
	120	Degrees		59	

Min.	30	Degrees		149	
	Sine	Coſine	Tangent	Cotangent	
30	9.7054689	9.9353204	9.7701485	10.2298515	30
31	6833	2459	4373	5627	29
32	8975	1715	7261	2739	28
33	9.7061116	0969	9.7710147	10.2289853	27
34	3256	0223	3033	6967	26
35	5394	9.9349477	5917	4083	25
36	9.7067531	8730	8801	1199	24
37	9667	7983	9.7721684	10.2278316	23
38	9.7071801	7235	4566	5434	22
39	3933	6486	7447	2553	21
40	6064	5738	0327	10.2269673	20
41	8194	9.9344988	9.7733206	6794	19
42	9.7080323	4238	6084	3916	18
43	2450	3488	8961	1039	17
44	4575	2737	9.7741838	10.2258162	16
45	6699	1986	4713	5287	15
46	8822	9.9341234	7588	2412	14
47	9.7090943	4482	9.7750462	10.2249538	13
48	3063	9.9339729	3334	6666	12
49	5182	8976	6206	3794	11
50	7299	8222	9077	0923	10
51	9415	9.9337467	9.7761947	10.2238053	9
52	9.7101529	6713	4816	5184	8
53	3642	5957	7685	2315	7
54	5753	5201	9.7770552	10.2229448	6
55	7863	4445	3418	6582	5
56	9972	9.9333688	6284	3716	4
57	9.7112080	2931	9149	0851	3
58	4186	2173	9.7782012	10.2217988	2
59	6290	1415	4875	5125	1
60	8393	0656	7737	2263	0
	Coſine	Sine	Cotangent	Tangent	Min.
	120	Degrees		59	

Min.	31	*Degrees*		148	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.7118393	9.9330656	9.7787737	10.2212263	60
1	9.7120495	9.9329897	9.7790599	10.2209401	59
2	2596	9137	3459	6541	58
3	4695	8376	6318	3682	57
4	6792	7616	9177	0823	56
5	8889	6854	9.7802034	10.2197966	55
6	9.7130983	9.9326092	4891	5109	54
7	3077	5330	7747	2253	53
8	5169	4567	9.7810602	10.2189398	52
9	7260	3804	3456	6544	51
10	9349	3040	6309	3691	50
11	9.7141437	9.9322276	9162	0838	49
12	3524	1511	9.7822013	10.2177987	48
13	5609	0746	4864	5136	47
14	7693	9.9319980	7713	2287	46
15	9776	9213	9.7830562	10.2169438	45
16	9.7151857	9.9318447	3410	6590	44
17	3937	7679	6258	3742	43
18	6015	6911	9104	0896	42
19	8092	6143	9.7841949	10.2158051	41
20	9.7160168	5374	4794	5206	40
21	2243	9.9314605	7638	2362	39
22	4316	3835	9.7850481	10.2149519	38
23	6387	3065	3323	6677	37
24	8458	2294	6164	3836	36
25	9.7170526	1522	9004	0996	35
26	2594	0750	9.7861844	10.2138156	34
27	4660	9.9309978	4682	5318	33
28	6725	9205	7520	2480	32
29	8789	8432	9.7870357	10.2129643	31
30	9.7180851	7658	3193	6807	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	*Min.*
	121	*Degrees*		58	

Min.	31	Degrees		148	
	Sine	Coſine	Tangent	Cotangent	
30	9.7180851	9.9307658	9.7873193	10.2126807	30
31	2912	6883	6028	3972	29
32	4971	6109	8863	1137	28
33	7030	5333	9.7881696	10.2118304	27
34	9086	4557	4529	5471	26
35	9.7191142	3781	7361	2639	25
36	3196	9.9303004	9.7890192	10.2109808	24
37	5249	2226	3023	6977	23
38	7300	1448	5852	4148	22
39	9350	0670	8681	1319	21
40	9.7201399	9.9299891	9.7901508	10.2098492	20
41	3447	9112	4335	5665	19
42	5493	8332	7161	2839	18
43	7538	7551	9987	0013	17
44	9581	6770	9.7912811	10.2087189	16
45	9.7211623	5989	5635	4365	15
46	3664	9.9295207	8458	1542	14
47	5704	4424	9.7921280	10.2078720	13
48	7742	3641	4101	5899	12
49	9779	2857	6921	3079	11
50	9.7221814	2073	9741	0259	10
51	3848	1289	9.7932560	10.2067440	9
52	5881	0504	5378	4622	8
53	7913	9.9289718	8195	1805	7
54	9943	8932	9.7941011	10.2058989	6
55	9.7231972	8145	3827	6173	5
56	4000	9.9287358	6641	3359	4
57	6026	6571	9455	0545	3
58	8051	5783	9.7952268	10.2047732	2
59	9.7240075	4994	5081	4919	1
60	2097	4205	7892	2108	0
	Coſine	Sine	Cotangent	Tangent	Min.
	121	Degrees		58	

Min.	32	Degrees		147	
	Sine	Coſine	Tangent	Cotangent	
0	9.7242097	9.9284205	9.7957892	10.2042108	60
1	4118	3415	9.7960703	10.2039297	59
2	6138	2625	3513	6487	58
3	8156	1834	6322	3678	57
4	9.7250174	1043	9130	0870	56
5	2189	0251	9.7971938	10.2028062	55
6	4204	9.9279459	4745	5255	54
7	6217	8666	7551	2449	53
8	8229	7873	9.7980356	10.2019644	52
9	9.7260240	7079	3160	6840	51
10	2249	6285	5964	4036	50
11	4257	9.9275490	8767	1233	49
12	6264	4695	9.7991569	10.2008431	48
13	8269	3899	4370	5630	47
14	9.7270273	3103	7170	2830	46
15	2276	2306	9970	0030	45
16	4278	9.9271509	9.8002769	10.1997231	44
17	6278	0711	5567	4433	43
18	8277	9.9269913	8365	1635	42
19	9.7280275	9114	9.8011161	10.1988839	41
20	2271	8314	3957	6043	40
21	4267	9.9267514	6752	3248	39
22	6260	6714	9546	0454	38
23	8253	5913	9.8022340	10.1977660	37
24	9.7290244	5112	5133	4867	36
25	2234	4310	7925	2075	35
26	4223	9.9263507	9.8030716	10.1969284	34
27	6211	2704	3506	6494	33
28	8197	1901	6296	3704	32
29	9.7300182	1096	9085	0915	31
30	2165	0292	9.8041873	10.1958127	30
	Coſine	Sine	Cotangent	Tangent	Min.
	122	Degrees		57	

Min.	32	*Degrees*		147	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	9.7302165	9.9260292	9.8041873	10.1958127	30
31	4148	9.9259487	4661	5339	29
32	6129	8681	7447	2553	28
33	8109	7875	9.8050233	10.1949767	27
34	9.7310087	7069	3019	6981	26
35	2064	6261	5803	4197	25
36	4040	9.9255454	8587	1413	24
37	6015	4646	9.8061370	10.1938630	23
38	7989	3837	4152	5848	22
39	9961	3028	6933	3067	21
40	9.7321932	2218	9714	0286	20
41	3902	9.9251408	9.8072494	10.1927506	19
42	5870	0597	5273	4727	18
43	7837	9.9249786	8052	1948	17
44	9803	8974	9.8080829	10.1919171	16
45	9.7331768	8161	3606	6394	15
46	3731	9.9247349	6383	3617	14
47	5693	6535	9158	0842	13
48	7654	5721	9.8091933	10.1908067	12
49	9614	4907	4707	5293	11
50	9.7341572	4092	7480	2520	10
51	3529	9.9243277	9.8100253	10.1899747	9
52	5485	2461	3025	6975	8
53	7440	1644	5796	4204	7
54	9393	0827	8566	1434	6
55	9.7351345	0010	9.8111336	10.1888664	5
56	3296	9.9239191	4105	5895	4
57	5246	8373	6873	3127	3
58	7195	7554	9641	0359	2
59	9142	6734	9.8122408	10.1877592	1
60	9.7361088	5914	5174	4826	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	122	*Degrees*		57	

Min.	33	Degrees		146	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.7361088	9.9235914	9.8125174	10.1874826	60
1	3032	5093	7939	2061	59
2	4976	4272	9.8130704	10.1869296	58
3	6918	3450	3468	6532	57
4	8859	2628	6231	3769	56
5	9.7370799	1805	8993	1007	55
6	2737	9.9230982	9.8141755	10.1858245	54
7	4675	0158	4516	5484	53
8	6611	9.9229334	7277	2723	52
9	8546	8509	9.8150036	10.1849964	51
10	9.7380479	7684	2795	7205	50
11	2412	9.9226858	5554	4446	49
12	4343	6032	8311	1689	48
13	6273	5205	9.8161068	10.1838932	47
14	8201	4377	3824	6176	46
15	9.7390129	3549	6580	3420	45
16	2055	9.9222721	9335	0665	44
17	3980	1891	9.8172089	10.1827911	43
18	5904	1062	4842	5158	42
19	7827	0232	7595	2405	41
20	9748	9.9219401	9.8180347	10.1819653	40
21	9.7401668	8570	3098	6902	39
22	3587	7738	5849	4151	38
23	5505	6906	8599	1401	37
24	7421	6073	9.8191348	10.1808652	36
25	9337	5240	4096	5904	35
26	9.7411251	9.9214406	6844	3156	34
27	3164	3572	9592	0408	33
28	5075	2737	9.8202338	10.1797662	32
29	6986	1902	5084	4916	31
30	8895	1066	7829	2171	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	123	Degrees		56	

Min.	33	Degrees		146	
	Sine	Coſine	Tangent	Cotangent	
30	9.7418895	9.9211066	9.8207829	10.1792171	30
31	9.7420803	0229	9.8210574	10.1789426	29
32	2710	9.9209393	3317	6683	28
33	4616	8555	6060	3940	27
34	6520	7717	8803	1197	26
35	8423	6878	9.8221545	10.1778455	25
36	9.7430325	9.9206039	4286	5714	24
37	2226	5200	7026	2974	23
38	4126	4360	9766	0234	22
39	6024	3519	9.8232505	10.1767495	21
40	7921	2678	5244	4756	20
41	9817	1836	7981	2019	19
42	9.7441712	0994	9.8240719	10.1759281	18
43	3606	0151	3455	6545	17
44	5498	9.9199308	6191	3809	16
45	7390	8464	8926	1074	15
46	9280	9.9197619	9.8251660	10.1748340	14
47	9.7451169	6775	4394	5606	13
48	3056	5929	7127	2873	12
49	4943	5083	9860	0140	11
50	6828	4237	9.8262592	10.1737408	10
51	8712	9.9193390	5323	4677	9
52	9.7460595	2542	8053	1947	8
53	2477	1694	9.8270783	10.1729217	7
54	4358	0845	3513	6487	6
55	6237	9.9189996	6241	3759	5
56	8115	9146	8969	1031	4
57	9992	8296	9.8281696	10.1718304	3
58	9.7471868	7445	4423	5577	2
59	3743	6594	7149	2851	1
60	5617	5742	9874	0126	0
	Coſine	Sine	Cotangent	Tangent	
	123	Degrees		56	Min.

Min.	34	*Degrees*		145	
	Sine	*Coſine*	*Tangent*	*Cotangent.*	
0	9.7475617	9.9185742	9.8289874	10.1710126	60
1	7489	4890	9.8292599	10.1707401	59
2	9360	4037	5323	4677	58
3	9.7481230	3183	8047	1953	57
4	3099	2329	9.8300769	10.1699231	56
5	4967	1475	3492	6508	55
6	6833	0620	6213	3787	54
7	8698	9.9179764	8934	1066	53
8	9.7490562	8908	9.8311654	10.1688346	52
9	2425	8051	4374	5626	51
10	4287	7194	7093	2907	50
11	6148	9.9176336	9811	0189	49
12	8007	5478	9.8322529	10.1677471	48
13	9866	4619	5246	4754	47
14	9.7501723	3760	7963	2037	46
15	3579	2900	9.8330679	10.1669321	45
16	5434	9.9172040	3394	6606	44
17	7287	1179	6109	3891	43
18	9140	0317	8823	1177	42
19	9.7510991	9.9169455	9.8341536	10.1658464	41
20	2842	8593	4249	5751	40
21	4691	9.9167730	6961	3039	39
22	6538	6866	9673	0327	38
23	8385	6002	9.8352384	10.1647616	37
24	9.7520231	5137	5094	4906	36
25	2075	4272	7804	2196	35
26	3919	9.9163406	9.8360513	10.1639487	34
27	5761	2539	3221	6779	33
28	7602	1673	5929	4071	32
29	9442	0805	8636	1364	31
30	9.7531280	9.9159937	9.8371343	10.1628657	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	*Min.*
	124	*Degrees*		55	

Min.	34	Degrees		145	
	Sine	Coſine	Tangent	Cotangent	
30	9.7531280	9.9159937	9.8371343	10.1628657	30
31	3118	9069	4049	5951	29
32	4954	8200	6755	3245	28
33	6790	7330	9460	0540	27
34	8624	6460	9.8382164	10.1617836	26
35	9.7540457	5589	4867	5133	25
36	2288	9.9154718	7571	2429	24
37	4119	3846	9.8390273	10.1609727	23
38	5949	2974	2975	7025	22
39	7777	2101	5676	4324	21
40	9604	1228	8377	1623	20
41	9.7551431	0354	9.8401077	10.1598923	19
42	3256	9.9149479	3776	6224	18
43	5080	8604	6475	3525	17
44	6902	7729	9174	0826	16
45	8724	6852	9.8411871	10.1588129	15
46	9.7560544	9.9145976	4569	5431	14
47	2364	5099	7265	2735	13
48	4182	4221	9961	0039	12
49	5999	3342	9.8422657	10.1577343	11
50	7815	2464	5351	4649	10
51	9630	9.9141584	8046	1954	9
52	9.7571444	0704	9.8430739	10.1569261	8
53	3256	9.9139824	3432	6568	7
54	5068	8943	6125	3875	6
55	6878	8061	8817	1183	5
56	8687	9.9137179	9.8441508	10.1558492	4
57	9.7580495	6296	4199	5801	3
58	2302	5413	6889	3111	2
59	4108	4530	9579	0421	1
60	5913	3645	9.8452268	10.1547732	0
	Coſine	Sine	Cotangent	Tangent	Min.
	124	Degrees		55	

Min.	35	Degrees		144	
	Sine	Coſine	Tangent	Cotangent	
0	9.7585913	9.9133645	9.8452268	10.1547732	60
1	7717	2760	4956	5044	59
2	9519	1875	7644	2356	58
3	9.7591321	0989	9.8460332	10.1539668	57
4	3121	0102	3018	6982	56
5	4920	9.9129215	5705	4295	55
6	6718	8328	8390	1610	54
7	8515	7440	9.8471075	10.1528925	53
8	9.7600311	6551	3760	6240	52
9	2106	5662	6444	3556	51
10	3899	4772	9127	0873	50
11	5692	9.9123882	9.8481810	10.1518190	49
12	7483	2991	4492	5508	48
13	9274	2099	7174	2826	47
14	9.7611063	1207	9855	0145	46
15	2851	0315	9.8492536	10.1507464	45
16	4638	9.9119422	5216	4784	44
17	6424	8528	7896	2104	43
18	8208	7634	9.8500575	10.1499425	42
19	9992	6739	3253	6747	41
20	9.7621775	5844	5931	4069	40
21	3556	9.9114948	8608	1392	39
22	5337	4051	9.8511285	10.1488715	38
23	7116	3155	3961	6039	37
24	8894	2257	6637	3363	36
25	9.7630671	1359	9312	0688	35
26	2447	0460	9.8521987	10.1478013	34
27	4222	9.9109561	4661	5339	33
28	5996	8661	7335	2665	32
29	7769	7761	9.8530008	10.1469992	31
30	9540	6860	2680	7320	30
	Coſine	Sine	Cotangent	Tangent	Min.
	125	Degrees		54	

Min.	35 Degrees Sine	Coſine	Tangent	Cotangent 144	Min.
30	9.7639540	9.9106860	9.8532680	10.1467320	30
31	9.7641311	5959	5352	4648	29
32	3080	5057	8023	1977	28
33	4849	4155	9.8540694	10.1459306	27
34	6616	3251	3365	6635	26
35	8382	2348	6034	3966	25
36	9.7650147	9.9101444	8704	1296	24
37	1911	0539	9.8551372	10.1448628	23
38	3674	9.9099634	4041	5959	22
39	5436	8728	6708	3292	21
40	7197	7821	9376	0624	20
41	8957	9.9096915	9.8562042	10.1437958	19
42	9.7660715	6007	4708	5292	18
43	2473	5099	7374	2626	17
44	4229	4190	9.8570039	10.1429961	16
45	5985	3281	2704	7296	15
46	7739	9.9092371	5368	4632	14
47	9492	1461	8031	1969	13
48	9.7671244	0550	9.8580694	10.1419306	12
49	2996	9.9089639	3357	6643	11
50	4746	8727	6019	3981	10
51	9.7676494	7814	8680	1320	9
52	8242	6901	9.8591341	10.1408659	8
53	9989	5988	4002	5998	7
54	9.7681735	5073	6661	3339	6
55	3480	4159	9321	0679	5
56	9.7685223	9.9083243	9.8601980	10.1398020	4
57	6966	2327	4638	5362	3
58	8707	1411	7296	2704	2
59	9.7690448	0494	9954	0046	1
60	2187	9.9079576	9.8612610	10.1387390	0
	Coſine	Sine	Cotangent	Tangent	Min.
	125	Degrees		54	

Min.	36	Degrees		143	
	Sine	Coſine	Tangent	Cotangent	
0	9.7692187	9.9079576	9.8612610	10.1387390	60
1	3925	8658	5267	4733	59
2	5662	7740	7923	2077	58
3	7398	6820	9.8620578	10.1379422	57
4	9134	5901	3233	6767	56
5	9.7700868	4980	5887	4113	55
6	2601	9.9074059	8541	1459	54
7	4332	3138	9.8631195	10.1368805	53
8	6063	2216	3848	6152	52
9	7793	1293	6500	3500	51
10	9522	0370	9152	0848	50
11	9.7711249	9.9069446	9.8641803	10.1358197	49
12	2976	8522	4454	5546	48
13	4702	7597	7105	2895	47
14	6426	6671	9755	0245	46
15	8150	5745	9.8652404	10.1347596	45
16	9872	9.9064819	5053	4947	44
17	9.7721593	3892	7702	2298	43
18	3314	2964	9.8660350	10.1339650	42
19	5033	2036	2997	7003	41
20	6751	1107	5644	4356	40
21	8468	0177	8291	1709	39
22	9.7730185	9.9059247	9.8670937	10.1329063	38
23	1900	8317	3583	6417	37
24	3614	7386	6228	3772	36
25	5327	6454	8873	1127	35
26	7039	9.9055522	9.8681517	10.1318483	34
27	8749	4589	4160	5840	33
28	9.7740459	3656	6804	3196	32
29	2168	2722	9446	0554	31
30	3876	1787	9.8692089	10.1307911	30
	Coſine	Sine	Cotangent	Tangent	Min.
	126	Degrees		53	

Min.	36	Degrees		143	
	Sine	Cosine	Tangent	Cotangent	
30	9.7743876	9.9051787	9.8692089	10.1307911	30
31	5583	0852	4731	5269	29
32	7288	9.9049916	7372	2628	28
33	8993	8980	9.8700013	10.1299987	27
34	9.7750697	8043	2653	7347	26
35	2399	7106	5293	4707	25
36	4101	9.9046168	7933	2067	24
37	5801	5230	9.8710572	10.1289428	23
38	7501	4291	3210	6790	22
39	9199	3351	5848	4152	21
40	9.7760897	2411	8486	1514	20
41	2593	9.9041470	9.8721123	10.1278877	19
42	4289	0529	3760	6240	18
43	5983	9.9039587	6396	3604	17
44	7676	8644	9032	0968	16
45	9369	7701	9.8731668	10.1268332	15
46	9.7771060	9.9036757	4302	5698	14
47	2750	5813	6937	3063	13
48	4439	4868	9571	0429	12
49	6128	3923	9.8742204	10.1257796	11
50	7815	2977	4838	5162	10
51	9501	9.9032031	7470	2530	9
52	9.7781186	1084	9.8750102	10.1249898	8
53	2870	0136	2734	7266	7
54	4553	9.9029188	5365	4635	6
55	6235	8239	7996	2004	5
56	7916	9.9027289	9.8760627	10.1239373	4
57	9596	6339	3257	6743	3
58	9.7791275	5389	5880	4114	2
59	2953	4438	8515	1485	1
60	4630	3486	9.8771144	10.1228856	0
	Cosine	Sine	Cotangent	Tangent	Min.
	126	Degrees		53	

Min.	37	Degrees		142	
	Sine	Coſine	Tangent	Cotangent	
0	9.7794630	9.9023486	9.8771144	10.1228856	60
1	6306	2534	3772	6228	59
2	7981	1581	6400	3600	58
3	9655	0628	9027	0973	57
4	9.7801328	9.9019674	9.8781654	10.1218346	56
5	3000	8719	4281	5719	55
6	9.7804671	9.9017764	6907	3093	54
7	6341	6808	9533	0467	53
8	8010	5852	9.8792158	10.1207842	52
9	9677	4895	4782	5218	51
10	9.7811344	3938	7407	2593	50
11	3010	9.9012980	9.8800031	10.1199969	49
12	4675	2021	2654	7346	48
13	6339	1062	5277	4723	47
14	8002	0102	7900	2100	46
15	9664	9.9009142	9.8810522	10.1189478	45
16	9.7821324	8181	3144	6856	44
17	2984	7219	5765	4235	43
18	4643	6257	8386	1614	42
19	6301	5294	9.8821007	10.1178993	41
20	7958	4331	3627	6373	40
21	9614	9.9003367	6246	3754	39
22	9.7831268	2403	8866	1134	38
23	2922	1438	9.8831484	10.1168516	37
24	4575	0472	4103	5897	36
25	6227	9.8999506	6721	3279	35
26	7878	8539	9338	0662	34
27	9528	7572	9.8841956	10.1158044	33
28	9.7841177	6604	4572	5428	32
29	2824	5636	7189	2811	31
30	4471	4667	9805	0195	30
	Coſine	Sine	Cotangent	Tangent	Min.
	127	Degrees		52	

Min.	37	Degrees		142	
	Sine	Coſine	Tangent	Cotangent	
30	9.7844471	9.8994667	9.8849805	10.1150195	30
31	6117	3697	9.8852420	10.1147580	29
32	7762	2727	5035	4965	28
33	9406	1756	7650	2350	27
34	9.7851049	0784	9.8860264	10.1139736	26
35	2691	9.8989812	2878	7122	25
36	4332	8840	5492	4508	24
37	5972	7867	8105	1895	23
38	7611	6893	9.8870718	10.1129282	22
39	9249	5919	3330	6670	21
40	9.7860886	4944	5942	4058	20
41	2522	9.8983968	8554	1446	19
42	4157	2992	9.8881165	10.1118835	18
43	5791	2015	3775	6225	17
44	7424	1038	6386	3614	16
45	9056	0060	8996	1004	15
46	9.7870687	9.8979082	9.8891605	10.1108395	14
47	2317	8103	4214	5786	13
48	3946	7123	6823	3177	12
49	5574	6143	9432	0568	11
50	7202	5162	9.8902040	10.1097960	10
51	8828	9.8974181	4647	5353	9
52	9.7880453	3199	7254	2746	8
53	2077	2216	9861	0139	7
54	3701	1233	9.8912468	10.1087532	6
55	5323	0249	5074	4926	5
56	6944	9.8969265	7679	2321	4
57	8565	8280	9.8920285	10.1079715	3
58	9.7890184	7294	2890	7110	2
59	1802	6308	5494	4506	1
60	3420	5321	8098	1902	0
	Coſine	Sine	Cotangent	Tangent	Min.
	127	Degrees		52	

Min.	38	Degrees		141	
	Sine	Coſine	Tangent	Cotangent	
0	9.7893420	9.8965321	9.8928098	10.1071902	60
1	5036	4334	9.8930702	10.1069298	59
2	6652	3346	3306	6694	58
3	8266	2358	5909	4091	57
4	9880	1369	8511	1489	56
5	9.7901493	0379	9.8941114	10.1058886	55
6	3104	9.8959389	3715	6285	54
7	4715	8398	6317	3683	53
8	6325	7406	8918	1082	52
9	7933	6414	9.8951519	10.1048481	51
10	9541	5422	4119	5881	50
11	9.7911148	9.8954429	6719	3281	49
12	2754	3435	9319	0681	48
13	4359	2440	9.8961918	10.1038082	47
14	5963	1445	4517	5483	46
15	7566	0450	7116	2884	45
16	9168	9.8949453	9714	0286	44
17	9.7920769	8457	9.8972312	10.1027688	43
18	2369	7459	4910	5090	42
19	3968	6461	7507	2493	41
20	5566	5463	9.8980104	10.1019896	40
21	7163	9.8944463	2700	7300	39
22	8760	3464	5296	4704	38
23	9.7930355	2463	7892	2108	37
24	1949	1462	9.8990487	10.1009513	36
25	3543	0461	3082	6918	35
26	9.7935135	9.8939458	5677	4323	34
27	6727	8456	8271	1729	33
28	8317	7452	9.9000865	10.0999135	32
29	9907	6448	3459	6541	31
30	9.7941496	5444	6052	3948	30
	Coſine	Sine	Cotangent	Tangent	Min.
	128	Degrees		51	

Min.	38	Degrees		141	
	Sine	Coſine	Tangent	Cotangent	
30	9.7941496	9.8935444	9.9006052	10.0993948	30
31	3083	4439	8645	1355	29
32	4670	3433	9.9011237	10.0988763	28
33	6256	2426	3830	6170	27
34	7841	1419	6422	3578	26
35	9425	0412	9013	0987	25
36	9.7951008	9.8929404	9.9021604	10.0978396	24
37	2590	8395	4195	5805	23
38	4171	7385	6786	3214	22
39	5751	6375	9376	0624	21
40	7330	5365	9.9031966	10.0968034	20
41	8909	9.8924354	4555	5445	19
42	9.7960486	3342	7144	2856	18
43	2062	2329	9733	0267	17
44	3638	1316	9.9042321	10.0957679	16
45	5212	0303	4910	5090	15
46	6786	9.8919289	7497	2503	14
47	8359	8274	9.9050085	10.0949915	13
48	9930	7258	2672	7328	12
49	9.7971501	6242	5259	4741	11
50	3071	5226	7845	2155	10
51	4640	9.8914208	9.9060431	10.0939569	9
52	6208	3191	3017	6983	8
53	7775	2172	5603	4397	7
54	9341	1153	8188	1812	6
55	9.7980906	0133	9.9070773	10.0929227	5
56	2470	9.8909113	3357	6643	4
57	4034	8092	5941	4059	3
58	5596	7071	8525	1475	2
59	7158	6049	9.9081109	10.0918891	1
60	8718	5026	3692	6308	0
	Coſine	Sine	Cotangent	Tangent	Min.
	128	Degrees		51	

Min.	39	Degrees		140	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.7988718	9.8905026	9.9083692	10.0916308	60
1	9.7990278	4003	6275	3725	59
2	1836	2979	8858	1142	58
3	3394	1954	9.9091440	10.0908560	57
4	4951	0929	4022	5978	56
5	6507	9.8899903	6603	3397	55
6	8062	8877	9185	0815	54
7	9616	7850	9.9101766	10.0898234	53
8	9.8001169	6822	4347	5653	52
9	2721	5794	6927	3073	51
10	4272	4765	9507	0493	50
11	9.8005823	9.8893736	9.9112087	10.0887913	49
12	7372	2706	4666	5334	48
13	8921	1675	7245	2755	47
14	9.8010468	0644	9824	0176	46
15	2015	9.8889612	9.9122403	10.0877597	45
16	3561	8580	4981	5019	44
17	5106	7547	7559	2441	43
18	6649	6513	9.9130137	10.0869863	42
19	8192	5479	2714	7286	41
20	9735	4444	5291	4709	40
21	9.8021276	9.8883408	7868	2132	39
22	2816	2372	9.9140444	10.0859556	38
23	4355	1335	3020	6980	37
24	5894	0298	5596	4404	36
25	7431	9.8879260	8171	1829	35
26	8968	8221	9.9150747	10.0849253	34
27	9.8030504	7182	3322	6678	33
28	2038	6142	5896	4104	32
29	3572	5102	8471	1529	31
30	5105	4061	9.9161045	10.0838955	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	129	Degrees		50	

Min.	39	Degrees		140	
	Sine	Coſine	Tangent	Cotangent	
30	9.8035105	9.8874061	9.9161045	10.0838955	30
31	6637	3019	3618	6382	29
32	8168	1977	6192	3808	28
33	9699	0934	8765	1235	27
34	9.8041228	9.8869890	9.9171338	10.0828662	26
35	2757	8846	3911	6089	25
36	4284	7801	6483	3517	24
37	5811	6756	9055	0945	23
38	7336	5710	9.9181627	10.0818373	22
39	8861	4663	4198	5802	21
40	9.8050385	3616	6769	3231	20
41	1908	9.8862568	9340	0660	19
42	3430	1519	9.9191911	10.0808089	18
43	5451	0470	4481	5519	17
44	6472	9.8859420	7051	2949	16
45	7991	8370	9621	0379	15
46	9510	7319	9.9202191	10.0797809	14
47	9.8061027	6267	4760	5240	13
48	2544	5215	7329	2671	12
49	4060	4162	9898	0102	11
50	5575	3109	9.9212466	10.0787534	10
51	7089	9.8852055	5034	4966	9
52	8602	1000	7602	2398	8
53	9.8070114	9.8849945	9.9220170	10.0779830	7
54	1626	8889	2737	7263	6
55	3136	7832	5304	4696	5
56	9.8074646	6775	7871	2129	4
57	6154	5717	9.9230437	10.0769563	3
58	7662	4659	3004	6996	2
59	9169	3599	5570	4430	1
60	9.8080675	2540	8135	1865	0
	Coſine	Sine	Cotangent	Tangent	Min.
	129	Degrees		50	

Min.	40	Degrees		139	
	Sine	Coſine	Tangent	Cotangent.	
0	9.8080675	9.8842540	9.9238135	10.0761865	60
1	2180	1479	9.9240701	10.0759299	59
2	3684	0418	3266	6734	58
3	5188	9.8839357	5831	4169	57
4	6690	8294	8396	1604	56
5	8192	7232	9.9250960	10.0749040	55
6	9692	9.8836168	3524	6476	54
7	9.8091192	5104	6088	3912	53
8	2691	4039	8652	1348	52
9	4189	2974	9.9261215	10.0738785	51
10	5686	1908	3778	6222	50
11	7182	0841	6341	3659	49
12	8678	9.8829774	8904	1096	48
13	9.8100172	8706	9.9271466	10.0728534	47
14	1666	7638	4028	5972	46
15	3159	6568	6590	3410	45
16	9.8104650	9.8825499	9152	0848	44
17	6141	4428	9.9281713	10.0718287	43
18	7631	3357	4274	5726	42
19	9121	2285	6835	3165	41
20	9.8110609	1213	9396	0604	40
21	2096	0140	9.9291956	10.9708044	39
22	3583	9.8819067	4516	5484	38
23	5069	7992	7076	2924	37
24	6554	6918	9636	0364	36
25	8038	5842	9.9302195	10.0697805	35
26	9521	9.8814766	4755	5245	34
27	9.8121003	3689	7314	2686	33
28	2484	2612	9872	0128	32
29	3965	1534	9.9312431	10.0687569	31
30	5444	0455	4989	5011	30
	Coſine	Sine	Cotangent	Tangent	Min.
	130	Degrees		49	

Min.	40	Degrees		139	
	Sine	Coſine	Tangent	Cotangent	
30	9.8125444	9.8810455	9.9314989	10.0685011	30
31	6923	9.8809376	7547	2453	29
32	8401	8296	9.9320105	10.0679895	28
33	9878	7215	2662	7338	27
34	9.8131354	6134	5220	4780	26
35	2829	5052	7777	2223	25
36	4303	9.8803970	9.9330334	10.0669666	24
37	5777	2886	2890	7110	23
38	7250	1803	5446	4554	22
39	8721	0719	8003	1997	21
40	9.8140192	9.8799634	9.9340559	10.0659441	20
41	1662	8548	3114	6886	19
42	3131	7462	5670	4330	18
43	4600	6375	8225	1775	17
44	6067	5287	9.9350780	10.0649220	16
45	7534	4199	3335	6665	15
46	8999	9.8793110	5889	4111	14
47	9.8150464	2021	8444	1556	13
48	1928	0930	9.9360998	10.0639002	12
49	3391	9.8789840	3552	6448	11
50	4854	8748	6105	3895	10
51	9.8156315	7656	8659	1341	9
52	7776	6563	9.9371212	10.0628788	8
53	9235	5470	3765	6235	7
54	9.8160694	4376	6318	3682	6
55	2152	3281	8871	1129	5
56	3609	9.8782186	9.9381423	10.0618577	4
57	5066	1090	3975	6025	3
58	6521	9.8779994	6527	3473	2
59	7975	8896	9079	0921	1
60	9429	7799	9.9391631	10.0608369	0
	Coſine	Sine	Cotangent	Tangent	
	130	Degrees		49	Min.

Min.	41	Degrees		138	
	Sine	Coſine	Tangent	Cotangent	
0	9.8169429	9.8777799	9.9391631	10.0608369	60
1	9.8170882	6700	4182	5818	59
2	2334	5601	6733	3267	58
3	3785	4501	9284	0716	57
4	5235	3401	9.9401835	10.0598165	56
5	6685	2300	4385	5615	55
6	9.8178133	9.8771198	9636	3064	54
7	9581	0096	9486	0514	53
8	9.8181028	9.8768993	9.9412036	10.0587964	52
9	2474	7889	4585	5415	51
10	3919	6785	7135	2865	50
11	9.8185364	9.8765680	9684	0316	49
12	6807	4574	9.9422233	10.0577767	48
13	8250	3468	4782	5218	47
14	9692	9.8762361	7331	2669	46
15	9.8191133	1253	9879	0121	45
16	2573	0145	9.9432428	10.0567572	44
17	4012	9.8759036	4976	5024	43
18	5450	7927	7524	2476	42
19	9.8196888	6816	9.9440072	10.0559928	41
20	8325	5706	2619	7381	40
21	9761	9.8754594	5166	4834	39
22	9.8201196	3482	7714	2286	38
23	2630	2369	9.9450261	10.0549739	37
24	4063	1256	2807	7193	36
25	5496	0142	5354	4646	35
26	9.8206927	9.8749027	7900	2100	34
27	8358	7912	9.9460447	10.0539553	33
28	9788	6795	2993	7007	32
29	9.8211217	5679	5539	4461	31
30	2646	4561	8084	1916	30
	Coſine	Sine	Cotangent	Tangent	Min.
	131	Degrees		48	

Min.	41	Degrees		138	
	Sine	Coſine	Tangent	Cotangent	
30	9.8212646	9.8744561	9.9468084	10.0531916	30
31	4073	3443	9.9470630	10.0529370	29
32	5500	2325	3175	6825	28
33	6926	1205	5720	4280	27
34	8351	0085	8265	1735	26
35	9775	9.8738965	9.9480810	10.0519190	25
36	9.8221198	7844	3355	6645	24
37	2621	6722	5899	4101	23
38	4042	5599	8443	1557	22
39	5463	4476	9.9490987	10.0509013	21
40	6883	3352	3531	6469	20
41	9.8228302	9.8732227	6075	3925	19
42	9721	1102	8619	1381	18
43	9.8231138	9.8729976	9.9501162	10.0498838	17
44	2555	8849	3705	6295	16
45	3971	7722	6248	3752	15
46	9.8235386	9.8726594	8791	1209	14
47	6800	5466	9.9511334	10.0488666	13
48	8213	4337	3876	6124	12
49	9626	3207	6419	3581	11
50	9.8241037	2076	8961	1039	10
51	2448	0945	9.9521503	10.0478497	9
52	3858	9.8719813	4045	5955	8
53	5267	8681	6587	3413	7
54	6676	7548	9128	0872	6
55	8083	6414	9.9531670	10.0468330	5
56	9490	9.8715279	4211	5789	4
57	9.8250896	4144	6752	3248	3
58	2301	3008	9293	0707	2
59	3705	1872	9.9541834	10.0458166	1
60	5109	0735	4374	5626	0
	Coſine	Sine	Cotangent	Tangent	Min.
	131	Degrees		48	

Min.	42	Degrees		137	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
0	9.8255109	9.8710735	9.9544374	10.0455626	60
1	6512	9.8709597	6915	3085	59
2	7913	8458	9455	0545	58
3	9314	7319	9.9551995	10.0448005	57
4	9.8260715	6179	4535	5465	56
5	2114	5039	7075	2925	55
6	9.8263512	9.8703898	9615	0385	54
7	4910	2756	9.9562154	10.0437846	53
8	6307	1613	4694	5306	52
9	7703	0470	7233	2767	51
10	9098	9.8699326	9772	0228	50
11	9.8270493	3182	9.9572311	10.0427689	49
12	1887	7037	4850	5150	48
13	3279	5891	7389	2611	47
14	4671	4744	9927	0073	46
15	6063	3597	9.9582465	10.0417535	45
16	9.8277453	9.8692449	5004	4996	44
17	8843	1301	7542	2458	43
18	9.8280231	0152	9.9590080	10.0409920	42
19	1619	9.8689002	2618	7382	41
20	3006	7851	5155	4845	40
21	9.8284393	9.8686700	7693	2307	39
22	5778	5548	9.9600230	10.0399770	38
23	7163	4396	2767	7233	37
24	8547	3242	5305	4695	36
25	9930	2088	7842	2158	35
26	9.8291312	0934	9.9610378	10.0389622	34
27	2694	9.8679779	2915	7085	33
28	4075	8623	5452	4548	32
29	5454	7466	7988	2012	31
30	6833	6309	9.9620525	10.0379475	30
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	132	Degrees		47	

Min.	42	Degrees		137	
	Sine	Coſine	Tangent	Cotangent	
30	9.8296833	9.8676309	9.9620525	10.0379475	30
31	8212	5151	3061	6939	29
32	9589	3992	5597	4403	28
33	9.8300966	2833	8133	1867	27
34	2342	1673	9.9630669	10.0369331	26
35	3717	0512	3204	6796	25
36	9.8305091	9.8669351	5740	4260	24
37	6464	8189	8275	1725	23
38	7837	7026	9.9640811	10.0359189	22
39	9209	5863	3346	6654	21
40	9.8310580	4699	5881	4119	20
41	1950	9.8663534	8416	1584	19
42	3320	2369	9.9650951	10.0349049	18
43	4688	1203	3486	6514	17
44	6056	0036	6020	3980	16
45	7423	9.8658868	8555	1445	15
46	8789	7700	9.9661089	10.0338911	14
47	9.8320155	6531	3623	6377	13
48	1519	5362	6157	3843	12
49	2883	4192	8692	1308	11
50	4246	3021	9.9671225	10.0328775	10
51	9.8325609	9.8651849	3759	6241	9
52	6970	0677	6293	3707	8
53	8331	9.8649504	8827	1173	7
54	9691	8331	9.9681360	10.0318640	6
55	9.8331050	7156	3893	6107	5
56	2408	9.8645981	6427	3573	4
57	3766	4806	8960	1040	3
58	5122	3629	9.9691493	10.0308507	2
59	6478	2452	4026	5974	1
60	7833	1275	6559	3441	0
	Coſine	Sine	Cotangent	Tangent	Min.
	132	Degrees		47	

Min.	43	Degrees		136	
	Sine	Cosine	Tangent	Cotangent	
0	9.8337833	9.8641275	9.9696559	10.0303441	60
1	9188	0096	9091	0909	59
2	9.8340541	9.8638917	9.9701624	10.0298376	58
3	1894	7737	4157	5843	57
4	3246	6557	6689	3311	56
5	4597	5376	9221	0779	55
6	9.8345948	9.8634194	9.9711754	10.0288246	54
7	7297	3011	4286	5714	53
8	8646	1828	6818	3182	52
9	9994	0644	9350	0650	51
10	9.8351341	9.8629460	9.9721882	10.0278118	50
11	2688	8274	4413	5587	49
12	4033	7088	6945	3055	48
13	5378	5902	9477	0523	47
14	6722	4714	9.9732008	10.0267992	46
15	8066	3526	4539	5461	45
16	9408	9.8622338	7071	2929	44
17	9.8360750	1148	9602	0398	43
18	2091	9.8619958	9.9742133	10.0257867	42
19	3431	8767	4664	5336	41
20	4771	7576	7195	2805	40
21	9.8366109	9.8616383	9726	0274	39
22	7447	5190	9.9752257	10.0247743	38
23	8784	3997	4787	5213	37
24	9.8370121	2803	7318	2682	36
25	1456	1608	9849	0151	35
26	9.8372791	0412	9.9762379	10.0237621	34
27	4125	9.8609215	4909	5091	33
28	5458	8018	7440	2560	32
29	6790	6821	9970	0030	31
30	8122	5622	9.9772500	10.0227500	30
	Coſine	Sine	Cotangent	Tangent	Min.
	133	Degrees		46	

Min.	43	Degrees		136	
	Sine	*Coſine*	*Tangent*	*Cotangent*	
30	9.8378122	9.8605622	9.9772500	10.0227500	30
31	9453	4423	5030	4970	29
32	9.8380783	3223	7560	2440	28
33	2112	2022	9.9780090	10.0219910	27
34	3441	0821	2620	7380	26
35	4769	9.8599619	5149	4851	25
36	9.8386096	8416	7679	2321	24
37	7422	7213	9.9790209	10.0209791	23
38	8747	6009	2738	7262	22
39	9.8390072	4804	5268	4732	21
40	1396	3599	7797	2203	20
41	2719	9.8592393	9.9800326	10.0199674	19
42	4041	1186	2856	7144	18
43	5363	9.8589978	5385	4615	17
44	6684	8770	7914	2086	16
45	8004	7561	9.9810443	10.0189557	15
46	9323	9.8586351	2972	7028	14
47	9.8400642	5141	5501	4499	13
48	1959	3929	8030	1970	12
49	3276	2718	9.9820559	10.0179441	11
50	4593	1505	3087	6913	10
51	9.8405908	0292	5616	4384	9
52	7223	9.8579078	8145	1855	8
53	8537	7863	9.9830673	10.0169327	7
54	9850	6648	3202	6798	6
55	9.8411162	5432	5730	4270	5
56	2474	9.8574215	8259	1741	4
57	3785	2998	9.9840787	10.0159213	3
58	5095	1779	3315	6685	2
59	6404	0561	5844	4156	1
60	7713	9.8569341	8372	1628	0
	Coſine	*Sine*	*Cotangent*	*Tangent*	Min.
	133	Degrees		46	

Min.	44	Degrees		135	
	Sine	Coſine	Tangent	Cotangent	
0	9.8417713	9.8569341	9.9848372	10.0151628	60
1	9021	8121	9.9850900	10.0149100	59
2	9.8420328	6900	3428	6572	58
3	1634	5678	5956	4044	57
4	2939	4455	8484	1516	56
5	4244	3232	9.9861012	10.0138988	55
6	9.8425548	9.8562008	3540	6460	54
7	6851	0784	6068	3932	53
8	8154	9.8559558	8596	1404	52
9	9456	8332	9.9871123	10.0128877	51
10	9.8430757	7106	3651	6349	50
11	2057	9.8555878	6179	3821	49
12	3356	4650	8706	1294	48
13	4655	3421	9.9881234	10.0118766	47
14	5953	2192	3761	6239	46
15	7250	0961	6289	3711	45
16	9.8438547	9.8549730	8816	1184	44
17	9842	8499	9.9891344	10.0108656	43
18	9.8441137	7266	3871	6129	42
19	2432	6033	6399	3601	41
20	3725	4799	8926	1074	40
21	9.8445018	9.8543564	9.9901453	10.0098547	39
22	6310	2329	3981	6019	38
23	7601	1093	6508	3492	37
24	8891	9.8539856	9035	0965	36
25	9.8450181	8619	9.9911562	10.0088438	35
26	1470	7381	4089	5911	34
27	2758	6142	6616	3384	33
28	4045	4902	9143	0857	32
29	5332	3662	9.9921670	10.0078330	31
30	6618	2421	4197	5803	30
	Coſine	Sine	Cotangent	Tangent	Min.
	134	Degrees		45	

Min.	44	Degrees		135	
	Sine	Coſine	Tangent	Cotangent	
30	9.8456618	9.8532421	9.9924197	10.0075803	30
31	7903	1179	6724	3276	29
32	9188	9.8529936	9251	0749	28
33	9.8460471	8693	9.9931778	10.0068222	27
34	1754	7449	4305	5695	26
35	3036	6204	6832	3168	25
36	9.8464318	9.8524959	9359	0641	24
37	5599	3713	9.9941886	10.0058114	23
38	6879	2466	4413	5587	22
39	8158	1218	6940	3060	21
40	9436	9.8519970	9466	0534	20
41	9.8470714	8721	9.9951993	10.0048007	19
42	1991	7471	4520	5480	18
43	3267	6220	7047	2953	17
44	4543	4969	9573	0427	16
45	5817	3717	9.9962100	10.0037900	15
46	9.8477091	9.8512465	4627	5373	14
47	8365	1211	7154	2846	13
48	9637	9.8509957	9680	0320	12
49	9.8480909	8702	9.9972207	10.0027793	11
50	2180	7446	4734	5266	10
51	3450	9.8506190	7260	2740	9
52	4720	4933	9787	0213	8
53	5989	3675	9.9982314	10.0017686	7
54	7257	2417	4840	5160	6
55	8524	1157	7367	2633	5
56	9791	9.8499897	9893	0107	4
57	9.8491057	8637	9.9992420	10.0007580	3
58	2322	7375	4947	5053	2
59	3586	6113	7473	2527	1
60	4850	4850	10.0000000	10.0000000	0
	Coſine	Sine	Cotangent	Tangent	Min.
	134	Degrees		45	

Here followeth

A TABLE

OF THE

LOGARITHMS

OF

ABSOLUTE NUMBERS

From 1 to 10000.

LOGARITHMS.

Num	Logarithms	Num	Logarithms	Num	Logarithms
1	0.0000000	36	1.5563025	71	1.8512583
2	0.3010300	37	1.5682017	72	1.8573325
3	0.4771212	38	1.5797836	73	1.8633229
4	0.6020600	39	1.5910646	74	1.8692317
5	0.6989700	40	1.6020600	75	1.8750613
6	0.7781512	41	1.6127839	76	1.8808136
7	0.8450980	42	1.6232493	77	1.8864907
8	0.9030900	43	1.6334685	78	1.8920946
9	0.9542425	44	1.6434527	79	1.8976271
10	1.0000000	45	1.6532125	80	1.9030900
11	1.0413927	46	1.6627578	81	1.9084850
12	1.0791812	47	1.6720979	82	1.9138138
13	1.1139433	48	1.6812412	83	1.9190781
14	1.1461280	49	1.6901961	84	1.9242793
15	1.1760913	50	1.6989700	85	1.9294189
16	1.2041200	51	1.7075702	86	1.9344984
17	1.2304489	52	1.7160033	87	1.9395192
18	1.2552725	53	1.7242759	88	1.9444827
19	1.2787536	54	1.7323938	89	1.9493900
20	1.3010300	55	1.7403627	90	1.9542425
21	1.3222193	56	1.7481880	91	1.9590414
22	1.3424227	57	1.7558748	92	1.9637878
23	1.3617278	58	1.7634280	93	1.9684829
24	1.3802112	59	1.7708520	94	1.9731278
25	1.3979400	60	1.7781512	95	1,9777236
26	1.4149733	61	1.7853298	96	1.9822712
27	1.4313638	62	1.7923917	97	1.9867717
28	1.4471580	63	1.7993405	98	1.9912261
29	1.4623980	64	1.8061800	99	1.9956352
30	1.4771212	65	1.8129133	100	2.0000000
31	1.4913617	66	1.8195439		
32	1.5051500	67	1.8260748		
33	1.5185139	68	1.8325089		
34	1.5314789	69	1.8388491		
35	1.5440680	70	1.8450980		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
101	2.0043214	136	2.1335389	171	2.2329961
102	86002	137	67206	172	55284
103	2.0128372	138	98791	173	80461
104	70333	139	2.1430148	174	2.2405492
105	2.0211893	140	61280	175	30380
106	53059	141	92191	176	55127
107	93838	142	2.1522883	177	79733
108	2.0334238	143	53360	178	2.2504200
109	74265	144	83625	179	28530
110	2.0413927	145	2.1613680	180	52725
111	53230	146	43528	181	76786
112	92180	147	73173	182	2.2600714
113	2.0530784	148	2.1702617	183	24511
114	69048	149	31863	184	48178
115	2.0606978	150	60913	185	71717
116	44580	151	89769	186	95129
117	81859	152	2.1818436	187	2.2718416
118	2.0718820	153	46914	188	41578
119	55470	154	75207	189	64618
120	91812	155	2.1903317	190	87536
121	2.0827854	156	31246	191	2.2810334
122	63598	157	58996	192	33012
123	99051	158	86571	193	55573
124	2.0934217	159	2.2013971	194	78017
125	69100	160	41200	195	2.2900346
126	2.1003705	161	68259	196	22561
127	38037	162	95150	197	44662
128	72100	163	2.2121876	198	66652
129	2.1105897	164	48438	199	88531
130	39433	165	74839	200	2.3010300
131	72713	166	2.2201081		
132	2.1205739	167	27165		
133	38516	168	53093		
134	71048	169	78867		
135	2.1303338	170	2.2304489		

LOGARITHMS.

Num	Logarithms	Num	Logarithms	Num	Logarithms
201	2.3031961	236	2.3729120	271	2.4329693
202	53514	237	47483	272	45689
203	74960	238	65770	273	61620
204	96302	239	83979	274	77506
205	2.3117539	240	2.3802112	275	93327
206	38672	241	20170	276	2.4409091
207	59703	242	38154	277	24798
208	80633	243	56063	278	40448
209	2.3201463	244	73898	279	56042
210	22193	245	91661	280	71580
211	42824	246	2.3909351	281	87063
212	63359	247	26969	282	2.4502491
213	83796	248	44517	283	17864
214	2.3304138	249	61993	284	33183
215	24385	250	79400	285	48449
216	44537	251	96737	286	63660
217	64597	252	2.4014005	287	78819
218	84565	253	31205	288	93925
219	2.3404441	254	48337	289	2.4608978
220	24227	255	65402	290	23980
221	43923	256	82400	291	38930
222	63530	257	99331	292	53828
223	83049	258	2.4116197	293	68676
224	2.3502480	259	32998	294	83473
225	21825	260	49733	295	98220
226	41084	261	66405	296	2.4712917
227	60259	262	83013	297	27564
228	79348	263	99557	298	42163
229	98355	264	2.4216039	299	56712
230	2.3617278	265	32459	300	71212
231	36120	266	48816		
232	54880	267	65113		
233	73559	268	81348		
234	92159	269	97523		
235	2.3710679	270	2.4313638		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
301	2.4785665	336	2.5263393	371	2.5693739
302	2.4800069	337	76299	372	2.5705429
303	14426	338	89167	373	17088
304	28736	339	2.5301997	374	28716
305	42998	340	14789	375	40313
306	57214	341	27544	376	2.5751878
307	71384	342	40261	377	63413
308	85507	343	52941	378	74918
309	99585	344	65584	379	86392
310	2.4913617	345	78191	380	97836
311	27604	346	90761	381	2.5809250
312	41546	347	2.5403295	382	20634
313	55443	348	15792	383	31988
314	69296	349	28254	384	43312
315	83105	350	40680	385	54607
316	96871	351	2.5453071	386	2.5865873
317	2.5010593	352	65427	387	77110
318	24271	353	77747	388	88317
319	37907	554	90033	389	99496
320	51500	355	2.5502283	390	2.5910646
321	65050	356	14500	391	21768
322	78559	357	26682	392	32861
323	92025	358	38830	393	43925
324	2.5105450	359	50944	394	54962
325	18834	360	63025	395	65971
326	32176	361	2.5575072	396	2.5976952
327	45477	362	87086	397	87905
328	58738	363	99066	398	98831
329	71959	364	2.5611014	399	2.6009729
330	85139	365	22929	400	20600
331	98280	366	34811		
332	2.5211381	367	46661		
333	24442	368	58478		
334	37465	369	70264		
335	50448	370	82017		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
401	2.6031444	436	2.6394865	471	2.6730209
402	42260	437	2.6404814	472	39420
403	53050	438	14741	473	48611
404	63814	439	24645	474	57783
405	74550	440	34527	475	66936
406	2.6085260	441	2.6444386	476	2.6776069
407	95944	442	54223	477	85184
408	2.6106602	443	64037	478	94279
409	17233	444	73830	479	2.6803355
410	27839	445	83600	480	12412
411	2.6138418	446	93349	481	21451
412	48972	447	2.6503075	482	30470
413	59500	448	12780	483	39471
414	70003	449	22463	484	48454
415	80481	450	32125	485	57417
416	90933	451	2.6541765	486	2.6866363
417	2.6201360	452	51384	487	75290
418	11763	453	60982	488	84198
419	22140	454	70558	489	93089
420	32493	455	80114	490	2.6901961
421	2.6242821	456	89648	491	10815
422	53124	457	99162	492	19651
423	63404	458	2.6608655	493	28469
424	73659	459	18127	494	37269
425	83889	460	27578	495	46052
426	94096	461	37009	496	2.6954817
427	2.6304279	462	46420	497	63564
428	14438	463	55810	498	72293
429	24573	464	65180	499	81005
430	34685	465	74529	500	89700
431	2.6344773	466	83859		
432	54837	467	93169		
433	64879	468	2.6702458		
434	74897	469	11728		
435	84893	470	20979		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
501	2.6998377	536	2.7291648	571	2.7566361
502	2.7007037	537	99743	572	73960
503	15680	538	2.7307823	573	81546
504	24305	539	15888	574	89119
505	32914	540	23938	575	96678
506	41505	541	2.7331973	576	2.7604225
507	50080	542	39993	577	11758
508	58637	543	47998	578	19278
509	67178	544	55989	579	26786
510	75702	545	63965	580	34280
511	84209	546	2.7371926	581	2.7641761
512	92700	547	79873	582	49230
513	2.7101174	548	87806	583	56685
514	09631	549	95723	584	64128
515	18072	550	2.7403627	585	71559
516	2.7126497	551	11516	586	2.7678976
517	34905	552	19391	587	86381
518	43298	553	27251	588	93773
519	51674	554	35098	589	2.7701153
520	60033	555	42930	590	08520
521	2.7168377	556	2.7450748	591	15875
522	76705	557	58552	592	23217
523	85017	558	66342	593	30547
524	93313	559	74118	594	37864
525	2.7201593	560	81880	595	45170
526	09857	561	89629	596	2.7752463
527	18106	562	97363	597	59743
528	26339	563	2.7505084	598	67012
529	34557	564	12791	599	74268
530	42759	565	20484	600	81512
531	2.7250945	566	2.7528164		
532	59116	567	35831		
533	67272	568	43483		
534	75413	569	51123		
535	83538	570	58748		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
601	2.7788745	636	2.8034571	671	2.8267225
602	95965	637	41394	672	73693
603	2.7803173	638	48207	673	80151
604	10369	639	55009	674	86599
605	17554	640	61800	675	93038
606	2.7824726	641	2.8068580	676	99467
607	31887	642	75350	677	2.8305887
608	39036	643	82110	678	12297
609	46173	644	88859	679	18698
610	53298	645	95597	680	25089
611	2.7860412	646	2.8102325	681	2.8331471
612	67514	647	09043	682	37844
613	74605	648	15750	683	44207
614	81684	649	22447	684	50561
615	88751	650	29134	685	56906
616	95807	651	2.8135810	686	2.8363241
617	2.7902852	652	42476	687	69567
618	09885	653	49132	688	75884
619	16906	654	55777	689	82192
620	23917	655	62413	690	88491
621	2.7930916	656	2.8169038	691	94780
622	37904	657	75654	692	2.8401061
623	44880	658	82259	693	07332
624	51846	659	88854	694	13595
625	58800	660	95439	695	19848
626	2.7965743	661	2.8202015	696	2.8426092
627	72675	662	08580	697	32328
628	79596	663	15135	698	38554
629	86506	664	21681	699	44772
630	93405	665	28216	700	50980
631	2.8000294	666	2.8234742		
632	07171	667	41258		
633	14037	668	47765		
634	20893	669	54261		
635	27737	670	60748		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
701	2.8457180	736	2.8668778	771	2.8870544
702	63371	737	74675	772	76173
703	69553	738	80564	773	81795
704	75727	739	86444	774	87410
705	81891	740	92317	775	93017
706	88047	741	98182	776	98617
707	94194	742	2.8704039	777	2.8904210
708	2.8500333	743	09888	778	09796
709	06462	744	15729	779	15375
710	12583	745	21563	780	20946
711	2.8518696	746	2.8727388	781	2.8926510
712	24800	747	33206	782	32067
713	30895	748	39016	783	37618
714	36982	749	44818	784	43161
715	43060	750	50613	785	48696
716	2.8549130	751	2.8756399	786	2.8954225
717	55191	752	62178	787	59747
718	61244	753	67950	788	65262
719	67289	754	73713	789	70770
720	73325	755	79469	790	76271
721	2.8579353	756	2.8785218	791	2.8981765
722	85372	757	90959	792	87252
723	91383	758	96692	793	92732
724	97386	759	2.8802418	794	98205
725	2.8603380	760	08136	795	2.9003671
726	09366	761	13847	796	09131
727	15344	762	19550	797	14583
728	21314	763	25245	798	20029
729	27275	764	30934	799	25468
730	33229	765	36614	800	30900
731	2.8639174	766	2.8842288		
732	45111	767	47954		
733	51040	768	53612		
734	56961	769	59263		
735	62873	770	64907		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
801	2.9036325	836	2.9222063	871	2.9400181
802	41744	837	27254	872	05165
803	47155	838	32440	873	10142
804	52560	839	37620	874	15114
805	57959	840	42793	875	20080
806	2.9063350	841	2.9247960	876	2.9425041
807	68735	842	53121	877	29996
808	74114	843	58276	878	34945
809	79485	844	63424	879	39889
810	84850	845	68567	880	44827
811	90208	846	2.9273704	881	2.9449759
812	95560	847	78834	882	54686
813	2.9100905	848	83958	883	59607
814	06244	849	89077	884	64523
815	11576	850	94189	885	69433
816	2.9116901	851	99296	886	2.9474337
817	22220	852	2.9304396	887	79236
818	27533	853	09490	888	84130
819	32839	854	14579	889	89018
820	38138	855	19661	890	93900
821	2.9143431	856	2.9324738	891	98777
822	48718	857	29808	892	2.9503648
823	53998	858	34873	893	08514
824	59272	859	39932	894	13375
825	64539	860	44984	895	18230
826	2.9169800	861	2.9350031	896	2.9523080
827	75055	862	55073	897	27924
828	80303	863	60108	898	32763
829	85545	864	65137	899	37597
830	90781	865	70161	900	42425
831	96010	866	2.9375179		
832	2.9201233	867	80191		
833	06450	868	85197		
834	11660	869	90198		
835	16865	870	95192		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logaritdms*
901	2.9547248	936	2.9712758	971	2.9872192
902	52065	937	17396	972	76663
903	56877	938	22028	973	81128
904	61684	939	26656	974	85589
905	66486	940	31278	975	90046
906	2.9571282	941	2.9735896	976	94498
907	76073	942	40509	977	98946
908	80858	943	45117	978	2.9903388
909	85639	944	49720	979	07827
910	90414	945	54318	980	12261
911	95184	946	2.9758911	981	2.9916690
912	99948	947	63500	982	21115
913	2.9604708	948	68083	983	25535
914	09462	949	72662	984	29951
915	14211	950	77236	985	34362
916	2.9618955	951	2.9781805	986	2.9938769
917	23693	952	86369	687	43171
918	28427	953	90929	988	47569
919	33155	954	95484	989	51963
920	37878	955	2.9800034	990	56352
921	2.9642596	956	04579	991	2.9960736
922	47309	957	09119	992	65117
923	52017	958	13655	993	69492
924	56720	959	18186	994	73864
925	61417	960	22712	995	78231
926	2.9666110	961	2.9827234	996	2.9982593
927	70797	962	31751	997	86951
928	75480	963	36263	998	91305
929	80157	964	40770	999	95655
930	84829	965	45273	1000	3.0000000
931	2.9689497	966	2.9849771		
932	94159	967	54265		
933	98816	968	58753		
934	2.9703469	969	63238		
935	08116	970	67717		

Num	Logarithms	Num	Logarithms	Num	Logarithms
1001	3.0004341	1036	3.0153597	1071	3.0297895
1002	8677	1037	7787	1072	3.0301948
1003	3.0013009	1038	3.0161973	1073	5997
1004	7337	1039	6155	1074	3.0310043
1005	3.0021661	1040	3.0170333	1075	4085
1006	5980	1041	4507	1076	8123
1007	3.0030295	1042	8677	1077	3.0322157
1008	4605	1043	3.0182843	1078	6188
1009	8912	1044	7005	1079	3.0330214
1010	3.0043214	1045	3.0191163	1080	4237
1011	7511	1046	5317	1081	8257
1012	3.0051805	1047	9467	1082	3.0342273
1013	6094	1048	3.0203613	1083	6284
1014	3.0060379	1049	7755	1084	3.0350293
1015	4660	1050	3.0211893	1085	4297
1016	8937	1051	6027	1086	8298
1017	3.0073209	1052	3.0220157	1087	3.0362295
1018	7478	1053	4284	1088	6289
1019	3.0081742	1054	8406	1089	3.0370279
1020	6002	1055	3.0232524	1090	4265
1021	3.0090257	1056	6639	1091	8247
1022	4509	1057	3.0240750	1092	3.0382226
1023	8756	1058	4857	1093	6201
1024	3.0102999	1059	8960	1094	3.0390173
1025	7239	1060	3.0253059	1095	4141
1026	3.0111474	1061	7154	1096	8105
1027	5704	1062	3.0261245	1097	3.0402066
1028	9931	1063	5333	1098	6023
1029	3.0124154	1064	9416	1099	9977
1030	8372	1065	3.0273496	1100	3.0413927
1031	3.0132587	1066	7572		
1032	6797	1067	3.0281644		
1033	3.0141003	1068	5712		
1034	5205	1069	9777		
1035	9403	1070	3.0293838		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1101	3.0417873	1136	3.0553783	1171	3.0685569
1102	3.0421816	1137	7604	1172	9276
1103	5755	1138	3.0561423	1173	3.0692980
1104	9691	1139	5237	1174	6681
1105	3.0433623	1140	9048	1175	3.0700379
1106	7551	1141	3.0572856	1176	4073
1107	3.0441476	1142	6661	1177	7765
1108	5398	1143	3.0580462	1178	3.0711453
1109	9315	1144	4260	1179	5138
1110	3.0453230	1145	8055	1180	8820
1111	7140	1146	3.0591846	1181	3.0722499
1112	3.0461048	1147	5634	1182	6175
1113	4952	1148	9419	1183	9847
1114	8852	1149	3.0603200	1184	3.0733517
1115	3.0472749	1150	6978	1185	7183
1116	6642	1151	3.0610753	1186	3.9740847
1117	3.0480532	1152	4525	1187	4507
1118	4418	1153	8293	1188	8164
1119	8301	1154	3.0622058	1189	3.0751818
1120	3.0492180	1155	5820	1190	5470
1121	6056	1156	9578	1191	9118
1122	9928	1157	3.0633334	1192	3.0762762
1123	3.0503797	1158	7085	1193	6404
1124	7663	1159	3.0640834	1194	3.0770043
1125	3.0511525	1160	4580	1195	3679
1126	5384	1161	8322	1196	7312
1127	9239	1162	3.0652061	1197	3.0780941
1128	3.0523091	1163	5797	1198	4568
1129	6939	1164	9530	1199	8192
1130	3.0530784	1165	3.0663259	1200	3.0791812
1131	4626	1166	6985		
1132	8464	1167	3.0670708		
1133	3.0542299	1168	4428		
1134	6130	1169	8145		
1135	9958	1170	3.0681859		

Num	Logarithms	Num	Logarithms	Num	Logarithms
1201	3.0795430	1236	3.0920185	1271	3.1041455
1202	9045	1237	3697	1272	4871
1203	3.0802655	1238	7206	1273	8284
1204	6265	1239	3.0930713	1274	3.1051694
1205	9870	1240	4217	1275	5102
1206	3.0813473	1241	7718	1276	8507
1207	7073	1242	3.0941216	1277	3.1061909
1208	3.0820669	1243	4711	1278	5308
1209	4263	1244	8204	1279	8705
1210	7854	1245	3.0951693	1280	3.1072100
1211	3.0831441	1246	5180	1281	5491
1212	5026	1247	8664	1282	8880
1213	8608	1248	3.0962146	1283	3.1082266
1214	3.0842187	1249	5624	1284	5650
1215	5763	1250	9100	1285	9031
1216	9336	1251	3.0972573	1286	3.1092410
1217	3.0852906	1252	6043	1287	5785
1218	6473	1253	9511	1288	9159
1219	3.0860037	1254	3.0982975	1289	3.1102529
1220	3598	1255	6437	1290	5897
1221	7156	1256	9896	1291	9262
1222	3.0870712	1257	3.0993353	1292	3.1112625
1223	4264	1258	6806	1293	5985
1224	7814	1259	3.1000257	1294	9343
1225	3.0881361	1260	3705	1295	3.1122698
1226	4905	1261	7151	1296	6050
1227	8446	1262	3.1010593	1297	9400
1228	3.0891984	1263	4033	1298	3.1132747
1229	5519	1264	7471	1299	6091
1230	9051	1265	3.1020905	1300	9433
1231	3.0902580	1266	4337		
1232	6107	1267	7766		
1233	9631	1268	3.1031192		
1234	3.0913151	1269	4616		
1235	6669	1270	8037		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1301	3.1142773	1336	3.1258064	1371	3.1370374
1302	6110	1337	3.1261314	1372	3541
1303	9444	1338	4561	1373	6705
1304	3.1152776	1339	7806	1374	9867
1305	6105	1340	3.1271048	1375	3.1383027
1306	9432	1341	4288	1376	6184
1307	3.1162756	1342	7525	1377	9339
1308	6077	1343	3.1280760	1378	3.1392492
1309	9396	1344	3993	1379	5643
1310	3.1172713	1345	7223	1380	8791
1311	6027	1346	3.1290450	1381	3.1401937
1312	9338	1347	3676	1382	5080
1313	3.1182647	1348	6899	1383	8222
1314	5954	1349	3.1300119	1384	3.1411361
1315	9257	1350	3338	1385	4498
1316	3.1192559	1351	6553	1386	7632
1317	5858	1352	9767	1387	3.1420765
1318	9154	1353	3.1312978	1388	3895
1319	3.1202448	1354	6187	1389	7022
1320	5739	1355	9393	1390	3.1430148
1321	9028	1356	3.1322597	1391	3271
1322	3.1212314	1357	5798	1392	6392
1323	5598	1358	8998	1393	9511
1324	8880	1359	3.1332195	1394	3.1442628
1325	3.1222159	1360	5389	1395	5742
1326	5435	1361	8581	1396	8854
1327	8709	1362	3.1341771	1397	3.1451964
1328	3.1231981	1363	4958	1398	5072
1329	5250	1364	8144	1399	8177
1330	8516	1365	3.1351326	1400	3.1461280
1331	3.2241780	1366	4507		
1332	5042	1367	7685		
1333	8301	1368	3.1360861		
1334	3.1251558	1369	4034		
1335	4813	1370	7206		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1401	3.1464381	1436	3.1571544	1471	3.1676127
1402	7480	1437	4568	1472	9078
1403	3.1470577	1438	7589	1473	3.1682027
1404	3671	1439	3.1580608	1474	4975
1405	6763	1440	3625	1475	7920
1406	9853	1441	6640	1476	3.1690863
1407	3.1482941	1442	9653	1477	3805
1408	6026	1443	3.1592663	1478	6744
1409	9110	1444	5672	1479	9682
1410	3.1492191	1445	8678	1480	3.1702617
1411	5270	1446	3.1601683	1481	5550
1412	8347	1447	4685	1482	8482
1413	3.1501422	1448	7686	1483	3.1711411
1414	4494	1449	3.1610684	1484	4339
1415	7564	1450	3680	1485	7264
1416	3.1510632	1451	6674	1486	3.1720188
1417	3698	1452	9666	1487	3110
1418	6762	1453	3.1622656	1488	6029
1419	9824	1454	5644	1489	8947
1420	3.1522883	1455	8630	1490	3.1731863
1421	5941	1456	3.1631614	1491	4776
1422	8996	1457	4595	1492	7688
1423	3.1532049	1458	7575	1493	3.1740598
1424	5100	1459	3.1640553	1494	3506
1425	8149	1460	3528	1495	6412
1426	3.1541195	1461	6502	1496	9316
1427	4240	1462	9471	1497	3.1752218
1428	7282	1463	3.1652443	1498	5118
1429	3.1550322	1464	5411	1499	8016
1430	3360	1465	8376	1500	3.1760913
1431	6396	1466	3.1661340		
1432	9430	1467	4301		
1433	3.1562462	1468	7260		
1434	5491	1469	3.1670218		
1435	8519	1470	3173		

Num	Logarithms	Num	Logarithms	Num	Logarithms
1501	3.1763807	1536	3.1863912	1571	3.1961762
1502	6699	1537	6739	1572	4525
1503	9590	1538	9563	1573	7287
1504	3.1772478	1539	3.1872386	1574	3.1970047
1505	5365	1540	5207	1575	2806
1506	8250	1541	8026	1576	5562
1507	3.1781132	1542	3.1880844	1577	8317
1508	4013	1543	3659	1578	3.1981070
1509	6892	1544	6473	1579	3821
1510	9769	1545	9285	1580	6571
1511	3.1792645	1546	3.1892095	1581	9319
1512	5518	1547	4903	1582	3.1992065
1513	8389	1548	7709	1583	4809
1514	3.1801259	1549	3.1900514	1584	7552
1515	4126	1550	3317	1585	3.2000293
1516	6992	1551	6118	1586	3032
1517	9856	1552	8917	1587	5769
1518	3.1812718	1553	3.1911714	1588	8505
1519	5578	1554	4510	1589	3.2011239
1520	8436	1555	7304	1590	3971
1521	3.1821292	1556	3.1920096	1591	6702
1522	4146	1557	2886	1592	9431
2523	6999	1558	5674	1593	3.2022158
1524	9850	1559	8461	1594	4883
1525	3.1832698	1560	3.1931246	1595	7607
1526	5545	1561	4029	1596	3.2030329
1527	8390	1562	6810	1597	3049
1528	3.1841233	1563	9590	1598	5768
1529	4075	1564	3.1942367	1599	8485
1530	6914	1565	5143	1600	3.2041200
1531	9752	1566	7917		
1532	3.1852588	1567	3.1950690		
1533	5421	1568	3460		
1534	8253	1569	6229		
1535	3.1861084	1570	8996		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1601	3.2043913	1636	3.2137833	1671	3.2229764
1602	6625	1637	3.2140487	1672	3.2232363
1603	9335	1638	3139	1673	4959
1604	3.2052044	1639	5789	1674	7555
1605	4750	1640	8438	1675	3.2240148
1606	7455	1641	3.2151086	1676	2740
1607	3.2060159	1642	3732	1677	5331
1608	2860	1643	6376	1678	7920
1609	5560	1644	9018	1679	3.2250507
1610	8259	1645	3.2161659	1680	3093
1611	3.2070955	1646	4298	1681	5677
1612	3650	1647	6936	1682	8260
1613	6344	1648	9572	1683	3.2260841
1614	9035	1649	3.2172206	1684	3421
1615	3.2081725	1650	4839	1685	5999
1616	4414	1651	7471	1686	8576
1617	7100	1652	3.2180100	1687	3.2271151
1618	9785	1653	2728	1688	3724
1619	3.2092468	1654	5355	1689	6296
1620	5150	1655	7980	1690	8867
1621	7830	1656	3.2190603	1691	3.2281436
1622	3.2100508	1657	3225	1692	4004
1623	3185	1658	5845	1693	6570
1624	5860	1659	8464	1694	9134
1625	8534	1660	3.2201081	1695	3.2291697
1626	3.2111205	1661	3696	1696	4258
1627	3876	1662	6310	1697	6818
1628	6544	1663	8922	1698	9377
1629	9211	1664	3.2211533	1699	3.2301934
1630	3.2121876	1665	4142	1700	4489
1631	4540	1666	6750		
1632	7201	1667	9356		
1633	9862	1668	3.2221960		
1634	3.2132521	1669	4563		
1635	5178	1670	7165		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1701	3.2307043	1736	3.2395497	1771	3.2482186
1702	9596	1737	7998	1772	4637
1703	3.2312146	1738	3.2400498	1773	7087
1704	4696	1739	2996	1774	9536
1705	7244	1740	5492	1775	3.2491984
1706	9790	1741	7988	1776	4430
1707	3.2322335	1742	3.2410481	1777	6874
1708	4879	1743	2974	1778	9318
1709	7421	1744	5465	1779	3.2501759
1710	9961	1745	7954	1780	4200
1711	3.2332500	1746	3.2420442	1781	6639
1712	5038	1747	2929	1782	9077
1713	7574	1748	5414	1783	3.2511513
1714	3.2340108	1749	7898	1784	3948
1715	2641	1750	3.2430380	1785	6382
1716	5173	1751	2861	1786	8815
1717	7703	1752	5341	1787	3.2521246
1718	3.2350232	1753	7819	1788	3675
1719	2759	1754	3.2440296	1789	6103
1720	5284	1755	2771	1790	8530
1721	7809	1756	5245	1791	3.2530956
1722	3.2360331	1757	7718	1792	3380
1723	2853	1758	3.2450189	1793	5803
1724	5373	1759	2658	1794	8224
1725	7891	1760	5127	1795	3.2540645
1726	3.2370408	1761	7594	1796	3063
1727	2923	1762	3.2460059	1797	5481
1728	5437	1763	2523	1798	7897
1729	7950	1764	4986	1799	3.2550312
1730	3.2380461	1765	7447	1800	2725
1731	2971	1766	9907		
1732	5479	1767	3.2472365		
1733	7986	1768	4823		
1734	3.2390491	1769	7278		
1735	2995	1770	9733		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1801	3.2555137	1836	3.2638727	1871	3.2720738
1802	7548	1837	3.2641092	1872	3058
1803	9957	1838	3455	1873	5378
1804	3.2562365	1839	5817	1874	7696
1805	4772	1840	8178	1875	3.2730013
1806	7177	1841	3.2650538	1876	2328
1807	9582	1842	2896	1877	4643
1808	3.2571984	1843	5253	1878	6956
1809	4386	1844	7609	1879	9268
1810	6786	1845	9964	1880	3.2741578
1811	9184	1846	3.2662317	1881	3888
1812	3.2581582	1847	4669	1882	6196
1813	3978	1848	7020	1883	8503
1814	6373	1849	9369	1884	3.2750809
1815	8766	1850	3.2671717	1885	3113
1816	3.2591158	1851	4064	1886	5417
1817	3549	1852	6410	1887	7719
1818	5939	1853	8754	1888	3.2760020
1819	8327	1854	3.2681097	1889	2320
1820	3.2600714	1855	3439	1890	4618
1821	3099	1856	5780	1891	6915
1822	5484	1857	8119	1892	9211
1823	7867	1858	3.2690457	1893	3.2771506
1824	3.2610248	1859	2794	1894	3800
1825	2629	1860	5129	1895	6092
1826	5008	1861	7464	1896	8383
1827	7385	1862	9797	1897	3.2780673
1828	9762	1863	3.2702128	1898	2962
1829	3.2622137	1864	4459	1899	5250
1830	4511	1865	6788	1900	7536
1831	6883	1866	9116		
1832	9255	1867	3.2711443		
1833	3.2631625	1868	3769		
1834	3993	1869	6093		
1835	6361	1870	8416		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
1901	3.2789821	1936	3.2869054	1971	3.2946866
1902	3.2792105	1937	3.2871296	1972	9069
1903	4388	1938	3538	1973	3.2951271
1904	6669	1939	5778	1974	3471
1905	8950	1940	8017	1975	5671
1906	3.2801229	1941	3.2880255	1976	7869
1907	3507	1942	2492	1977	3.2960067
1908	5784	1943	4728	1978	2263
1909	8059	1944	6963	1979	4458
1910	3.2810334	1945	9196	1980	6652
1911	2607	1946	3.2891428	1981	8845
1912	4879	1947	3659	1982	3.2971036
1913	7150	1948	5889	1983	3227
1914	9419	1949	8118	1984	5417
1915	3.2821688	1950	3.2900346	1985	7605
1916	3955	1951	2573	1986	9792
1917	6221	1952	4798	1987	3.2981979
1918	8486	1953	7022	1988	4164
1919	3.2830750	1954	9246	1989	6348
1920	3012	1955	3.2911468	1990	8531
1921	5274	1956	3688	1991	3.2990713
1922	7534	1957	5908	1992	2893
1923	9793	1958	8127	1993	5073
1924	3.2842051	1959	3.2920344	1994	7251
1925	4307	1960	2561	1995	9429
1926	6563	1961	4776	1996	3.3001605
1927	5817	1962	6990	1997	3781
1928	3.2851070	1963	9203	1998	5955
1929	3322	1964	3.2931415	1999	8128
1930	5573	1965	3626	2000	3.3010300
1931	7823	1966	5835		
1932	3.2860071	1967	8044		
1933	2318	1968	3.2940251		
1934	4565	1969	2457		
193	6810	1970	4662		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
2001	3.3012471	2036	3.3087778	2071	3.3161801
2002	4641	2037	9910	2072	3897
2003	6809	2038	3.3092042	2073	5993
2004	8977	2039	4172	2074	8087
2005	3.3021144	2040	6302	2075	3.3170181
2006	3309	2041	8430	2076	2273
2007	5474	2042	3.3100557	2077	4365
2008	7637	2043	2684	2078	6455
2009	9799	2044	4809	2079	8545
2010	3.3031961	2045	6933	2080	3.3180633
2011	4121	2046	9056	2081	2721
2012	6280	2047	3.3111178	2082	4807
2013	8438	2048	3299	2083	6893
2014	3.3040595	2049	5420	2084	8977
2015	2751	2050	7539	2085	3.3191061
2016	4905	2051	9657	2086	3143
2017	7059	2052	3.3121774	2087	5224
2018	9212	2053	3889	2088	7305
2019	3.3051363	2054	6004	2089	9384
2020	3514	2055	8118	2090	3.3201463
2021	5663	2056	3.3130231	2091	3540
2022	7812	2057	2343	2092	5617
2023	9959	2058	4454	2093	7692
2024	3.3062105	2059	6563	2094	9767
2025	4250	2060	8672	2095	3.3211840
2026	6394	2061	3.3140780	2096	3913
2027	8537	2062	2887	2097	5984
2028	3.3070679	2063	4992	2098	8055
2029	2820	2064	7097	2099	3.3220124
2030	4960	2065	9200	2100	2193
2031	7099	2066	3.3151303		
2032	9237	2067	3405		
2033	3.3081374	2068	5505		
2034	3509	2069	7605		
2035	5644	2070	9703		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
2101	3·3224260	2136	3·3296012	2171	3·3366598
2102	6327	2137	8045	2172	8598
2103	8393	2138	3·3300077	2173	3·3370597
2104	3·3230457	2139	2108	2174	2595
2105	2521	2140	4138	2175	4593
2106	4584	2141	6167	2176	6589
2107	6645	2142	8195	2177	8584
2108	8706	2143	3·3310222	2178	3·3380579
2109	3·3240766	2144	2248	2179	2572
2110	2825	2145	4273	2180	4565
2111	4882	2146	6297	2181	6557
2112	6939	2147	8320	2182	8547
2113	8995	2148	3·3320343	2183	3·3390537
2114	3·3251050	2149	2364	2184	2526
2115	3104	2150	4385	2185	4514
2116	5157	2151	6404	2186	6501
2117	7209	2152	8423	2187	8488
2118	9260	2153	3·3330440	2188	3·3400473
2119	3·3261310	2154	2457	2189	2458
2120	3359	2155	4473	2190	4441
2121	5407	2156	6488	2191	6424
2122	7454	2157	8501	2192	8405
2123	9500	2158	3·3340514	2193	3·3410386
2124	3·3271545	2159	2526	2194	2366
2125	3589	2160	4537	2195	4345
2126	5633	2161	6548	2196	6323
2127	7675	2162	8557	2197	8301
2128	9716	2163	3·3350565	2198	3·3420277
2129	3·3281757	2164	2572	2199	2252
2130	3796	2165	4579	2200	4227
2131	5834	2166	6585		
2132	7872	2167	8589		
2133	9909	2168	3·3360593		
2134	3·3291944	2169	2596		
2135	3979	2170	4597		

Num	Logarithms	Num	Logarithms	Num	Logarithms
2201	3.3426200	2236	3.3494718	2271	3.3562171
2202	8173	2237	6660	2272	4083
2203	3.3430145	2238	8601	2273	5994
2204	2116	2239	3.3500541	2274	7905
2205	4086	2240	2480	2275	9814
2206	6055	2241	4419	2276	3.3571723
2207	8023	2242	6356	2277	3630
2208	9991	2243	8293	2278	5537
2209	3.3441957	2244	3.3510228	2279	7443
2210	3923	2245	2163	2280	9348
2211	5887	2246	4098	2281	3.3581253
2212	7851	2247	6031	2282	3156
2213	9814	2248	7963	2283	5059
2214	3.3451776	2249	9895	2284	6961
2215	3737	2250	3.3521825	2285	8862
2216	5698	2251	3755	2286	3.3590762
2217	7657	2252	5684	2287	2662
2218	9615	2253	7612	2288	4560
2219	3.3461573	2254	9539	2289	6458
2220	3530	2255	3.3531465	2290	8355
2221	5486	2256	3391	2291	3.3600251
2222	7441	2257	5316	2292	2146
2223	9395	2258	7239	2293	4041
2224	3.3471348	2259	9162	2294	5934
2225	3300	2260	3.3541084	2295	7827
2226	5252	2261	3006	2296	9719
2227	7202	2262	4926	2297	3.3611610
2228	9152	2263	6846	2298	3500
2229	3.3481101	2264	8764	2299	5390
2230	3049	2265	3.3550682	2300	7278
2231	4996	2266	2599		
2232	6942	2267	4515		
2233	8887	2268	6430		
2234	3.3490832	2269	8345		
2235	2775	2270	3.3560259		

Num	Logarithms	Num	Logarithms	Num	Logarithms
2301	3·3619166	2336	3·3684728	2371	3·3749316
2302	3·3621053	2337	6587	2372	3·3751147
2303	2939	2338	8445	2373	2977
2304	4825	2339	3·3690302	2374	4807
2305	6709	2340	2159	2375	6636
2306	8593	2341	4014	2376	8464
2307	3·3630476	2342	5869	2377	3·3760292
2308	2358	2343	7723	2378	2118
2309	4239	2344	9576	2379	3944
2310	6120	2345	3·3701428	2380	5769
2311	7999	2346	3280	2381	7594
2312	9878	2347	5131	2382	9418
2313	3·3641756	2348	6981	2383	3·3771240
2314	3633	2349	8830	2384	3062
2315	5510	2350	3·3710679	2385	4884
2316	7386	2351	2526	2386	6704
2317	9260	2352	4373	2387	8524
2318	3·3651134	2353	6219	2388	3·3780343
2319	3007	2354	8065	2389	2161
2320	4880	2355	9909	2390	3979
2321	6751	2356	3·3721753	2391	5796
2322	8622	2357	3596	2392	7612
2323	3·3660492	2358	5438	2393	9427
2324	2361	2359	7279	2394	3·3791241
2325	4230	2360	9120	2395	3055
2326	6097	2361	3·3730960	2396	4868
2327	7964	2362	2799	2397	6680
2328	9830	2363	4637	2398	8492
2329	3·3671695	2364	6475	2399	3·3800302
2330	3559	2365	8311	2400	2112
2331	5423	2366	3·3740147		
2332	7285	2367	1983		
2333	9147	2368	3817		
2334	3·3681008	2369	5651		
2335	2869	2370	7483		

Num	Logarithms	Num	Logarithms	Num	Logarithms
2401	3.3803922	2436	3.3866773	2471	3.3928727
2402	5730	2437	8555	2472	3.3930485
2403	7538	2438	3.3870337	2473	2241
2404	9345	2439	2118	2474	3997
2405	3.3811151	2440	3898	2475	5752
2406	2956	2441	5678	2476	7506
2407	4761	2442	7457	2477	9260
2408	6565	2443	9235	2478	3.3941013
2409	8368	2444	3.3881012	2479	2765
2410	3.3820170	2445	2789	2480	4517
2411	1972	2446	4565	2481	6268
2412	3773	2447	6340	2482	8018
2413	5573	2448	8114	2483	9767
2414	7373	2449	9888	2484	3.3951516
2415	9171	2450	3.3891661	2485	3264
2416	3.3830969	2451	3433	2486	5011
2417	2766	2452	5205	2487	6758
2418	4563	2453	6975	2488	8504
2419	6359	2454	8746	2489	3.3960249
2420	8154	2455	3.3900515	2490	1993
2421	9948	2456	2284	2491	3737
2422	3.3841741	2457	4052	2492	5480
2423	3534	2458	5819	2493	7223
2424	5326	2459	7585	2494	8964
2425	7117	2460	9351	2495	3.3970705
2426	8908	2461	3.3911116	2496	2446
2427	3.3850698	2462	2880	2497	4185
2428	2487	2463	4644	2498	5924
2429	4275	2464	6407	2499	7662
2430	6063	2465	8169	2500	9400
2431	7850	2466	9931		
2432	9636	2467	3.3921691		
2433	3.3861421	2468	3452		
2434	3206	2469	5211		
2435	4990	2470	6969		

Num	Logarithms	Num	Logarithms	Num	Logarithms
2501	3.3981137	2536	3.4041492	2571	3.4101021
2502	2873	2537	3205	2572	2710
2503	4608	2538	4916	2573	4398
2504	6343	2539	6627	2574	6085
2505	8077	2540	8337	2575	7772
2506	9811	2541	3.4050047	2576	9459
2507	3.3991543	2542	1755	2577	3.4111144
2508	3275	2543	3464	2578	2829
2509	5007	2544	5171	2579	4513
2510	6737	2545	6878	2580	6197
2511	8467	2546	8584	2581	7880
2512	3.4000196	2547	3.4060289	2582	9562
2513	1925	2548	1994	2583	3.4121244
2514	3653	2549	3698	2584	2925
2515	5380	2550	5402	2585	4605
2516	7106	2551	7105	2586	6285
2517	8832	2552	8807	2587	7964
2518	3.4010557	2553	3.4070508	2588	9643
2519	2282	2554	2209	2589	3.4131320
2520	4005	2555	3909	2590	2998
2521	5728	2556	5608	2591	4674
2522	7451	2557	7307	2592	6350
2523	9173	2558	9005	2593	8025
2524	3.4020893	2559	3.4080703	2594	9700
2525	2614	2560	2400	2595	3.4141374
2526	4333	2561	4096	2596	3047
2527	6052	2562	5791	2597	4719
2528	7771	2563	7486	2598	6391
2529	9488	2564	9180	2599	8063
2530	3.4031205	2565	3.4090874	2600	9733
2531	2921	2566	2567		
2532	4637	2567	4259		
2533	6352	2568	5950		
2534	8066	2569	7641		
2535	9780	2570	9331		

Num	Logarithms	Num	Logarithms	Num	Logarithms
2601	3.4151404	2636	3.4209454	2671	3.4266739
2602	3073	2637	3.4211101	2672	8365
2603	4742	2638	2748	2673	9990
2604	6410	2639	4394	2674	3.4271614
2605	8077	2640	6039	2675	3238
2606	9744	2641	7684	2676	4861
2607	3.4161410	2642	9328	2677	6484
2608	3076	2643	3.4220972	2678	8106
2609	4741	2644	2614	2679	9727
2610	6405	2645	4257	2680	3.4281348
2611	8069	2646	5898	2681	2968
2912	9732	2647	7539	2682	4588
2613	3.4171394	2648	9180	2683	6207
2614	3056	2649	3.4230820	2684	7825
2615	4717	2650	2459	2685	9443
2616	6377	2651	4097	2686	3.4291060
2617	8037	2652	5735	2687	2677
2618	9696	2653	7372	2688	4293
2619	3.4181355	2654	9009	2689	5908
2620	3013	2655	3.4240645	2690	7523
2621	4670	2656	2281	2691	9137
2622	6327	2657	3916	2692	3.4300751
2623	7983	2658	5550	2693	2364
2624	9638	2659	7183	2694	3976
2625	3.4191293	2660	8816	2695	5588
2626	2947	2661	3.4250449	2696	7199
2627	4601	2662	2080	2697	8809
2628	6254	2663	3712	2698	3.4310419
2629	7906	2664	5342	2699	2029
2630	9557	2665	6972	2700	3638
2631	3.4201208	2666	8601		
2632	2859	2667	0230		
2633	4509	2668	3.4261858		
2634	6158	2669	3486		
2635	7806	2670	5113		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
2701	3.4315426	2736	3.4371161	2771	3.4426365
2702	6853	2737	2748	2772	7932
2703	8460	2738	4334	2773	9499
2704	3.4320067	2739	5920	2774	3.4431065
2705	1673	2740	7506	2775	2630
2706	3278	2741	9090	2776	4195
2707	4882	2742	3.4380674	2777	5759
2708	6487	2743	2258	2778	7322
2709	8090	2744	3841	2779	8885
2710	9693	2745	5423	2780	3.4440448
2711	3.4331295	2746	7005	2781	2010
2712	2897	2747	8587	2782	3571
2713	4498	2748	3.4390167	2783	5132
2714	6098	2749	1747	2784	6692
2715	7698	2750	3327	2785	8252
2716	9298	2751	4906	2786	9811
2717	3.4340896	2752	6484	2787	3.4451370
2718	2494	2753	8062	2788	2928
2719	4092	2754	9639	2789	4485
2720	5689	2755	3.4401216	2790	6042
2721	7285	2756	2792	2791	7598
2722	8881	2757	4368	2792	9154
2723	3.4350476	2758	5943	2793	3.4460709
2724	2071	2759	7517	2794	2264
2725	3665	2760	9091	2795	3818
2726	5258	2761	3.4410664	2796	5372
2727	6851	2762	2237	2797	6925
2728	8444	2763	3809	2798	8477
2729	3.4360035	2764	5380	2799	3.4470029
2730	1626	2765	6951	2800	1580
2731	3217	2766	8522		
2732	4807	2767	3.4420092		
2733	6396	2768	1661		
2734	7985	2769	3229		
2735	9573	2770	4798		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
2801	3.4473131	2836	3.4527062	2871	3.4580332
2802	4681	2837	8593	2872	1844
2803	6231	2838	3.4530124	2873	3356
2304	7780	2839	1654	2874	4867
2805	9329	2840	3183	2875	6378
2806	3.4480877	2841	4712	2876	7889
2807	2424	2842	6241	2877	9399
2808	3971	2843	7769	2878	3.4590908
2809	5517	2844	9296	2879	2417
2810	7063	2845	3.4540823	2880	3925
2811	8608	2846	2349	2881	5433
2812	3.4490153	2847	3875	2882	6940
2813	1697	2848	5400	2883	8446
2814	3241	2849	6924	2884	9953
2815	4784	2850	8449	2885	3.4601458
2816	6326	2851	9972	2886	2963
2817	7868	2852	3.4551495	2887	4468
2818	9410	2853	3018	2888	5972
2819	3.4500951	2854	4540	2889	7475
2820	2491	2855	6061	2890	8978
2821	4031	2856	7582	2891	3.4610481
2822	5570	2857	9102	2892	1983
2823	7109	2858	3.4560622	2893	3484
2824	8647	2859	2142	2894	4985
2825	3.4510184	2860	3660	2895	6486
2826	1721	2861	5179	2896	7986
2827	3258	2862	6696	2897	9485
2828	4794	2863	8213	2898	3.4620984
2829	6329	2864	9730	2899	2482
2830	7864	2865	3.4571246	2900	3980
2831	9399	2866	2762		
2832	3.4520932	2867	4277		
2833	2466	2868	5791		
2834	3998	2869	7305		
2835	5531	2870	8819		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
2901	3.4625477	2936	3.4677560	2971	3.4729027
2902	6974	2937	9039	2972	3.4730488
2903	8470	2938	3.4680518	2973	1949
2904	9966	2939	1996	2974	3410
2905	3.4631461	2940	3473	2975	4870
2906	2956	2941	4950	2976	6329
2907	4450	2942	6427	2977	7788
2908	5944	2943	7903	2978	9247
2909	7437	2944	9378	2979	3.4740705
2910	8930	2945	3.4690853	2980	2163
2911	3.4640422	2946	2327	2981	3620
2912	1914	2947	3801	2982	5076
2913	3405	2948	5275	2983	6533
2914	4895	2949	6748	2984	7988
2915	6386	2950	8220	2985	9443
2916	7875	2951	9692	2986	3.4750898
2917	9364	2952	3.4701163	2987	2352
2918	3.4650853	2953	2634	2988	3806
2919	2341	2954	4105	2989	5259
2920	3828	2955	5575	2990	6712
2921	5316	2956	7044	2991	8164
2922	6802	2957	8513	2992	9616
2923	8288	2958	9982	2993	3.4761067
2924	9774	2959	3.4711450	2994	2518
2925	3.4661259	2960	2917	2995	3968
2926	2743	2961	4384	2996	5418
2927	4227	2962	5851	2997	6867
2928	5711	2963	7317	2998	8316
2929	7194	2964	8782	2999	9765
2930	8676	2965	3.4720247	3000	3.4771212
2931	3.4670158	2966	1711		
2932	1640	2967	3175		
2933	3121	2968	4639		
2934	4601	2969	6102		
2935	6081	2970	7564		

LOGARITHMS.

Num	Logarithms	Num	Logarithms	Num	Logarithms
3001	3.4772660	3036	3.4823018	3071	3.4872798
3002	4107	3037	4448	3072	4212
3003	5553	3038	5878	3073	5626
3004	6999	3039	7307	3074	7039
3005	8445	3040	8736	3075	8451
3006	9890	3041	3.4830164	3076	9863
3007	3.4781334	3042	1592	3077	3.4881275
3008	2778	3043	3019	3078	2686
3009	4222	3044	4446	3079	4097
3010	5665	3045	5873	3080	5507
3011	7108	3046	7299	3081	6917
3012	8550	3047	8725	3082	8326
3013	9991	3048	3.4840150	3083	9735
3014	3.4791432	3049	1574	3084	3.4891144
3015	2873	3050	2998	3085	2552
3016	4313	3051	4422	3086	3959
3017	5753	3052	5845	3087	5366
3018	7192	3053	7268	3088	6773
3019	8631	3054	8690	3089	8179
3020	3.4800069	3055	3.4850112	3090	9585
3021	1507	3056	1533	3091	3.4900990
3022	2945	3057	2954	3092	2395
3023	4381	3058	4375	3093	3799
3024	5818	3059	5795	3094	5203
3025	7254	3060	7214	3095	6607
3026	8689	3061	8633	3096	8009
3027	3.4810124	3062	3.4860052	3097	9412
3028	1559	3063	1470	3098	3.4910814
3029	2993	3064	2888	3099	2216
3030	4426	3065	4305	3100	3617
3031	5859	3066	5721		
3032	7292	3067	7138		
3033	8724	3068	8554		
3034	3.4820156	3069	9969		
3035	1587	3070	3.4871384		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
3101	3.4915018	3136	3.4963761	3171	3.5011962
3102	6418	3137	5145	3172	3332
3103	7818	3138	6529	3173	4701
3104	9217	3139	7913	3174	6069
3105	3.4920616	3140	9296	3175	7437
3106	2014	3141	3.4970679	3176	8805
3107	3413	3142	2062	3177	3.5020172
3108	4810	3143	3444	3178	1539
3109	6207	3144	4825	3179	2905
3110	7604	3145	6206	3180	4271
3111	9000	3146	7587	3181	5637
3112	3.4930396	3147	8967	3182	7002
3113	1791	3148	3.4980347	3183	8366
3114	3186	3149	1727	3184	9731
3115	4580	3150	3106	3185	3.5031094
3116	5974	3151	4484	3186	2458
3117	7368	3152	5862	3187	3821
3118	8761	3153	7240	3188	5183
3119	3.4940154	3154	8617	3189	6545
3120	1546	3155	9994	3190	7907
3121	2938	3156	3.4991370	3191	9268
3122	4329	3157	2746	3192	3.5040629
3123	5720	3158	4121	3193	1989
3124	7110	3159	5496	3194	3349
3125	8500	3160	6871	3195	4709
3126	9890	3161	8245	3196	6068
3127	3.4951279	3162	9619	3197	7426
3128	2667	3163	3.5000992	3198	8785
3129	4056	3164	2365	3199	3.5050142
3130	5443	3165	3737	3200	1500
3131	6831	3166	5109		
3132	8218	3167	6481		
3133	9604	3168	7851		
3134	3.4960990	3169	9222		
3135	2375	3170	3.5010593		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
3201	3.5052857	3236	3.5100085	3271	3.5146805
3202	4213	3237	1427	3272	8133
3203	5569	3238	2768	3273	9460
3204	6925	3239	4109	3274	3.5150787
3205	8280	3240	5450	3275	2113
3206	9635	3241	6790	3276	3439
3207	3.5060990	3242	8130	3277	4764
3208	2344	3243	9469	3278	6089
3209	3697	3244	3.5110808	3279	7414
3210	5050	3245	2147	3280	8738
3211	6403	3246	3485	3281	3.5160062
3212	7755	3247	4823	3282	1386
3213	9107	3248	6160	3283	2709
3214	3.5070459	3249	7497	3284	4031
3215	1810	3250	8834	3285	5354
3216	3160	3251	3.5120170	3286	6676
3217	4511	3252	1505	3287	7997
3218	5860	3253	2841	3288	9318
3219	7210	3254	4175	3289	3.5170639
3220	8559	3255	5510	3290	1959
3221	9907	3256	6844	3291	3279
3222	3.5081255	3257	8178	3292	4598
3223	2603	3258	9511	3293	5917
3224	3950	3259	3.5130844	3294	7236
3225	5297	3260	2176	3295	8554
3226	6644	3261	3508	3296	9872
3227	7990	3262	4840	3297	3.5181189
3228	9335	3263	6171	3298	2506
3229	3.5090680	3264	7501	3299	3823
3230	2025	3265	8832	3300	5139
3231	3370	3266	3.5140162		
3232	4713	3267	1491		
3233	6057	3268	2820		
3234	7400	3269	4149		
3235	8743	3270	5478		

Num	Logarithms	Num	Logarithms	Num	Logarithms
3301	3.5186455	3336	3.5232260	3371	3.5277588
3302	7771	3337	3562	3372	8876
3303	9086	3338	4863	3373	3.5280163
3304	3.5190400	3339	6164	3374	1451
3305	1715	3340	7465	3375	2738
3306	3028	3341	8765	3376	4024
3307	4342	3342	3.5240064	3377	5311
3308	5655	3343	1364	3378	6596
3309	6968	3344	2663	3379	7882
3310	8280	3345	3961	3380	9167
3311	9592	3346	5259	3381	3.5290452
3312	3.5200903	3347	6557	3382	1736
3313	2214	3348	7854	3383	3020
3314	3525	3349	9151	3384	4303
3315	4835	3350	3.5250448	3385	5587
3316	6145	3351	1744	3386	6869
3317	7455	3352	3040	3387	8152
3318	8764	3353	4335	3388	9434
3319	3.5210073	3354	5631	3389	3.5300716
3320	1381	3355	6925	3390	1997
3321	2689	3356	8219	3391	3278
3322	3996	3357	9513	3392	4558
3323	5303	3358	3.5260807	3393	5839
3324	6610	3359	2100	3394	7118
3325	7916	3360	3393	3395	8398
3326	9222	3361	4685	3396	9677
3327	3.5220528	3362	5977	3397	3.5310955
3328	1833	3363	7269	3398	2234
3329	3138	3364	8560	3399	3512
3330	4442	3365	9851	3400	4789
3331	5746	3366	3.5271141		
3332	7056	3367	2431		
3333	8353	3368	3721		
3334	9656	3369	5010		
3335	3.5230958	3370	6299		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
3401	3.5316066	3436	3.5360532	3471	3.5404546
3402	7343	3437	1795	3472	5797
3403	8619	3438	3059	3473	7048
3404	9895	3439	4322	3474	8298
3405	3.5321171	3440	5584	3475	9548
3406	2446	3441	6847	3476	3.5410798
3407	3721	3442	8109	3477	2047
3408	4996	3443	9370	3478	3296
3409	6270	3444	3.5370631	3479	4544
3410	7544	3445	1892	3480	5792
3411	8817	3446	3153	3481	7040
3412	3.5330090	3447	4413	3482	8288
3413	1363	3448	5672	3483	9535
3414	2635	3449	6932	3484	3.5420781
3415	3907	3450	8191	3485	2028
3416	5179	3451	9450	3486	3274
3417	6450	3452	3.5380708	3487	4519
3418	7721	3453	1966	3488	5765
3419	8991	3454	3223	3489	7010
3420	3.5340261	3455	4481	3490	8254
3421	1531	3456	5737	3491	9498
3422	2800	3457	6994	3492	3.5430742
3423	4069	3458	8250	3493	1986
3424	5338	3459	9506	3494	3229
3425	6606	3460	3.5390761	3495	4472
3426	7874	3461	2016	3496	5714
3427	9141	3462	3271	3497	6956
3428	3.5350408	3463	4525	3498	8198
3429	1675	3464	5779	3499	9439
3430	2941	3465	7032	3500	3.5440680
3431	4207	3466	8286		
3432	5473	3467	9538		
3433	6738	3468	3.5400791		
3434	8003	3469	2043		
3435	9267	3470	3295		

Num	Logarithms	Num	Logarithms	Num	Logarithms
3501	3·5441921	3536	3·5485123	3571	3·5527898
3502	3161	3537	6351	3572	9114
3503	4401	3538	7578	3573	3·5530330
3504	5641	3539	8806	3574	1545
3505	6880	3540	3·5490033	3575	2760
3506	3·5448119	3541	1259	3576	3·5533975
3507	9358	3542	2486	3577	5189
3508	3·5450596	3543	3712	3578	6403
3509	1834	3544	4937	3579	7617
3510	3071	3545	6162	3580	8830
3511	3·5454308	3546	3·5497387	3581	3·5540043
3512	5545	3547	8612	3582	1256
3513	6781	3548	9836	3583	2468
3514	8018	3549	3·5501060	3584	3680
3515	9253	3550	2283	3585	4892
3516	3·5460489	3551	3·5503507	3586	3·5546103
3517	1724	3552	4730	3587	7314
3518	2958	3553	5952	3588	8524
3519	4193	3554	7174	3589	9735
3520	5427	3555	8396	3590	3·5550944
3521	3·5466660	3556	9618	3591	2154
3522	7894	3557	3·5510839	3592	3363
3523	9126	3558	2059	3593	4572
3524	3·5470359	3559	3280	3594	5781
3525	1591	3560	4500	3595	6989
3526	3·5472823	3561	3·5515720	3596	3·5558197
3527	4055	3562	6939	3597	9404
3528	5286	3563	8158	3598	3·5560612
3529	6517	3564	9377	3599	1818
3530	7747	3565	3·5520595	3600	3025
3531	8977	3566	1813		
3532	3·5480207	3567	3031		
3533	1436	3568	4248		
3534	2665	3569	5465		
3535	3894	3570	6682		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
3601	3.5564231	3636	3.5606239	3671	3.5647844
3602	5437	3637	7433	3672	9027
3603	6643	3638	8627	3673	3.5650209
3604	7848	3639	9820	3674	1392
3605	9053	3640	3.5611014	3675	2573
3606	3.5570257	3641	2207	3676	3.5653755
3607	1461	3642	3399	3677	4936
3608	2665	3643	4592	3678	6117
3609	3869	3644	5784	3679	7298
3610	5072	3645	6975	3680	8478
3611	3.5576275	3646	3.5618167	3681	9658
3612	7477	3647	9358	3682	3.5660838
3613	8680	3648	3.5620548	3683	2017
3614	9881	3649	1739	3684	3196
3615	3.5581083	3650	2929	3685	4375
3616	2284	3651	3.5624118	3686	3.5665553
3617	3485	3652	5308	3687	6731
3618	4686	3653	6497	3688	7909
3619	5886	3654	7685	3689	9087
3620	7086	3655	8874	3690	3.5670264
3621	3.5588285	3656	3.5630062	3691	1440
3622	9484	3657	1250	3692	2617
3623	3.5590683	3658	2437	3693	3793
3624	1882	3659	3624	3694	4969
3625	3080	3660	4811	3695	6144
3626	4278	3661	3.5635997	3696	3.5677320
3627	5476	3662	7183	3697	8494
3628	6673	3663	8369	3698	9669
3629	7870	3664	9555	3699	3.5680843
3630	9066	3665	3.5640740	3700	2017
3631	3.5600262	3666	1925		
3632	1458	3667	3109		
3633	2654	3668	4293		
3934	3849	3669	5477		
3635	5044	3670	6661		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
3701	3.5683191	3736	3.5724069	3771	3.5764565
3702	4364	3737	5231	3772	5717
3703	5537	3738	6393	3773	6868
3704	6710	3739	7555	3774	8019
3705	7882	3740	8716	3775	9169
3706	9054	3741	9877	3776	3.5770320
3707	3.5690226	3742	3.5731038	3777	1470
3708	1397	3743	2198	3778	2620
3709	2568	3744	3358	3779	3769
3710	3739	3745	4518	3780	4918
3711	4910	3746	5678	3781	6067
3712	6080	3747	6837	3782	7215
3713	7249	3748	7996	3783	8363
3714	8419	3749	9154	3784	9511
3715	9588	3750	3.5740313	3785	3.5780659
3716	3.5700757	3751	1471	3786	1806
3717	1926	3752	2628	3787	2953
3718	3094	3753	3786	3788	4100
3719	4262	2754	4943	3789	5246
3720	5429	3755	6099	3790	6392
3721	6597	3756	7256	3791	7538
3722	7764	3757	8412	3792	8683
3723	8930	3758	9568	3793	9828
3724	3.5710097	3759	3.5750723	3794	3.5790973
3725	1263	3760	1878	3795	2118
3726	2428	3761	3033	3796	3262
3727	3594	3762	4188	3797	4406
3728	4759	3763	5342	3798	5550
3729	5924	3764	6496	3799	6693
3730	7088	3765	7650	3800	7836
3731	8252	3766	8803		
3732	9416	3767	9956		
3733	3.5720580	3768	3.5761109		
3734	1743	3769	2261		
3735	2906	3770	3413		

Num	Logarithms	Num	Logarithms	Num	Logarithms
3801	3.5798979	3836	3.5838786	3871	3.5878232
3802	3.5800121	3837	9918	3872	9353
3803	1263	3838	3.5841050	3873	3.5880475
3804	2405	3839	2181	3874	1596
3805	3547	3840	3312	3875	2717
3806	3.5804688	3841	4443	3876	3838
3807	5829	3842	5574	3877	4958
3808	6969	3843	6704	3878	6078
3809	8110	3844	7834	3879	7198
3810	9250	3845	8963	3880	8317
3811	3.5810389	3846	3.5850093	3881	9436
3812	1529	3847	1222	3882	3.5890555
3813	2668	3848	2351	3883	1674
3814	3807	3849	3479	3884	2792
3815	4945	3850	4607	3885	3910
3816	3.5816084	3851	5735	3886	5028
3817	7222	3852	6863	3887	6145
3818	8359	3853	7990	3888	7262
3819	9497	3854	9117	3889	8379
3820	3.5820634	3855	3.5860244	3890	9496
3821	1770	3856	1370	3891	3.5900612
3822	2907	3857	2496	3892	1728
3823	4043	3858	3622	3893	2844
3824	5179	3859	4748	3894	3959
3825	6314	3860	5873	3895	5075
3826	3.5827450	3861	6698	3896	6189
3827	8585	3862	8123	3897	7304
3828	9719	3863	9247	3898	8418
3829	3.5830854	3864	3.5870371	3899	9532
3830	1988	3865	1495	3900	3.5910646
3831	3122	3866	2618		
3832	4255	3867	3742		
3833	5388	3868	4865		
3834	6521	3869	5987		
3835	7654	3870	7110		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
3901	3·5911759	3936	3·5950551	3971	3·5988999
3902	2873	3937	1654	3972	3·5990092
3903	3985	3938	2757	3973	1186
3904	5098	3939	3860	3974	2279
3905	6210	3940	4962	3975	3371
3906	3·5917322	3941	3·5956064	3976	3·5994464
3907	8434	3942	7166	3977	5556
3908	9546	3943	8268	3978	6648
3909	3·5920657	3944	9369	3979	7739
3910	1768	3945	3·5960470	3980	8831
3911	3·5922878	3946	1571	3981	9922
3912	3988	3947	2671	3982	3.6001013
3913	5098	3948	3771	3983	2103
3914	6208	3949	4871	3984	3193
3915	7318	3950	5971	3985	4283
3916	3·5928427	3951	3·5967070	3986	3.6005373
3917	9536	3952	8169	3987	6462
3918	3·5930644	3953	9268	3988	7551
3919	1753	3954	3·5970367	3989	8640
3920	2861	3955	1465	3990	9729
3921	3·5933968	3956	3·5972563	3991	3.6010817
3022	5076	3957	3660	3992	1905
3923	6183	3958	4758	3993	2993
3924	7290	3959	5855	3994	4080
3925	8397	3960	6952	3995	5168
3926	9503	3961	3·5978048	3996	3.6016255
3927	3·5940609	3962	9145	3997	7341
2928	1715	3963	3·5980241	3998	8428
3929	2820	3964	1336	3999	9514
3930	3925	3965	2432	4000	3.6020600
3931	3·5945030	3966	3·5983527		
3932	6135	3967	4622		
3933	7239	3968	5717		
3934	8344	3969	6811		
3935	9447	3970	7905		

Num	Logarithms	Num	Logarithms	Num	Logarithms
4001	3.6021685	4036	3.6059512	4071	3.6097011
4002	2771	4037	3.6060587	4072	8078
4003	3856	4038	1663	4073	9144
4004	4941	4039	2738	4074	3.6100210
4005	6025	4040	3814	4075	1276
4006	3.6027109	4041	3.6064888	4076	3.6102342
4007	8193	4042	5963	4077	3407
4008	9277	4043	7037	4078	4472
4009	3.6030361	4044	8111	4079	5537
4010	1444	4045	9185	4080	6602
4011	3.6032527	4046	3.6070259	4081	3.6107666
4012	3609	4047	1332	4082	8730
4013	4692	4048	2405	4083	9794
4014	5774	4049	3478	4084	3.6110857
4015	6855	4050	4550	4085	1921
4016	3.6037937	4051	3.6075622	4086	3.6112984
4017	9018	4052	6694	4087	4046
4018	3.6040099	4053	7766	4088	5109
4019	1180	4054	8837	4089	6171
4020	2261	4055	9909	4090	7233
4021	3.6043341	4056	3.6080979	4091	3.6118295
4022	4421	4057	2050	4092	9356
4023	5500	4058	3120	4093	3.6120417
4024	6580	4059	4190	4094	1478
4025	7659	4060	5260	4095	2539
4026	3.6048738	4061	3.6086330	4096	3.6123599
4027	9816	4062	7399	4097	4660
4028	3.6050895	4063	8468	4098	5720
4029	1973	4064	9537	4099	6779
4030	3050	4065	3.6090605	4100	7839
4031	3.6054128	4066	1674		
4032	5205	4067	2742		
4033	6282	4068	3809		
4034	7359	4069	4877		
4035	8435	4070	5944		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
4101	3.6128898	4136	3.6165805	4171	3.6202402
4102	9957	4137	6855	4172	3443
4103	3.6131015	4138	7905	4173	4484
4104	2073	4139	8954	4174	5524
4105	3132	4140	3.6170003	4175	6565
4106	4189	4141	1052	4176	7605
4107	5248	4142	2101	4177	8645
4108	6304	4143	3149	4178	9684
4109	7361	4144	4197	4179	3.6210724
4110	8418	4145	5245	4180	1763
4111	9475	4146	6293	4181	2802
4112	3.6140531	4147	7340	4182	3840
4113	1587	4148	8387	4183	4879
4114	2643	4149	9434	4184	5917
4115	3698	4150	3.6180481	4185	6955
4116	4754	4151	1527	4186	7992
4117	5809	4152	2573	4187	9030
4118	6863	5153	3619	4188	3.6220067
4119	7918	4154	4645	4189	1104
4120	8972	4155	5710	4190	2140
4121	3.6150026	4156	6755	4191	3177
4122	1080	4157	7800	4192	4213
4123	2133	5158	8845	4193	5249
4124	3187	4159	9889	4194	6284
4125	4240	4160	3.6190933	4195	7320
4126	5292	4161	1977	4196	8355
4127	6345	4162	3021	4197	9390
4128	7397	4163	4064	4198	3.6230424
4129	8449	4164	5107	4199	1459
4130	9501	4165	6150	4200	2493
4131	3.6160552	4166	7193		
4132	1603	4167	8235		
4133	2654	4168	9277		
4134	3705	4169	3.6200319		
4135	4755	4170	1360		

Num	Logarithms	Num	Logarithms	Num	Logarithms
4201	3.6233527	4236	3.6269559	4271	3.6305296
4202	4560	4237	3.6270585	4272	6312
4203	5594	4238	1610	4273	7329
4204	6627	4239	2634	4274	8345
4205	7660	4240	3659	4275	9361
4206	3.6238693	4241	3.6274683	4276	3.6310377
4207	9725	4242	5707	4277	1392
4208	3.6240757	4243	6730	4278	2408
4209	1789	4244	7754	4279	3423
4210	2821	4245	8777	4280	4438
4211	3.6243852	4246	9800	4281	3.6315452
4212	4884	4247	3.6280823	4282	6467
4213	5915	4248	1845	4283	7481
4214	6945	4249	2867	4284	8495
4215	7976	4250	3889	4285	9508
4216	9006	4251	3.6284911	4286	3.6320522
4217	3.6250036	4252	5933	4287	1535
4218	1066	5253	6954	4288	2548
4219	2095	4254	7975	4289	3560
4220	3124	4255	8996	4290	4573
4221	3.6254153	4256	3.6290016	4291	3.6325585
4222	5182	4257	1036	4292	6597
4223	6211	4258	2057	4293	7609
4224	7239	4259	3076	4294	8620
4225	8267	4260	4096	4295	9632
4226	9295	4261	3.6295115	4296	3.6330643
4227	3.6260322	4262	6134	4297	1653
4228	1350	4263	7153	4298	2664
4229	2377	4264	8172	4299	3674
4230	3404	4265	9190	4300	4685
4231	3.6264430	4266	3.6300208		
4232	5457	4267	1226		
4233	6483	4268	2244		
4234	7509	4269	3262		
4235	8534	4270	4279		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
4301	3.6335694	4336	3.6370893	4371	3.6405808
4302	6704	4337	1894	4372	6802
4303	7713	4338	2895	4373	7795
4304	8723	4339	3896	4374	8788
4305	9732	4340	4897	4375	9781
4306	3.6340740	4341	3.6375898	4376	3.6410773
4307	1749	4342	6898	4377	1765
4308	2757	4343	7898	4378	2758
4309	3765	4344	8898	4379	3749
4310	4773	4345	9898	4380	4741
4311	3.6345780	4346	3.6380897	4381	3.6415733
4312	6788	4347	1896	4382	6724
4313	7795	4348	2895	4383	7715
4314	8801	4349	3894	4384	8705
4315	9808	4350	4893	4385	9696
4316	3.6350814	4351	3.6385891	4386	3.6420686
4317	1820	4352	6889	4387	1676
4318	2826	4353	7887	4388	2666
4319	3832	4354	8884	4389	3656
4320	4837	4355	9882	4390	4645
4321	3.6355843	4356	3.6390879	4391	3.6425634
4322	6848	4357	1876	4392	6623
4323	7852	4358	2872	4393	7612
4324	8857	4359	3869	4394	8601
4325	9861	4360	3865	4395	9589
4326	3.6360865	4361	3.6395861	4396	3.6430577
4327	1869	4362	6857	4397	1565
4328	2872	4363	7852	4398	2552
4329	3876	4364	8847	4399	3540
4330	4879	4365	9842	4400	4527
4331	3.6365882	4366	3.6400837		
4332	6884	4367	1832		
4333	7887	4368	2826		
4334	8889	4369	3820		
4335	9891	4370	4814		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
4401	3.6435514	4436	3.6469915	4471	3.6504047
4402	6500	4437	3.6470894	4472	5018
4403	7487	4438	1873	4473	5989
4404	8473	4439	2851	4474	6960
4405	9459	4440	3830	4475	7930
4406	3.6440445	4441	3.6474808	4476	3.6508901
4407	1430	4442	5785	4477	9871
4408	2416	4443	6763	4478	3.6510841
4409	3401	4444	7740	4479	1811
4410	4386	4445	8718	4480	2780
4411	3.6445371	4446	9695	4481	3.6513749
4412	6355	4447	3.6480671	4482	4719
4413	7339	4448	1648	4483	5687
4414	8323	4449	2624	4484	6656
4415	9307	4450	3600	4485	7624
4416	3.6450291	4451	3.6484576	4486	3.6518593
4417	1274	4452	5552	4487	9561
4418	2257	4453	6527	4488	3.6520528
4419	3240	4454	7502	4489	1496
4420	4223	4455	8477	4490	2463
4421	3.6455205	4456	9452	4491	3.6523430
4422	6187	4457	3.6490426	4492	4397
4423	7169	4458	1401	4493	5364
4424	8151	4459	2375	4494	6331
4425	9133	4460	3349	4495	7297
4426	3.6460114	4461	3.6494322	4496	3.6528263
4427	1095	4462	5296	4497	9229
4428	2076	4463	6269	4498	3.6530195
4429	3057	4464	7242	4499	1160
4430	4037	4465	8215	4500	2125
4431	3.6465017	4466	9187		
4432	5997	4467	3.6500160		
4433	6977	4468	1132		
4434	7957	4469	2104		
4435	8936	4470	3075		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
4501	3.6533090	4536	3.6566730	4571	3.6600112
4502	4055	4537	7688	4572	1062
4503	5019	4538	8645	4573	2012
4504	5984	4539	9602	4574	2962
4505	6948	4540	3.6570559	4575	3911
4506	3.6537912	4541	1515	4576	3.6604860
4507	8876	4542	2471	4577	5809
4508	9839	4543	3427	4578	6758
4509	3.6540802	4544	4383	4579	7706
4510	1765	4545	5339	4580	8655
4511	3.6542728	4546	3.6576294	4581	9603
4512	3691	4547	7250	4582	3.6610551
4513	4653	4548	8205	4583	1499
4514	5616	4549	9159	4584	2446
4515	6578	4550	3.6580114	4585	3393
4516	3.6547539	4551	1068	4586	3.6614340
4517	8501	4552	2023	4587	5287
4518	9462	4553	2976	4588	6234
4519	3.6550423	4554	3930	4589	7181
4520	1384	4555	4884	4590	8127
4521	3.6552345	4556	3.6585837	4591	9073
4522	3306	4557	6790	4592	3.6620019
4523	4266	4558	7743	4593	0964
4524	5226	4559	8696	4594	1910
4525	6186	4560	9648	4595	2855
4526	3.6557145	4561	3.6590601	4596	3.6623800
4527	8105	4562	1553	4597	4745
4528	9064	4563	2505	4598	5690
4529	3.6560023	4564	3456	4599	6634
4530	0982	4565	4408	4600	7578
4531	3.6561941	4566	3.6595359		
4532	2899	4567	6310		
4533	3857	4568	7261		
4534	4815	4569	8212		
4535	5773	4570	9162		

Num	Logarithms	Num	Logarithms	Num	Logarithms
4601	3.6628522	4636	3.6661434	4671	3.6694099
4602	9466	4637	2371	4672	5028
4603	3.6630410	4638	3307	4673	5958
4604	1353	4639	4244	4674	6887
4605	2296	4640	5180	4675	7816
4606	3.6633239	4641	3.6666116	4676	3.6698745
4607	4182	4642	7051	4677	9674
4608	5125	4643	7987	4678	3.6700602
4609	6067	4644	8922	4679	1530
4610	7009	4645	9857	4680	2459
4611	3.6637951	4646	3.6670792	4681	3.6703386
4612	8893	4647	1727	4682	4314
4613	9835	4648	2661	4683	5242
4614	3.6640776	4649	3.6673595	4684	6169
4615	1717	4650	4530	4685	7096
4616	3.6642658	4651	3.6675463	4686	3.6708023
4617	3599	4652	6397	4687	8950
4618	4539	4653	7331	4688	9876
4619	5480	4654	8264	4689	3.6710802
4620	6420	4655	9197	4690	1728
4621	3.6647360	4656	3.6680130	4691	3.6712654
4622	8299	4657	1062	4692	3580
4623	9239	4658	1995	4693	4506
4624	3.6650178	4659	2927	4694	5431
4625	1117	4660	3859	4695	6356
4626	3.6652056	4661	3.6684791	4696	3.6717281
4627	2995	4662	5723	4697	8206
4628	3933	4663	6654	4698	9130
4629	4872	4664	7585	4699	3.6720054
4630	5810	4665	3.6688516	4700	0979
4631	3.6656748	4666	9447		
4632	7685	4667	3.6690378		
4633	8623	4668	1308		
4634	9560	4669	2239		
4635	3.6660497	4670	3169		

Num	Logarithms	Num	Logarithms	Num	Logarithms
4701	3.6721903	4736	3.6754117	4771	3.6786094
4702	2826	4737	5034	4772	7004
4703	3750	4738	5951	4773	7914
4704	4673	4739	6867	4774	8824
4705	5596	4740	7783	4775	9734
4706	3.6726519	4741	3.6758700	4776	3.6790643
4707	7442	4742	9615	4777	1552
4708	8365	4743	3.6760531	4778	2461
4709	9287	4744	1447	4779	3370
4710	3.6730209	4745	2362	4780	4279
4711	1131	4746	3.6763277	4781	3.6795187
4712	2053	4747	4192	4782	6096
4713	2974	4748	5107	4783	7004
4714	3896	4749	6022	4784	7912
4715	4817	4750	6936	4785	8819
4716	3.6735738	4751	3.6767850	4786	9727
4717	6659	4752	8764	4787	3.6800634
4718	7579	4753	9678	4788	1541
4719	8500	4754	3.6770592	4789	2448
4720	9420	4755	1505	4790	3355
4721	3.6740340	4756	3.6772418	4791	3.6804262
4722	1260	4757	3332	4792	5168
4723	2179	4758	4244	4793	6074
4724	3099	4759	5157	4794	6980
4725	4018	4760	6069	4795	7886
4726	3.6744937	4761	3.6776982	4796	3.6808792
4727	5856	4762	7894	4797	9697
4728	6775	4763	8806	4798	3.6810602
4729	7693	4764	9718	4799	1507
4730	8611	4765	3.6780629	4800	2412
4731	9529	4766	1540		
4732	3.6750447	4767	2452		
4733	1365	4768	3362		
4734	2283	4769	4273		
4735	3200	4770	5184		

Num	Logarithms	Num	Logarithms	Num	Logarithms
4801	3.6813317	4836	3.6844863	4871	3.6876181
4802	4222	4837	5761	4872	7073
4803	5126	4838	6659	4873	7964
4804	6030	4839	7556	4874	8855
4805	6934	4840	8454	4875	9746
4806	3.6817838	4841	9351	4876	3.6880637
4807	8741	4842	3.6850248	4877	1528
4808	9645	4843	1145	4878	2418
4809	3.6820548	4844	2041	4879	3308
4810	1451	4845	2938	4880	4198
4811	3.6822354	4846	3.6853834	4881	3.6885088
4812	3256	4847	4730	4882	5978
4813	4159	4848	5626	4883	6867
4814	5061	4849	6522	4884	7756
4815	5963	4850	7417	4885	8646
4816	3.6826865	4851	3.6858313	4886	9535
4817	7766	4852	9208	4887	3.6890423
4818	8668	4853	3.6860103	4888	1312
4819	9569	4854	0998	4889	2200
4820	3.6830470	4855	1892	4890	3089
4821	1371	4856	3.6862787	4891	3.6893977
4822	2272	4857	3681	4892	4864
4823	3173	4858	4575	4893	5752
4824	4073	4859	5469	4894	6640
4825	4973	4860	6363	4895	7527
4826	3.6835873	4861	3.6867256	4896	3.6898414
4827	6773	4862	8149	4897	9301
4828	7673	4863	9043	4898	3.6900188
4829	8572	4864	9936	4899	1074
4830	9471	4865	3.6870828	4900	1961
4831	3.6840370	4866	1721		
4832	1269	4867	2613		
4833	2168	4868	3506		
4834	3066	4869	4398		
4835	3965	4870	5290		

Num	Logarithms	Num	Logarithms	Num	Logarithms
4901	3.6902847	4936	3.6933752	4971	3.6964438
4902	3733	4937	4631	4972	5311
4903	4619	4938	5511	4973	6185
4904	5505	4939	6390	4974	7058
4905	6390	4940	7269	4975	7931
4906	3.6907275	4941	3.6938148	4976	3.6968804
4907	8161	4942	9027	4977	9676
4908	9046	4943	9906	4978	3.6970549
4909	9930	4944	3.6940785	4979	1421
4910	3.6910815	4945	1663	4980	2293
4911	1699	4946	3.6942541	4981	3.6973165
4912	2584	4947	3419	4982	4037
4913	3468	4948	4297	4983	4909
4914	4352	4949	5174	4984	5780
4915	5235	4950	6052	4985	6652
4916	3.6916119	4951	3.6946929	4986	3.6977523
4917	7002	4952	7806	4987	8394
4918	7885	4953	8683	4988	9264
4919	8768	4954	9560	4989	3.6980135
4920	9651	4955	3.6950437	4990	1005
4921	3.6920534	4956	1313	4991	3.6981876
4922	1416	4957	2189	4992	2746
4923	2298	4958	3065	4993	3616
4924	3180	4959	3941	4994	4485
4925	4062	4960	4817	4995	5355
4926	3.6924944	4961	3.6955692	4996	3.6986224
4927	5826	4962	6568	4997	7093
4928	6707	4963	7443	4998	7963
4929	7588	4964	8318	4999	8831
4930	8469	4965	9193	5000	9700
4931	9350	4966	3.6960067		
4932	3.6930231	4967	0942		
4933	1111	4968	1816		
4934	1991	4969	2690		
4935	2872	4970	3564		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
5001	3.6990569	5036	3.7020857	5071	3.7050936
5002	1437	5037	1719	5072	1792
5003	2305	5038	2582	5073	2649
5004	3173	5039	3444	5074	3505
5005	4041	5040	4305	5075	4360
5006	3.6994908	5041	3.7025167	5076	3.7055216
5007	5776	5042	6028	5077	6072
5008	6643	5043	6890	5078	6927
5009	7510	5044	7751	5079	7782
5010	8377	5045	8612	5080	8637
5011	9244	5046	9472	5081	9492
5012	3.7000111	5047	3.7030333	5082	3.7060347
5013	0977	5048	1193	5083	1201
5014	1843	5049	2054	5084	2055
5015	2709	5050	2914	5085	2910
5016	3.7003575	5051	3.7033774	5086	3.7063764
5017	4441	5052	4633	5087	4617
5018	5307	5053	5493	5088	5471
5019	6172	5054	6352	5089	6324
5020	7037	5055	7212	5090	7178
5021	3.7007902	5056	3.7038071	5091	3.7068031
5022	8767	5057	8929	5092	8884
5023	9632	5058	9788	5093	9737
5024	3.7010496	5059	3.7040647	5094	3.7070589
5025	1361	5060	1505	5095	1442
5026	3.7012225	5061	3.7042363	5096	3.7072294
5027	3089	5062	3221	5097	3146
5028	3953	5063	4079	5098	3998
5029	4816	5064	4937	5099	4850
5030	5680	5065	5794	5100	5702
5031	3.7016543	5066	3.7046652		
5032	7406	5067	7509		
5033	8269	5068	8366		
5034	9132	5069	9223		
5035	9995	5070	3.7050080		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
5101	3.7076553	5136	3.7106250	5171	3.7135745
5102	7405	5137	7096	5172	6585
5103	8256	5138	7941	5173	7425
5104	9107	5139	8786	5174	8264
5105	9957	5140	9631	5175	9104
5106	3.7080808	5141	3.7110476	5176	9943
5107	1659	5142	1321	5177	3.7140782
5108	2509	5143	2165	5178	1620
5109	3559	5144	3010	5179	2459
5110	4209	5145	3854	5180	3298
5111	3.7085059	5146	3.7114698	5181	3.7144136
5112	5908	5147	5542	5182	4974
5113	6758	5148	6385	5183	5812
5114	7607	5149	7229	5184	6650
5115	8456	5150	8072	5185	7488
5116	9305	5151	3.7118915	5186	3.7148325
5117	3.7090154	5152	9759	5187	9162
5118	1003	5153	3.7120601	5188	3.7150000
5119	1851	5154	1444	5189	0837
5120	2700	5155	2287	5190	1674
5121	3.7093548	5156	3.7123129	5191	3.7152510
5122	4396	5157	3971	5192	3347
5123	5244	5158	4813	5193	4183
5124	6091	5159	5655	5194	5019
5125	6939	5160	6497	5195	5856
5126	3.7097786	5161	3.7127339	5196	3.7156691
5127	8633	5162	8180	5197	7527
5128	9480	5163	9021	5198	8363
5129	3.7100327	5164	9862	5199	9198
5130	1174	5165	3.7130703	5200	3.7160033
5131	3.7102020	5166	1544		
5132	2866	5167	2385		
5133	3713	5168	3225		
5134	4559	5169	4065		
5135	5404	5170	4905		

Num	Logarithms	Num	Logarithms	Num	Logarithms
5201	3·7160869	5236	3·7189996	5271	3·7218930
5202	1703	5237	3·7190826	5272	9754
5203	2538	5238	1655	5273	3·7220578
5204	3373	5239	2484	5274	1401
5205	4207	5240	3313	5275	2225
5206	3·7165042	5241	3·7194142	5276	3·7223048
5207	5876	5242	4970	5277	3871
5208	6710	5243	5799	5278	4694
5209	7544	5244	6627	5279	5517
5210	8377	5245	7455	5280	6339
5211	9211	5246	3·7198283	5281	3·7227162
5212	3·7170044	5247	9111	5282	7984
5213	0877	5248	9938	5283	8806
5214	1710	5249	3·7200766	5284	9628
5215	2543	5250	1593	5285	3·7230450
5216	3·7173376	5251	3·7202420	5286	1272
5217	4208	5252	3247	5287	2093
5218	5041	5253	4074	5288	2914
5219	5873	5254	4901	5289	3736
5220	6705	5255	5727	5290	4557
5221	3·7177537	5256	3·7206554	5291	3·7235378
5222	8369	5257	7380	5292	6198
5223	9200	5258	8206	5293	7019
5224	3·7180032	5259	9032	5294	7839
5225	0863	5260	9857	5295	8660
5226	3·7181694	5261	3·7210683	5296	9480
5227	2525	5262	1508	5297	3·7240300
5228	3356	5263	2334	5298	1120
5229	4186	5264	3159	5299	1939
5230	5017	5265	3984	5300	2759
5231	3·7185847	5266	3·7214809		
5232	6677	5267	5633		
5233	7507	5268	6458		
5234	8337	5269	7282		
5235	9167	5270	8106		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
5301	3.7243578	5336	3.7272158	5371	3.7300551
5302	4397	5337	2972	5372	1360
5303	5216	5338	3786	5373	2168
5304	6035	5339	4599	5374	2977
5305	6854	5340	5413	5375	3785
5306	3.7247672	5341	3.7276226	5376	3.7304593
5307	8491	5342	7039	5377	5400
5308	9309	5343	7852	5378	6208
5309	3.7250127	5344	8664	5379	7015
5310	0945	5345	9477	5380	7823
5311	3.7251763	5346	3.7280290	5381	3.7308630
5312	2581	5347	1102	5382	9437
5313	3398	5348	1914	5383	3.7310244
5314	4215	5349	2726	5384	1051
5315	5033	5350	3538	5385	1857
5316	3.7255850	5351	3.7284349	5386	3.7312663
5317	6667	5352	5161	5387	3470
5318	7483	5353	5972	5388	4276
5319	8300	5354	6784	5389	5082
5320	9116	5355	7595	5390	5888
5321	9933	5356	3.7288406	5391	3.7316693
5322	3.7260749	5357	9216	5392	7499
5323	1565	5358	3.7290027	5393	8304
5324	2380	5359	0838	5394	9109
5325	3196	5360	1648	5395	9914
5326	3.7264012	5361	3.7292458	5396	3.7320719
5327	4827	5362	3268	5397	1524
5328	5642	5363	4078	5398	2329
5329	6457	5364	4888	5399	3133
5330	7272	5365	5697	5400	3938
3331	3.7268087	5366	3.7296507		
3332	8901	5367	7316		
3333	9716	5368	8125		
3534	3.7270530	5369	8934		
5335	1344	5370	9743		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
5401	3.7324742	5436	3.7352794	5471	3.7380667
5402	5546	5437	3593	5472	1461
5403	6350	5438	4392	5473	2254
5404	7153	5439	5191	5474	3048
5405	7957	5440	5989	5475	3841
5406	3.7328760	5441	3.7356787	5476	3.7384634
5407	9564	5442	7585	5477	5427
5408	3.7330367	5443	8383	5478	6220
5409	1170	5444	9181	5479	7013
5410	1973	5445	9979	5480	7806
5411	3.7332775	5446	3.7360776	5481	3.7388598
5412	3578	5447	1574	5482	9390
5413	4380	5448	2371	5483	3.7390182
5414	5182	5449	3168	5484	0974
5415	5985	5450	3965	5485	1766
5416	3.7336787	5451	3.7364762	5486	3.7392558
5417	7588	5452	5558	5487	3350
5418	8390	5453	6355	5488	4141
5419	9191	5454	7151	5489	4932
5420	9993	5455	7948	5490	5723
5421	3.7340794	5456	3.7368744	5491	3.7396514
5422	1595	5457	9540	5492	7305
5423	2396	5458	3.7370335	5493	8069
5424	3197	5459	1131	5494	8886
5425	3997	5460	1926	5495	9677
5426	3.7344798	5461	3.7372722	5496	3.7400467
5427	5598	5462	3517	5497	1257
5428	6398	5463	4312	5498	2047
5429	7198	5464	5107	5499	2837
5430	7998	5465	5902	5500	3627
5431	3.7348798	5466	3.7376696		
5432	9598	5467	7491		
5433	3.7350397	5468	8285		
5434	1196	5469	9079		
5435	1995	5470	9873		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
5501	3·7404416	5536	3·7431961	5571	3·7459332
5502	5206	5537	2745	5572	3·7460111
5503	5995	5538	3530	5573	0890
5504	6784	5539	4314	5574	1670
5505	7573	5540	5098	5575	2449
5506	3·7408362	5541	3·7435881	5576	3·7463228
5507	9151	5542	6665	5577	4006
5508	9939	5543	7449	5578	4785
5509	3·7410728	5544	8232	5579	5564
5510	1516	5545	9015	5580	6342
5511	3·7412304	5546	9799	5581	3·7467120
5512	3092	5547	3·7440582	5582	7898
5513	3880	5548	1365	5583	8676
5514	4668	5549	2147	5584	9454
5515	5455	5550	2930	5585	3·7470232
5516	3·7416243	5551	3·7443712	5586	1009
5517	7030	5552	4495	5587	1787
5518	7817	5553	5277	5588	2564
5519	8604	5554	6059	5589	3341
5520	9391	5555	6841	5590	4118
5521	3·7420177	5556	3·7447622	5591	3·7474895
5522	0964	5557	8404	5592	5672
5523	1750	5558	9185	5593	6448
5524	2537	5559	9967	5594	7225
5525	3323	5560	3·7450748	5595	8001
5526	3·7424109	5561	1529	5596	3·7478777
5527	4895	5562	2310	5597	9553
5528	5680	5563	3091	5598	3·7480329
5529	6466	5564	3871	5599	1105
5530	7251	5565	4652	5600	1880
5531	3·7428037	5566	3·7455432		
5532	8822	5567	6212		
5533	9607	5568	6992		
5534	3·7430392	5569	7772		
5535	1176	5570	8552		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
5601	3·7482656	5636	3·7509710	5671	3·7536596
5602	3431	5637	3·7510480	5672	7362
5603	4206	5638	1251	5673	8128
5604	4981	5639	2021	5674	8893
5605	5756	5640	2791	5675	9659
5606	3·7486531	5641	3·7513561	5676	3·7540424
5607	7306	5642	4331	5677	1189
5608	8080	5643	5100	5678	1954
5609	8854	5644	5870	5679	2719
5610	9629	5645	6639	5680	3483
5611	3·7490403	5646	3·7517409	5681	3·7544248
5612	1177	5647	8178	5682	5012
5613	1950	5648	8947	5683	5777
5614	2724	5649	9716	5684	6541
5615	3498	5650	3·7520484	5685	7305
5616	3·7494271	5651	1253	5686	3·7548069
5617	5044	5652	2022	5687	8832
5618	5817	5653	2790	5688	9596
5619	6590	5654	3558	5689	3·7550359
5620	7363	5655	4326	5690	1123
5621	3·7498136	5656	3·7525094	5691	3·7551886
5622	8908	5657	5862	5692	2649
5623	9681	5658	6629	5693	3412
5624	3·7500453	5659	7397	5694	4175
5625	1225	5660	8164	5695	4937
5626	3·7501997	5661	3·7528932	5696	3·7555700
5627	2769	5662	9699	5697	6462
5628	3541	5663	3·7530466	5698	7224
5629	4312	5664	1232	5699	7987
5630	5084	5665	1999	5700	8749
5631	3·7505855	5666	3·7532766		
5632	6626	5667	3532		
5633	7398	5668	4298		
5634	8168	5669	5065		
5635	8939	5670	5831		

Num	Logarithms	Num	Logarithms	Num	Logarithms
5701	3·7559510	5736	3·7586091	5771	3·7612511
5702	3·7560272	5737	6848	5772	3263
5703	1034	5738	7605	5773	4016
5704	1795	5739	8362	5774	4768
5705	2556	5740	9119	5775	5520
5706	3·7563318	5741	9875	5776	3·7616272
5707	4079	5742	3·7590632	5777	7024
5708	4840	5743	1388	5778	7775
5709	5600	5744	2144	5779	8527
5710	6361	5745	2900	5780	9278
5711	3·7567122	5746	3·7593656	5781	3·7620030
5712	7882	5747	4412	5782	0781
5713	8642	5748	5168	5783	1532
5714	9402	5749	5923	5784	2283
5715	3·7570162	5750	6678	5785	3034
5716	0922	5751	3·7597434	5786	3·7623784
5717	1682	5752	8189	5787	4535
5718	2441	5753	8944	5788	5285
5719	3201	5754	9699	5789	6035
5720	3960	5755	3·7600453	5790	6786
5721	3·7574719	5756	1208	5791	3·7627536
5722	5479	5757	1962	5792	8286
5723	6237	5758	2717	5793	9035
5724	6996	5759	3471	5794	9785
5725	7755	5760	4225	5795	3·7630534
5726	3·7578513	5761	3·7604979	5796	1284
5727	9272	5762	5733	5797	2033
5728	3·7580030	5763	6486	5798	2782
5729	0788	5764	7240	5799	3531
5730	1546	5765	7993	5800	4280
5731	3·7582304	5766	3·7608746		
5732	3062	5767	9500		
5733	3819	5768	3·7610253		
5734	4577	5769	1005		
5735	5334	5770	1758		

Num	Logarithms	Num	Logarithms	Num	Logarithms
5801	3.7635029	5836	3.7661153	5871	3.7687121
5802	5777	5837	1897	5872	7860
5803	6526	5838	2641	5873	8600
5804	7274	5839	3385	5874	9339
5805	8022	5840	4128	5875	3.7690079
5806	3.7638770	5841	3.7664872	5876	0818
5807	9518	5842	5616	5877	1557
5808	3.7640266	5843	6359	5878	2296
5809	1014	5844	7102	5879	3035
5810	1761	5845	7845	5880	3773
5811	3.7642509	5846	3.7668588	5881	3.7694512
5812	3256	5847	9331	5882	5250
5813	4003	5848	3.7670074	5883	5988
5814	4750	5849	0816	5884	6727
5815	5497	5850	1559	5885	7465
5816	3.7646244	5851	3.7672301	5886	3.7698203
5817	6991	5852	3043	5887	8940
5818	7737	5853	3785	5888	9678
5819	8484	5854	4527	5889	3.7700416
5820	9230	5855	5269	5890	1153
5821	9976	5856	3.7676011	5891	3.7701890
5822	3.7650722	5857	6752	5892	2627
5823	1468	5858	7494	5893	3364
5824	2214	5859	8235	5894	4101
5825	2959	5860	8976	5895	4838
5826	3.7653705	5861	9717	5896	3.7705575
5827	4450	5862	3.7680458	5897	6311
5828	5195	5863	1199	5898	7048
5829	5941	5864	1940	5899	7784
5830	6686	5865	2680	5900	8520
5831	3.7657430	5866	3.7683421		
5832	8175	5867	4161		
5833	8920	5868	4901		
5834	9664	5869	5641		
5835	3.7660409	5870	6381		

Num	Logarithms	Num	Logarithms	Num	Logarithms
5901	3·7709256	5936	3·7734939	5971	3·7760471
5902	9992	5937	5670	5972	1198
5903	3·7710728	5938	6402	5973	1925
5904	1463	5939	7133	5974	2652
5905	2199	5940	7864	5975	3379
5906	3·7712934	5941	3·7738596	5976	3·7764106
5907	3670	5942	9326	5977	4833
5908	4405	5943	3·7740057	5978	5559
5909	5140	5944	0788	5979	6286
5910	5875	5945	1519	5980	7012
5911	3·7716610	5946	3·7742249	5981	3·7767738
5912	7344	5947	2979	5982	8464
5913	8079	5948	3710	5983	9190
5914	8813	5949	4440	5984	9916
5915	9547	5950	5170	5985	3·7770642
5916	3·7720282	5951	3·7745899	5986	1367
5917	1016	5952	6629	5987	2093
5918	1750	5953	7359	5988	2818
5919	2483	5954	8088	5989	3543
5920	3217	5955	8818	5990	4268
5921	3·7723951	5956	9547	5991	3·7774993
5922	4684	5957	3·7750276	5992	5718
5923	5417	5958	1005	5993	6443
5924	6150	5959	1734	5994	7167
5925	6884	5960	2463	5995	7892
5926	3·7727616	5961	3·7753191	5996	3·7778616
5927	8349	5962	3920	5997	9340
5928	9082	5963	4648	5998	3·7780065
5929	9814	5964	5376	5999	0789
5930	3·7730547	5965	6104	6000	1512
5931	1279	5966	3·7756832		
5932	2011	5967	7560		
5933	2743	5968	8288		
5934	3475	5969	9016		
5935	4207	5970	9743		

Num	Logarithms	Num	Logarithms	Num	Logarithms
6001	3.7782236	6036	3.7807492	6071	3.7832602
6002	2960	6037	8212	6072	3318
6003	3683	6038	8931	6073	4033
6004	4407	6039	9650	6074	4748
6005	5130	6040	3.7810369	6075	5463
6006	3.7785853	6041	1088	6076	3.7836178
6007	6576	6042	1807	6077	6892
6008	7299	6043	2526	6078	7607
6009	8022	6044	3245	6079	8321
6010	8745	6045	3963	6080	9036
6011	3.7789467	6046	3.7814681	6081	9750
6012	3.7790190	6047	5400	6082	3.7840464
6013	0912	6048	6118	6083	1178
6014	1634	6049	6836	6084	1892
6015	2356	6050	7554	6085	2606
6016	3.7793078	6051	3.7818272	6086	3.7843319
6017	3800	6052	8989	6087	4033
6018	4522	6053	9707	6088	4746
6019	5243	6054	3.7820424	6089	5460
6020	5965	6055	1141	6090	6173
6021	3.7796686	6056	3.7821859	6091	3.7846886
6022	7408	6057	2576	6092	7599
6023	8129	6058	3293	6093	8312
6024	8850	6059	4010	6094	9024
6025	9571	6060	4726	6095	9737
6026	3.7800291	6061	3.7825443	6096	3.7850450
6027	1012	6062	6159	6097	1162
6028	1732	6063	6876	6098	1874
6029	2453	6064	7592	6099	2586
6030	3173	6065	8308	6100	3298
6031	3.7803893	6066	3.7829024		
6032	4613	6067	9740		
6033	5333	6068	3.7830456		
6034	6053	6069	1171		
6035	6773	6070	1887		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6101	3.7854010	6136	3.7878853	6171	3.7903555
6102	4722	6137	9561	6172	4259
6103	5434	6138	3.7880269	6173	4963
6104	6145	6139	0976	6174	5666
6105	6857	6140	1684	6175	6370
6106	3.7857568	6141	3.7882391	6176	3.7907073
6107	8279	6142	3098	6177	7776
6108	8990	6143	3805	6178	8479
6109	9701	6144	4512	6179	9182
6110	3.7860412	6145	5219	6180	9885
6111	1123	6146	3.7885926	6181	3.7910587
6112	1833	6147	6632	6182	1290
6113	2544	6148	7339	6183	1992
6114	3254	6149	8045	6184	2695
6115	3965	6150	8751	6185	3397
6116	3.7864675	6151	9457	6186	3.7914099
6117	5385	6152	3.7890163	6187	4801
6118	6095	6153	0869	6188	5503
6119	6805	6154	1575	6189	6205
6120	7514	6155	2281	6190	6906
6121	3.7868224	6156	3.7892986	6191	3.7917608
6122	8933	6157	3691	6192	8309
6123	9643	6158	4397	6193	9011
6124	3.7870352	6159	5102	6194	9712
6125	1061	6160	5807	6195	3.7920413
6126	3.7871770	6161	3.7896512	6196	1114
6127	2479	6162	7217	6197	1815
6128	3188	6163	7922	6198	2516
6129	3896	6164	8626	6199	3216
6130	4605	6165	9331	6200	3917
6131	3.7875313	6166	3.7900035		
6132	6021	6167	0739		
6133	6730	6168	1444		
6134	7438	6169	2148		
6135	8146	6170	2852		

Num	Logarithms	Num	Logarithms	Num	Logarithms
6201	3.7924617	6236	3.7949061	6271	3.7973368
6202	5318	6237	9757	6272	4060
6203	6018	6238	3.7950454	6273	4753
6204	6718	6239	1150	6274	5445
6205	7418	6240	1846	6275	6137
6206	3.7928118	6241	3.7952542	6276	3.7976829
6207	8817	6242	3238	6277	7521
6208	9517	6243	3933	6278	8213
6209	3.7930217	6244	4629	6279	8905
6210	0916	6245	5324	6280	9596
6211	3.7931615	6246	3.7956020	6281	3.7980288
6212	2314	6247	6715	6282	0979
6213	3014	6248	7410	6283	1671
6214	3712	6249	8105	6284	2362
6215	4411	6250	8800	6285	3053
6216	3.7935110	6251	9495	6286	3.7983744
6217	5809	6252	3.7960190	6287	4435
6218	6507	6253	0884	6288	5125
6219	7206	6254	1579	6289	5816
6220	7904	6255	2273	6290	6506
6221	3.7938602	6256	3.7962967	6291	3.7987197
6222	9300	6257	3662	6292	7887
6223	9998	6258	4356	6293	8577
6224	3.7940696	6259	5050	6294	9267
6225	1394	6260	5743	6295	9957
6226	3.7942091	6261	3.7966437	6296	3.7990647
6227	2789	6262	7131	6297	1337
6228	3486	6263	7824	6298	2027
6229	4183	6264	8517	6299	2716
6230	4880	6265	9211	6300	3405
6231	3.7945578	6266	9904		
6232	6274	6267	3.7970597		
6233	6971	6268	1290		
6234	7668	6269	1983		
6235	8365	6270	2675		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6301	3.7994095	6336	3.8018152	6371	3.8042076
6302	4784	6337	8837	6372	2758
6303	5473	6338	9522	6373	3439
6304	6162	6339	3.8020208	6374	4121
6305	6851	6340	0893	6375	4802
6306	3.7997540	6341	3.8021578	6376	3.8045483
6307	8228	6342	2262	6377	6164
6308	8917	6343	2947	6378	6845
6309	9605	6344	3632	6379	7526
6310	3.8000294	6345	4316	6380	8207
6311	0982	6346	3.8025001	6381	3.8048887
6312	1670	6347	5685	6382	9568
6313	2358	6348	6369	6383	3.8050248
6314	3046	6349	7053	6384	0929
6315	3734	6350	7737	6385	1609
6316	3.8004421	6351	3.8028421	6386	3.8052289
6317	5109	6352	9105	6387	2969
6318	5796	6353	9789	6388	3649
6319	6484	6354	3.8030472	6389	4329
6320	7171	6355	1156	6390	5009
6321	3.8007858	6356	3.8031839	6391	3.8055688
6322	8545	6357	2522	6392	6368
6323	9232	6358	3205	6393	7047
6324	9919	6359	3888	6394	7726
6325	3.8010605	6360	4571	6395	8405
6326	1292	6361	3.8035254	6396	3.8059085
6327	1978	6362	5937	6397	9763
6328	2665	6363	6619	6398	3.8060442
6329	3351	6364	7302	6399	1121
6330	4037	6365	7984	6400	1800
6331	3.8014723	6366	3.8038666		
6332	5409	6367	9348		
6333	6095	6368	3.8040031		
6334	6781	6369	0712		
6335	7466	6370	1394		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6401	3.8062478	6436	3.8086160	6471	3.8109714
6402	3157	6437	6835	6472	3.8110385
6403	3835	6438	7510	6473	1056
6404	4513	6439	8184	6474	1727
6405	5191	6440	8859	6475	2398
6406	3.8065869	6441	9533	6476	3.8113068
6407	6547	6442	3.8090207	6477	3739
6408	7225	6443	0881	6478	4409
6409	7903	6444	1555	6479	5080
6410	8580	6445	2229	6480	5750
6411	3.8069258	6446	3.8092903	6481	3.8116420
6412	9935	6447	3577	6482	7090
6413	3.8070612	6448	4250	6483	7760
6414	1290	6449	4924	6484	8430
6415	1967	6450	5597	6485	9100
6416	3.8072644	6451	3.8096270	6486	9769
6417	3320	6452	6944	6487	3.8120439
6418	3997	6453	7617	6488	1108
6419	4674	6454	8290	6489	1778
6420	5350	6455	8962	6490	2447
6421	3.8076027	6456	9635	6491	3.8123116
6422	6703	6457	3.8100308	6492	3785
6423	7379	6458	0980	6493	4454
6424	8055	6459	1653	6494	5123
6425	8731	6460	2325	6495	5792
6426	9407	6461	3.8102997	6496	6460
6427	3.8080083	6462	3670	6497	3.8127129
6428	0759	6463	4342	6498	7797
6429	1434	6464	5013	6499	8465
6430	2110	6465	5685	6500	9134
6431	3.8082785	6466	3.8106357		
6432	3460	6467	7029		
6433	4136	6468	7700		
6434	4811	6469	8371		
6435	5485	6470	9043		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6501	3.8129802	6536	3.8153120	6571	3.8176315
6502	3.8130470	6537	3785	6572	6976
6503	1138	6538	4449	6573	7636
6504	1805	6539	5113	6574	8297
6505	2473	6540	5777	6575	8958
6506	3.8133141	6541	3.8156441	6576	9618
6507	3808	6542	7105	6577	3.8180278
6508	4475	6543	7769	6578	0939
6509	5143	6544	8433	6579	1599
6510	5810	6545	9096	6580	2259
6511	3.8136477	6546	9760	6581	3.8182919
6512	7144	6547	3.8160423	6582	3579
6513	7811	6548	1087	6583	4239
6514	8478	6549	1750	6584	4898
6515	9144	6550	2413	6585	5558
6516	9811	6551	3.8163076	6586	3.8186217
6517	3.8140477	6552	3739	6587	6877
6518	1144	6553	4402	6588	7536
6519	1810	6554	5064	6589	8195
6520	2476	6555	5727	6590	8854
6521	3.8143142	6556	3.8166389	6591	9513
6522	3808	6557	7052	6592	3.8190172
6523	4474	6558	7714	6593	0831
6524	5140	6559	8376	6594	1489
6525	5805	6560	9038	6595	2148
6526	3.8146471	6561	9700	6596	3.8192806
6527	7136	6562	3.8170362	6597	3465
6528	7801	6563	1024	6598	4123
6529	8467	6564	1686	6599	4781
6530	9132	6565	2347	6600	5439
6531	9797	6566	3.8173009		
6532	3.8150462	6567	3670		
6533	1127	6568	4331		
6534	1791	6569	4993		
6535	2456	6570	5654		

Num	Logarithms	Num	Logarithms	Num	Logarithms
6601	3.8196097	6636	3.8219064	6671	3.8241909
6602	6755	6637	9718	6672	2560
6603	7413	6638	3.8220372	6673	3211
6604	8071	6639	1027	6674	3862
6605	8728	6640	1681	6675	4513
6606	9386	6641	3.8222335	6676	3.8245163
6607	3.8200043	6642	2989	6677	5814
6608	0700	6643	3643	6678	6464
6609	1358	6644	4296	6679	7114
6610	2015	6645	4950	6680	7765
6611	3.8202672	6646	3.8225603	6681	3.8248415
6612	3328	6647	6257	6682	9065
6613	3985	6648	6910	6683	9715
6614	4642	6649	7563	6684	3.8250364
6615	5298	6650	8216	6685	1014
6616	3.8205955	6651	3.8228869	6686	3.8251664
6617	6611	6652	9522	6687	2313
6618	7268	6653	3.8230175	6688	2963
6619	7924	6654	0828	6689	3612
6620	8580	6655	1481	6690	4261
6621	3.8209236	6656	3.8232133	6691	3.8254910
6622	9892	6657	2786	6692	5559
6623	3.8210548	6658	3438	6993	6208
6624	1203	6659	4090	6694	6857
6625	1859	6660	4742	6695	7506
6626	3.8212514	6661	3.8235394	6696	3.8258154
6627	3170	6662	6046	6697	8803
6628	3825	6663	6698	6698	9451
6629	4480	6664	7350	6699	3.8260100
6630	5135	6665	8002	6700	0748
6631	3.8215790	6666	3.8238653		
6632	6445	6667	9305		
6633	7100	6668	9956		
6634	7755	6669	3.8240607		
6635	8409	6670	1258		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6701	3.8261396	6736	3.8284021	6771	3.8306528
6702	2044	6737	4665	6772	7169
6703	2692	6738	5310	6773	7811
6704	3340	6739	5955	6774	8452
6705	3988	6740	6599	6775	9093
6706	3.8264635	6741	3.8287243	6776	9734
6707	5283	6742	7887	6777	3.8310375
6708	5931	6743	8532	7778	1016
6709	6578	6744	9176	6779	1656
6710	7225	6745	9820	6780	2297
6711	3.8267872	6746	3.8290463	6781	3.8312937
6712	8519	6747	1107	6782	3578
6713	9166	6748	1751	6783	4218
6714	9813	6749	2394	6784	4858
6715	3.8270460	6750	3038	6785	5499
6716	1107	6751	3.8293681	6786	3.8316139
6717	1753	6752	4324	6787	6778
6718	2400	6753	4967	6788	7418
6719	3046	6754	5611	6789	8058
6720	3693	6755	6254	6790	8698
6721	3.8274339	6756	3.8296896	6791	3.8319337
6722	4985	6757	7539	6792	9977
6723	5631	6758	8182	6793	3.8320616
6724	6277	6759	8824	6794	1255
6725	6923	6760	9467	6795	1895
6726	3.8277569	6761	3.8300109	6796	3.8322534
6727	8214	6762	0752	6797	3173
6728	8860	6763	1394	6798	3812
6729	9505	6764	2036	6799	4450
6730	3.8280151	6765	2678	6800	5089
6731	0796	6766	3.8303320		
6732	1441	6767	3962		
6733	2086	6768	4603		
6734	2731	6769	5245		
6735	3376	6770	5887		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6801	3.8325728	6836	3.8348021	6871	3.8370199
6802	6366	6837	8656	6872	0832
6803	7005	6838	9291	6873	1463
6804	7643	6839	9926	6874	2095
6805	8281	6840	3.8350561	6875	2727
6806	3.8328919	6841	1196	6876	3.8373359
6807	9558	6842	1831	6877	3990
6808	3.8330195	6843	2465	6878	4622
6809	0833	6844	3100	6879	5253
6810	1471	6845	3735	6880	5884
6811	3.8332109	6846	3.8354369	6881	3.8376516
6812	2746	6847	5003	6882	7147
6813	3384	6848	5638	6883	7778
6814	4021	6849	6272	6884	8409
6815	4659	6850	6906	6885	9039
6816	3.8335296	6851	3.8357540	6886	9670
6817	5933	6852	8174	6887	3.8380301
6818	6570	6853	8807	6888	0931
6819	7207	6854	9441	6889	1562
6820	7844	6855	3.8360075	6890	2192
6821	3.8338480	6856	0708	6891	3.8382822
6822	9117	6857	1341	6892	3453
6823	9754	6858	1975	6893	4083
6824	3.8340390	6859	2608	6894	4713
6825	1027	6860	3241	6895	5343
6826	3.8341663	6861	3.8363874	6896	3.8385973
6827	2299	6862	4507	6897	6602
6828	2935	6863	5140	6898	7232
6829	3571	6864	5773	6899	7861
6830	4207	6865	6405	6900	8491
6831	3.8344843	6866	3.8367038		
6832	5479	6867	7670		
6833	6114	6868	8303		
6834	6750	6869	8935		
6835	7385	6870	9567		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
6901	3.8389120	6936	3.8411091	6971	3.8432951
6902	9750	6937	1717	6972	3574
6903	3.8390379	6938	2343	6973	4197
6904	1008	6939	2969	6974	4819
6905	1637	6940	3595	6975	5442
6906	3.8392266	6941	3.8414220	6976	3.8436065
6907	2895	6942	4846	6977	6687
6908	3523	6943	5472	6978	7310
6909	4152	6944	6097	6979	7932
6910	4780	6945	6722	6980	8554
6911	3.8395409	6946	3.8417348	6981	3.8439176
6912	6037	6947	7973	6982	9798
6913	6666	6948	8598	6983	3.8440420
6914	7294	6949	9223	6984	1042
6915	7922	6950	9848	6985	1664
6916	3.8398550	6951	3.8420473	6986	3.8442286
6917	9178	6952	1098	6987	2907
6918	9806	6953	1722	6988	3529
6919	3.8400433	6954	2347	6989	4150
6920	1061	6955	2971	6990	4772
6921	3.8401688	6956	3.8423596	6991	3.8445393
6922	2316	6957	4220	6992	6014
6923	2943	6958	4844	6993	6635
6924	3571	6959	5468	6994	7256
6925	4198	6960	6092	6995	7877
6926	3.8404825	6961	3.8426716	6996	3.8448498
6927	5452	6962	7340	6997	9119
6928	6079	6963	7964	6998	9739
6929	6706	6964	8588	6999	3.8450360
6930	7332	6965	9211	7000	0980
6931	3.8407959	6966	9835		
6932	8586	6967	3.8430458		
6933	9212	6968	1081		
6934	9838	6969	1705		
6935	3.8410465	6970	2328		

Num	Logarithms	Num	Logarithms	Num	Logarithms
7001	3.8451601	7036	3.8473258	7071	3.8494808
7002	2221	7037	3876	7072	5423
7003	2841	7038	4493	7073	6037
7004	3461	7039	5110	7074	6651
7005	4081	7040	5727	7075	7264
7006	3.8454701	7041	3.8476343	7076	3.8497878
7007	5321	7042	6960	7077	8492
7008	5941	7043	7577	7078	9106
7009	6561	7044	8193	7079	9719
7010	7180	7045	8810	7080	3.8500333
7011	3.8457800	7046	9426	7081	0946
7012	8419	7047	3.8480043	7082	1559
7013	9038	7048	0659	7083	2172
7014	9658	7049	1275	7084	2786
7015	3.8460277	7050	1891	7085	3399
7016	0896	7051	3.8482507	7086	3.8504011
7017	1515	7052	3123	7087	4624
7018	2134	7053	3739	7088	5237
7019	2752	7054	4355	7089	5850
7020	3371	7055	4970	7090	6462
7021	3.8463990	7056	3.8485586	7091	3.8507075
7022	4608	7057	6201	7092	7687
7023	5227	7058	6817	7093	8300
7024	5845	7059	7432	7094	8912
7025	6463	7060	8047	7095	9524
7026	3.8467081	7061	3.8488662	7096	3.8510136
7027	7700	7062	9277	7097	0748
7028	8318	7063	9892	7098	1360
7029	8935	7064	3.8490507	7099	1972
7030	9553	7065	1122	7100	2583
7031	3.8470171	7066	3.8491736		
7032	0789	7067	2351		
7033	1406	7068	2965		
7034	2024	7069	3580		
7035	2641	7070	4194		

Num	Logarithms	Num	Logarithms	Num	Logarithms
7101	3.8513195	7136	3.8534548	7171	3.8555797
7102	3807	7137	5157	7172	6403
7103	4418	7138	5765	7173	7008
7104	5030	7139	6374	7174	7614
7105	5641	7140	6982	7175	8219
7106	3.8516252	7141	3.8537590	7176	3.8558824
7107	6863	7142	8198	7177	9429
7108	7474	7143	8806	7178	3.8560035
7109	8085	7144	9414	7179	0640
7110	8696	7145	3.8540022	7180	1244
7111	3.8519307	7146	0630	7181	3.8561849
7112	9917	7147	1238	7182	2454
7113	3.8520528	7148	1845	7183	3059
7114	1139	7149	2453	7184	3663
7115	1749	7150	3060	7185	4268
7116	3.8522359	7151	3.8543668	7186	3.8564872
7117	2970	7152	4275	7187	5476
7118	3580	7153	4882	7188	6081
7119	4190	7154	5489	7189	6685
7120	4800	7155	6096	7190	7289
7121	3.8525410	7156	3.8546703	7191	3.8567893
7122	6020	7157	7310	7192	8497
7123	6629	7158	7917	7193	9101
7124	7239	7159	8524	7194	9704
7125	7849	7160	9130	7195	3.8570308
7126	3.8528458	7161	9737	7196	0912
7127	9068	7162	3.8550343	7197	1515
7128	9677	7163	0949	7198	2118
7129	3.8530286	7164	1556	7199	2722
7130	0895	7165	2162	7200	3325
7131	3.8531504	7166	3.8552768		
7132	2113	7167	3374		
7133	2722	7168	3980		
7134	3331	7169	4586		
7135	3940	7170	5192		

Num	Logarithms	Num	Logarithms	Num	Logarithms
7201	3.8573928	7236	3.8594986	7271	3.8615941
7202	4531	7237	5586	7272	6539
7203	5134	7238	6186	7273	7136
7204	5737	7239	6786	7274	7733
7205	6340	7240	7386	7275	8330
7206	3.8576943	7241	3.8597985	7276	3.8618927
7207	7545	7242	8585	7277	9524
7208	8148	7243	9185	7278	3.8620120
7209	8750	7244	9784	7279	0717
7210	9353	7245	3.8600384	7280	1314
7211	9955	7246	0983	7281	3.8621910
7212	3.8580557	7247	1583	7282	2507
7213	1159	7248	2182	7283	3103
7214	1761	7249	2781	7284	3699
7215	2363	7250	3380	7285	4296
7216	3.8582965	7251	3.8603979	7286	3.8624892
7217	3567	7252	4578	7287	5488
7218	4169	7253	5177	7288	6084
7219	4770	7254	5776	7289	6679
7220	5372	7255	6374	7290	7275
7221	3.8585973	7256	3.8606973	7291	3.8627871
7222	6575	7257	7571	7292	8467
7223	7176	7258	8170	7293	9062
7224	7777	7259	8768	7294	9658
7225	8379	7260	9366	7295	3.8630253
7226	3.8588980	7261	9964	7296	0848
7227	9581	7262	3.8610562	7297	1443
7228	3.8590181	7263	1160	7298	2039
7229	0782	7264	1758	7299	2634
7230	1383	7265	2356	7300	3229
7231	3.8591984	7266	3.8612954		
7232	2584	7267	3552		
7233	3185	7268	4149		
7234	3785	7269	4747		
7235	4385	7270	5344		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
7301	3.8633823	7336	3.8654593	7371	3.8675264
7302	4418	7337	5185	7372	5853
7303	5013	7338	5777	7373	6442
7304	5608	7339	6369	7374	7031
7305	6202	7340	6961	7375	7620
7306	3.8636797	7341	3.8657552	7376	3.8678209
7307	7391	7342	8144	7377	8798
7308	7985	7343	8735	7378	9387
7309	8580	7344	9327	7379	9975
7310	9174	7345	9918	7380	3.8680564
7311	9768	7346	3.8660509	7381	1152
7312	3.8640362	7347	1100	7382	1740
7313	0956	7348	1691	7383	2329
7314	1550	7349	2282	7384	2917
7315	2143	7350	2873	7385	3505
7316	3.8642737	7351	3.8663464	7386	3.8684093
7317	3331	7352	4055	7387	4681
7318	3924	7353	4646	7388	5269
7319	4517	7354	5236	7389	5857
7320	5111	7355	5827	7390	6444
7321	3.8645704	7356	3.8666417	7391	3.8687032
7322	6297	7357	7008	7392	7620
7323	6890	7358	7598	7393	8207
7324	7483	7359	8188	7394	8794
7325	8076	7360	8778	7395	9382
7326	3.8648669	7361	3.8669368	7396	9969
7327	9262	7362	9958	7397	3.8690556
7328	9855	7363	3.8670548	7398	1143
7329	3.8650447	7364	1138	7399	1730
7330	1040	7365	1728	7400	2317
7331	3.8651632	7366	3.8672317		
7332	2225	7367	2907		
7333	2817	7368	3496		
7334	3409	7369	4086		
7335	4001	7370	4675		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
7401	3.8692904	7436	3.8713394	7471	3.8733787
7402	3491	7437	3978	7472	4369
7403	4077	7438	4562	7473	4950
7404	4664	7439	5146	7474	5531
7405	5251	7440	5729	7475	6112
7406	3.8695837	7441	3.8716313	7476	3.8736693
7407	6423	7442	6897	7477	7274
7408	7010	7443	7480	7478	7855
7409	7596	7444	8064	7479	8435
7410	8182	7445	8647	7480	9016
7411	3.8698768	7446	3.8719230	7481	9597
7412	9354	7447	9814	7482	3.8740177
7413	9940	7448	3.8720397	7483	0757
7414	3.8700526	7449	0980	7484	1338
7415	1112	7450	1563	7485	1918
7416	3.8701697	7451	3.8722146	7486	3.8742498
7417	2283	7452	2728	7487	3078
7418	2868	7453	3311	7488	3658
7419	3454	7454	3894	7489	4238
7420	4039	7455	4476	7490	4818
7421	3.8704624	7456	3.8725059	7491	3.8745398
7422	5209	7457	5641	7492	5978
7423	5795	7458	6224	7493	6557
7424	6380	7459	6806	7494	7137
7425	6965	7460	7388	7495	7716
7426	3.8707549	7461	3.8727970	7496	3.8748296
7427	8134	7462	8552	7497	8875
7428	8719	7463	9134	7498	9454
7429	9304	7464	9716	7499	3.8750034
7430	9888	7465	3.8730298	7500	0613
7431	3.8710473	7466	0880		
7432	1057	7467	1461		
7433	1641	7468	2043		
7434	2226	7469	2625		
7435	2810	7470	3206		

Num	Logarithms	Num	Logarithms	Num	Logarithms
7501	3.8751192	7536	3.8771409	7571	3.8791532
7502	1771	7537	1985	7572	2106
7503	2349	7538	2561	7573	2680
7504	2928	7539	3137	7574	3253
7505	3507	7540	3713	7575	3826
7506	3.8754086	7541	3.8774289	7576	3.8794400
7507	4664	7542	4865	7577	4973
7508	5243	7543	5441	7578	5546
7509	5821	7544	6017	7579	6119
7510	6399	7545	6592	7580	6692
7511	3.8756978	7546	3.8777168	7581	3.8797265
7512	7556	7547	7743	7582	7838
7513	8134	7548	8319	7583	8411
7514	8712	7549	8894	7584	8983
7515	9290	7550	9469	7585	9556
7516	9868	7551	3.8780045	7586	3.8800128
7517	3.8760445	7552	0620	7587	0701
7518	1023	7553	1195	7588	1273
7519	1601	7554	1770	7589	1846
7520	2178	7555	2345	7590	2418
7521	3.8762756	7556	3.8782919	7591	3.8802990
7522	3333	7557	3494	7592	3562
7523	3911	7558	4069	7593	4134
7524	4488	7559	4643	7594	4706
7525	5065	7560	5218	7595	5278
7526	3.8765642	7561	3.8785792	7596	3.8805850
7527	6219	7562	6367	7597	6421
7528	6796	7563	6941	7598	6993
7529	7373	7564	7515	7599	7564
7530	7950	7565	8089	7600	8136
7531	3.8768526	7566	3.8788663		
7532	9103	7567	9237		
7533	9680	7568	9811		
7534	3.8770256	7569	3.8790385		
7535	0833	7570	0959		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
7601	3.8808707	7636	3.8828659	7671	3.8848520
7602	9279	7637	9228	7672	9086
7603	9850	7638	9797	7673	9652
7604	3.8810421	7639	3.8830365	7674	3.8850218
7605	0992	7640	0934	7675	0784
7606	3.8811563	7641	3.8831502	7676	3.8851350
7607	2134	7642	2070	7677	1915
7608	2705	7643	2639	7678	2481
7609	3276	7644	3207	7679	3047
7610	3847	7645	3775	7680	3612
7611	3.8814417	7646	3.8834343	7681	3.8854178
7612	4988	7647	4911	7682	4743
7613	5558	7648	5479	7683	5308
7614	6129	7649	6047	7684	5874
7615	6699	7650	6614	7685	6439
7616	3.8817269	7651	3.8837182	7686	3.8857004
7617	7840	7652	7750	7687	7569
7618	8410	7653	8317	7688	8134
7619	8980	7654	8885	7689	8699
7620	9550	7655	9452	7690	9263
7621	3.8820120	7656	3.8840019	7691	9828
7622	0689	7657	0586	7692	3.8860393
7623	1259	7658	1154	7693	0957
7624	1829	7659	1721	7694	1522
7625	2398	7660	2288	7695	2086
7626	3.8822968	7661	3.8842855	7696	3.8862651
7627	3537	7662	3421	7697	3215
7628	4107	7663	3988	7698	3779
7629	4676	7664	4555	7699	4343
7630	5245	7665	5122	7700	4907
7631	3.8825815	7666	3.8845688		
7632	6384	7667	6255		
7633	6953	7668	6821		
7634	7522	7669	7387		
7635	8090	7670	7954		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
7701	3.8865471	7736	3.8885165	7771	3.8904769
7702	6035	7737	5726	7772	5328
7703	6599	7738	6287	7773	5887
7704	7163	7739	6848	7774	6445
7705	7726	7740	7410	7775	7004
7706	3.8868290	7741	3.8887971	7776	3.8907563
7707	8854	7742	8532	7777	8121
7708	9417	7743	9093	7778	8679
7709	9980	7744	9653	7779	9238
7710	3.8870544	7745	3.8890214	7780	9796
7711	1107	7746	0775	7781	3.8910354
7712	1670	7747	1336	7782	0912
7713	2233	7748	1896	7783	1470
7714	2796	7749	2457	7784	2028
7715	3359	7750	3017	7785	2586
7716	3.8873922	7751	3.8893577	7786	3.8913144
7717	4485	7752	4138	7787	3702
7718	5048	7753	4698	7788	4259
7719	5610	7754	5258	7789	4817
7720	6173	7755	5818	7790	5375
7721	3.8876736	7756	3.8896378	7791	3.8915932
7722	7298	7757	6938	7792	6489
7723	7860	7758	7498	7793	7047
7724	8423	7759	8058	7794	7604
7725	8985	7760	8617	7795	8161
7726	9547	7761	3.8899177	7796	3.8918718
7727	3.8880109	7762	9736	7797	9275
7728	0671	7763	3.8900296	7798	9832
7729	1233	7764	0855	7799	3.8920389
7730	1795	7765	1415	7800	0946
7731	3.8882357	7766	3.8901974		
7732	2918	7767	2533		
7733	3480	7768	3092		
7734	4042	7769	3651		
7735	4603	7770	4210		

Num	Logarithms	Num	Logarithms	Num	Logarithms
7801	3.8921503	7836	3.8940944	7871	3.8960299
7802	2059	7837	1498	7872	0851
7803	2616	7838	2053	7873	1403
7804	3173	7839	2607	7874	1954
7805	3729	7840	3161	7875	2506
7806	3.8924285	7841	3.8943715	7876	3.8963057
7807	4842	7842	4268	7877	3608
7808	5398	7843	4822	7878	4160
7809	5954	7844	5376	7879	4711
7810	6510	7845	5929	7880	5262
7811	3.8927066	7846	3.8946483	7881	3.8965813
7812	7622	7847	7037	7882	6364
7813	8178	7848	7590	7883	6915
7814	8734	7849	8143	7884	7466
7815	9290	7850	8697	7885	8017
7816	9846	7851	3.8949250	7886	3.8968568
7817	3.8930401	7852	9803	7887	9118
7818	0957	7853	3.8950356	7888	9669
7819	1512	7854	0909	7889	3.8970220
7820	2068	7855	1462	7890	0770
7821	3.8932623	7856	3.8952015	7891	3.8971320
7822	3178	7857	2568	7892	1871
7823	3733	7858	3120	7893	2421
7824	4288	7859	3673	7894	2971
7825	4843	7860	4225	7895	3521
7826	3.8935398	7861	3.8954778	7896	3.8974071
7827	5953	7862	5330	7897	4621
7828	6508	7863	5883	7898	5171
7829	7063	7864	6435	7899	5721
7830	7618	7865	6987	7900	6271
7831	3.8938172	7866	3 8957539		
7832	8727	7867	8091		
7833	9281	7868	8643		
7834	9836	7869	9195		
7835	3.8940390	7870	9747		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
7901	3.8976821	7936	3.8996017	7971	3.9015128
7902	7370	7937	6564	7972	5673
7903	7920	7938	7111	7973	6218
7904	8469	7939	7658	7974	6762
7905	9019	7940	8205	7975	7307
7906	9568	7941	3.8998752	7976	3.9017851
7907	3.8980117	7942	9299	7977	8396
7908	0667	7943	9846	7978	8940
7909	1216	7944	3.9000392	7979	9485
7910	1765	7945	0939	7980	3.9020029
7911	3.8982314	7946	3.9001486	7981	0573
7912	2863	7947	2032	7982	1117
7913	3412	7948	2579	7983	1661
7914	3960	7949	3125	7984	2205
7915	4509	7950	3671	7985	2749
7916	3.8985058	7951	3.9004218	7986	3.9023293
7917	5606	7952	4764	7987	3837
7918	6155	7953	5310	7988	4381
7919	6703	7954	5856	7989	4924
7920	7252	7955	6402	7990	5468
7921	3.8987800	7956	3.9006948	7991	3.9026011
7922	8348	7957	7494	7992	6555
7923	8897	7958	8039	7993	7098
7924	9445	7959	8585	7994	7641
7925	9993	7960	9131	7995	8185
7926	3.8990541	7961	9676	7996	3.9028728
7927	1089	7962	3.9010222	7997	9271
7928	1636	7963	0767	7998	9814
7929	2184	7964	1313	7999	3.9030357
7930	2732	7965	1858	8000	0900
7931	3.8993279	7966	3.9012403		
7932	3827	7967	2948		
7933	4375	7968	3493		
7934	4922	7969	4038		
7935	5469	7970	4583		

Num	Logarithms	Num	Logarithms	Num	Logarithms
8001	3.9031443	8036	3.9050399	8071	3.9069273
8002	1985	8037	0940	8072	9812
8003	2528	8038	1480	8073	3.9070350
8004	3071	8039	2020	8074	0887
8005	3613	8040	2560	8075	1425
8006	3.9034156	8041	3.9053101	8076	3.9071963
8007	4698	8042	3641	8077	2501
8008	5241	8043	4181	8078	3038
8009	5783	8044	4721	8079	3576
8010	6325	8045	5260	8080	4114
8011	3.9036867	8046	3.9055800	8081	3.9074651
8012	7409	8047	6340	8082	5188
8013	7951	8048	6880	8083	5726
8014	8493	8049	7419	8084	6263
8015	9035	8050	7959	8085	6800
8016	9577	8051	3.9058498	8086	3.9077337
8017	3.9040119	8052	9038	8087	7874
8018	0661	8053	9577	8088	8411
8019	1202	8054	3.9060116	8089	8948
8020	1744	8055	0655	8090	9485
8021	3.9042285	8056	3.9061195	8091	3.9080022
8022	2827	8057	1734	8092	0559
8023	3368	8058	2273	8093	1095
8024	3909	8059	2812	8094	1632
8025	4450	8060	3350	8095	2169
8026	3.9044992	8061	3.9063889	8096	3.9082705
8027	5533	8062	4428	8097	3241
8028	6074	8063	4967	8098	3778
8029	6615	8064	5505	8099	4314
8030	7155	8065	6044	8100	4850
8031	3.9047696	8066	3.9066582		
8032	8237	8067	7121		
8033	8778	8068	7659		
8034	9318	8069	8197		
8035	9859	8070	8735		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
8101	3.9085386	8136	3.9104109	8171	3.9122752
8102	5922	8137	4643	8172	3284
8103	6458	8138	5177	8173	3815
8104	6994	8139	5710	8174	4346
8105	7530	8140	6244	8175	4878
8106	3.9088066	8141	3.9106778	8176	3.9125409
8107	8602	8142	7311	8177	5940
8108	9137	8143	7844	8178	6471
8109	9673	8144	8378	8179	7002
8110	3.9090209	8145	8911	8180	7533
8111	0744	8146	3.9109444	8181	3.9128064
8112	1279	8147	9977	8182	8595
8113	1815	8148	3.9110510	8183	9126
8114	2350	8149	1043	8184	9656
8115	2885	8150	1576	8185	3.9130187
8116	3.9093420	8151	3.9112109	8186	0717
8117	3955	8152	2642	8187	1248
8118	4490	8153	3174	8188	1778
8119	5025	8154	3707	8189	2309
8120	5560	8155	4240	8190	2839
8121	3.9096095	8156	3.9114772	8191	3.9133369
8122	6630	8157	5305	8192	3899
8123	7165	8158	5837	8193	4430
8124	7699	8159	6369	8194	4960
8125	8234	8160	6902	8195	5490
8126	3.9098768	8161	3.9117434	8196	3.9136019
8127	9303	8162	7966	8197	6549
8128	9837	8163	8498	8198	7079
8129	3.9100371	8164	9030	8199	7609
8130	0905	8165	9562	8200	8139
8131	3.9101440	8166	3.9120094		
8132	1974	8167	0626		
8133	2508	8168	1157		
8134	3042	8169	1689		
8135	3576	8170	2221		

Num	Logarithms	Num	Logarithms	Num	Logarithms
8201	3.9138668	8236	3.9157163	8271	3.9175580
8202	9198	8237	7691	8272	6105
8203	9727	8238	8218	8273	6630
8204	3.9140257	8239	8745	8274	7155
8205	0786	8240	9272	8275	7680
8206	3.9141315	8241	3.9159799	8276	3.9178205
8207	1844	8242	3.9160326	8277	8730
8208	2373	8243	0853	8278	9254
8209	2903	8244	1380	8279	9779
8210	3432	8245	1907	8280	3.9180303
8211	3.9143961	8246	3.9162433	8281	0828
8212	4489	8247	2960	8282	1352
8213	5018	8248	3487	8283	1877
8214	5547	8249	4013	8284	2401
8215	6076	8250	4539	8285	2925
8216	3.9146604	8251	3.9165066	8286	3.9183449
8217	7133	8252	5592	8287	3973
8218	7661	8253	6118	8288	4497
8219	8190	8254	6645	8289	5021
8220	8718	8255	7171	8290	5545
8221	3.9149246	8256	3.9167697	8291	3.9186069
8222	9775	8257	8223	8292	6593
8223	3.9150303	8258	8749	8293	7117
8224	0831	8259	9275	8294	7640
8225	1359	8260	9800	8295	8164
8226	3.9151887	8261	3.9170326	8296	3.9188687
8227	2415	8262	0852	8297	9211
8228	2943	8263	1378	8298	9734
8229	3471	8264	1903	8299	3.9190258
8230	3998	8265	2429	8300	0781
8231	3.9154526	8266	3.9172954		
8232	5054	8267	3479		
8233	5581	8268	4005		
8234	6109	8269	4530		
8235	6636	8270	5055		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
8301	3.9191304	8336	3.9209577	8371	3.9227773
8302	1827	8337	3.9210098	8372	8292
8303	2350	8338	0619	8373	8811
8304	2873	8339	1140	8374	9330
8305	3396	8340	1661	8375	9848
8306	3.9193919	8341	3.9212181	8376	3.9230367
8307	4442	8342	2702	8377	0885
8308	4965	8343	3222	8378	1404
8309	5488	8344	3743	8379	1922
8310	6010	8345	4263	8380	2440
8311	3.9196533	8346	3.9214748	8381	3.9232958
8312	7055	8347	5304	8382	3477
8313	7578	8348	5824	8383	3995
8314	8100	8349	6345	8384	4513
8315	8623	8350	6865	8385	5031
8316	3.9199145	8351	3.9217385	8386	3.9235549
8317	9667	8352	7905	8387	6066
8318	3.9200189	8353	8425	8388	6584
8319	0711	8354	8945	8389	7102
8320	1233	8355	9465	8390	7620
8321	3.9201755	8356	9984	8391	3.9238137
8322	2277	8357	3.9220504	8392	8655
8323	2799	8358	1024	8393	9172
8324	3321	8359	1543	8394	9690
8325	3842	8360	2063	8395	3.9240207
8326	3.9204364	8361	3.9222582	8396	0724
8327	4886	8362	3102	8397	1242
8328	5407	8363	3621	8398	1759
8329	5929	8364	4140	8399	2276
8330	6450	8365	4659	8400	2793
8331	3.9206971	8366	3.9225179		
8332	7493	8367	5698		
8333	8014	8368	6217		
8334	8535	8369	6736		
8335	9056	8370	7255		

Num	Logarithms	Num	Logarithms	Num	Logarithms
8401	3.9243310	8436	3.9261366	8471	3.9279347
8402	3827	8437	1880	8472	9859
8403	4344	8438	2395	8473	3.9280372
8404	4860	8439	2910	8474	0885
8405	5377	8440	3424	8475	1397
8406	3.9245894	8441	3.9263939	8476	3.9281909
8407	6410	8442	4453	8477	2422
8408	6927	8443	4968	8478	2934
8409	7444	8444	5482	8479	3446
8410	7960	8445	5997	8480	3959
8411	3.9248476	8446	3.9266511	8481	3.9284471
8412	8993	8447	7025	8482	4983
8413	9509	8448	7539	8483	5495
8414	3.9250025	8449	8053	8484	6007
8415	0541	8450	8567	8485	6518
8416	3.9251057	8451	3.9269081	8486	3.9287030
8417	1573	8452	9595	8487	7542
8418	2089	8453	3.9270109	8488	8054
8419	2605	8454	0622	8489	8565
8420	3121	8455	1136	8490	9077
8421	3.9253637	8456	3.9271650	8491	9588
8422	4152	8457	2163	8492	3.9290100
8423	4668	8458	2677	8493	0611
8424	5184	8459	3190	8494	1123
8425	5699	8460	3704	8495	1634
8426	3.9256215	8461	3.9274217	8496	3.9292145
8427	6730	8462	4730	8497	2656
8428	7245	8463	5243	8498	3167
8429	7761	8464	5757	8499	3678
8430	8276	8465	6270	8500	4189
8431	3.9258791	8466	3.9276783		
8432	9306	8467	7296		
8433	9821	8468	7808		
8434	3.9260236	8469	8321		
8435	0851	8470	8834		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
8501	3.9294700	8536	3.9312544	8571	3.9330315
8502	5211	8537	3053	8572	0822
8503	5722	8538	3561	8573	1328
8504	6233	8539	4070	8574	1835
8505	6743	8540	4579	8575	2341
8506	3.9297254	8541	3.9315087	8576	3.9332848
8507	7764	8542	5596	8577	3354
8508	8275	8543	6104	8578	3860
8509	8785	8544	6612	8579	4367
8510	9296	8545	7121	8580	4873
8511	9806	8546	3.9317629	8581	3.9335379
8512	3.9300316	8547	8137	8582	5885
8513	0826	8548	8645	8583	6391
8514	1336	8549	9153	8584	6897
8515	1847	8550	9661	8585	7403
8516	3.9302357	8551	3.9320169	8586	3.9337909
8517	2866	8552	0677	8587	8415
8518	3376	8553	1185	8588	8920
8519	3886	8554	1692	8589	9426
8520	4396	8555	2200	8590	9932
8521	3.9304906	8556	3.9322708	8591	3.9340437
8522	5415	8557	3215	8592	0943
8523	5925	8558	3723	8593	1448
8524	6434	8559	4230	8594	1953
8525	6944	8560	4738	8595	2459
8526	3.9307453	8561	3.9325245	8596	3.9342964
8527	7963	8562	5752	8597	3469
8528	8472	8563	6259	8598	3974
8529	8981	8564	6767	8599	4479
8530	9490	8565	7274	8600	4984
8531	9999	8566	3.9327781		
8532	3.9310508	8567	8288		
8533	1017	8568	8795		
8534	1526	8569	9301		
8535	2035	8570	9808		

Num	Logarithms	Num	Logarithms	Num	Logarithms
8601	3.9345489	8636	3.9363126	8671	3.9380692
8602	5994	8637	3629	8672	1193
8603	6499	8638	4132	8673	1693
8604	7004	8639	4635	8674	2194
8605	7509	8640	5137	8675	2695
8606	3.9348013	8641	3.9365640	8676	3.9383195
8607	8518	8642	6143	8677	3696
8608	9023	8643	6645	8678	4196
8609	9527	8644	7148	8679	4697
8610	3.9350032	8645	7650	8680	5197
8611	0536	8646	3.9368152	8681	3.9385698
8612	1040	8647	8655	8682	6198
8613	1544	8648	9157	8683	6698
8614	2040	8649	9659	8684	7198
8615	2553	8650	3.9370161	8685	7698
8616	3.9353057	8651	0663	8686	3.9388198
8617	3561	8652	1165	8687	8698
8618	4065	8653	1667	8688	9198
8619	4569	8654	2169	8689	9698
8620	5073	8655	2671	8690	3.9390198
8621	3.9355576	8656	3.9373172	8691	0697
8622	6080	8657	3674	8692	1197
8623	6584	8658	4176	8693	1697
8624	7087	8659	4677	8694	2196
8625	7591	8660	5179	8695	2696
8626	3.9358095	8661	3.9375680	8696	3.9393195
8627	8598	8662	6182	8697	3695
8628	9101	8663	6683	8698	4194
8629	9605	8664	7184	8699	4693
8630	3.9360108	8665	7686	8700	5193
8631	0611	8666	3.9378187		
8632	1114	8667	8688		
8633	1617	8668	9189		
8634	2120	8669	9690		
8635	2623	8670	3.9380191		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
8701	3.9395692	8736	3.9413126	8771	3.9430491
8702	6191	8737	3623	8772	0986
8703	6690	8738	4120	8773	1481
8704	7189	8739	4617	8774	1976
8705	7688	8740	5114	8775	2471
8706	3.9398187	8741	3.9415611	8776	3.9432966
8707	8685	8742	6108	8777	3461
8708	9184	8743	6605	8778	3956
8709	9683	8744	7101	8779	4450
8710	3.9400182	8745	7598	8780	4945
8711	0680	8746	3.9418095	8781	3.9435440
8712	1179	8747	8591	8782	5934
8713	1677	8748	9088	8783	6429
8714	2176	8749	9584	8784	6923
8715	2674	8750	3.9420081	8785	7418
8716	3.9403172	8751	0577	8786	3.9437912
8717	3670	8752	1073	8787	8406
8718	4169	8753	1569	8788	8900
8719	4667	8754	2065	8789	9395
8720	5165	8755	2561	8790	9889
8721	3.9405663	8756	3.9423058	8791	3.9440383
8722	6161	8757	3553	8792	0877
8723	6659	8758	4049	8793	1371
8724	7157	8759	4545	8794	1865
8725	7654	8760	5041	8795	2358
8726	3.9408152	8761	3.9425537	8796	3.9442852
8727	8650	8762	6032	8797	3346
8728	9147	8763	6528	8798	3840
8729	9645	8764	7024	8799	4333
8730	3.9410142	8765	7519	8800	4827
8731	0640	8766	3.9428015		
8732	1137	8767	8510		
8733	1635	8768	9005		
8734	2132	8769	9501		
8735	2629	8770	9996		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
8801	3·9445320	8836	3·9462557	8871	3·9479726
8802	5814	8837	3048	8872	3·9480215
8803	6307	8838	3540	8873	0705
8804	6800	8839	4031	8874	1194
8805	7294	8840	4523	8875	1684
8806	3·9447787	8841	3·9465014	8876	3·9482173
8807	8280	8842	5505	8877	2662
8808	8773	8843	5996	8878	3151
8809	9266	8844	6487	8879	3641
8810	9759	8845	6978	8880	4130
8811	3·9450252	8846	3·9407469	8881	3·9484619
8812	0745	8847	7960	8882	5108
8813	1238	8848	8451	8883	5597
8814	1730	8849	8942	8884	6085
8815	2223	8850	9433	8885	6574
8816	3·9452716	8851	9923	8886	3·9487063
8817	3208	8852	3·9470414	8887	7552
8818	3701	8853	0905	8888	8040
8819	4193	8854	1395	8889	8529
8820	4686	8855	1886	8890	9018
8821	3·9455178	8856	3·9472376	8891	3·9489506
8822	5671	8857	2866	8892	9995
8823	6163	8858	3357	8893	3·9490483
8824	6655	8859	3847	8894	0971
8825	7147	8860	4337	8895	1460
8826	3·9457639	8861	3·9474827	8896	3·9491948
8827	8131	8862	5317	8897	2436
8828	8623	8863	5807	8898	2924
8829	9115	8864	6297	8899	3412
8830	9607	8865	6787	8900	3900
8831	3·9460099	8866	3·9477277		
8832	0591	8867	7767		
8833	1082	8868	8257		
8834	1574	8869	8747		
8835	2066	8870	9236		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
8901	3.9494388	8936	3.9511432	8971	3.9528409
8902	4876	8937	1918	8972	8893
8903	5364	8938	2404	8973	9377
8904	5852	8939	2889	8974	9861
8905	6339	8940	3375	8975	3.9530345
8906	3.9496827	8941	3.9513861	8976	0828
8907	7315	8942	4347	8977	1312
8908	7802	8943	4832	8978	1796
8909	8290	8944	5318	8979	2280
8910	8777	8945	5803	8980	2763
8911	3.9499264	8946	3.9516289	8981	3.9533247
8912	9752	8947	6774	8982	3730
8913	3.9500239	8948	7260	8983	4214
8914	0726	8949	7745	8984	4697
8915	1213	8950	8230	8985	5181
8916	3.9501701	8951	3.9518716	8986	3.9535664
8917	2188	8952	9201	8987	6147
8918	2675	8953	9686	8988	6631
8919	3162	8954	3.9520171	8989	7114
8920	3649	8955	0656	8990	7597
8921	3.9504135	8956	3.9521141	8991	3.9538080
8922	4622	8957	1626	8992	8563
8923	5109	8958	2111	8993	9046
8924	5596	8959	2595	8994	9529
8925	6082	8960	3080	8995	3.9540012
8926	3.9506569	8961	3.9523565	8996	0494
8927	7055	8962	4049	8997	0977
8928	7542	8963	4534	8998	1460
8929	8028	8964	5018	8999	1943
8930	8515	8965	5503	9000	2425
8931	3.9509001	8966	3.9525987		
8932	9487	8967	6472		
8933	9973	8968	6956		
8934	3.9510459	8969	7440		
8935	0946	8970	7924		

Num	Logarithms	Num	Logarithms	Num	Logarithms
9001	3·9542908	9036	3·9559762	9071	3·9576552
9002	3390	9037	3·9560243	9072	7030
9003	3872	9038	0723	9073	7509
9004	4355	9039	1204	9074	7988
9005	4837	9040	1684	9075	8466
9006	3·9545319	9041	3·9562165	9076	3·9578945
9007	5802	9042	2645	9077	6423
9008	6284	9043	3125	9078	9902
9009	6766	9044	3606	9079	3·9580380
9010	7248	9045	4086	9080	0858
9011	3·9547730	9046	3·9564566	9081	3·9581337
9012	8212	9047	5046	9082	1815
9013	8694	9048	5526	9083	2293
9014	9176	9049	6006	9084	2771
9015	9657	9050	6486	9085	3249
9016	3·9550139	9051	3·9566966	9086	3·9583727
9017	0621	9052	7445	9087	4205
9018	1102	9053	7925	9088	4683
9019	1584	9054	8405	9089	5161
9020	2065	9055	8885	9090	5639
9021	3·9552547	9056	3·9569364	9091	3·9586117
9022	3028	9057	9844	9092	6594
9023	3510	9058	3·9570323	9093	7072
9024	3991	9059	0803	9094	7549
9025	4472	9060	1282	9095	8027
9026	3·9554953	9061	3·9571761	9096	3·9588505
9027	5434	9062	2241	9097	8982
9028	5915	9063	2720	9098	9459
9029	6397	9064	3199	9099	9937
9030	6877	9065	3678	9100	3·9590414
9031	3·9557358	9066	3·9574157		
9032	7839	9067	4636		
9033	8320	9068	5115		
9034	8801	9069	5594		
9035	9282	9070	6073		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
9101	3.9590891	9136	3.9607561	9171	3.9624167
9102	1368	9137	8036	9172	4640
9103	1845	9138	8512	9173	5114
9104	2322	9139	8987	9174	5587
9105	2799	9140	9462	9175	6061
9106	3.9593276	9141	9937	9176	3.9626534
9107	3753	9142	3.9610412	9177	7007
9108	4230	9143	0887	9178	7481
9109	4707	9144	1362	9179	7954
9110	5184	9145	1837	9180	8427
9111	3.9595660	9146	3.9612312	9181	3.9628900
9112	6137	9147	2787	9182	9373
9113	6614	9148	3262	9183	9846
9114	7090	9149	3736	9184	3.9630319
9115	7567	9150	4211	9185	0792
9116	3.9598043	9151	3.9614686	9186	3.9631264
9117	8520	9152	5160	9187	1737
9118	8996	9153	5635	9188	2210
9119	9472	9154	6109	9189	2683
9120	9948	9155	6583	9190	3155
9121	3.9600425	9156	3.9617058	9191	3.9633628
9122	0901	9157	7532	9192	4100
9123	1377	9158	8006	9193	4573
9124	1853	9159	8481	9194	5045
9125	2329	9160	8955	9195	5517
9126	3.9602805	9161	3.9619429	9196	3.9635990
9127	3280	9162	9903	9197	6462
9128	3756	9163	3.9620377	9198	6934
9129	4232	9164	0851	9199	7406
9130	4708	9165	1325	9200	7878
9131	3.9605183	9166	3.9621799		
9132	5659	9167	2272		
9133	6135	9168	2746		
9134	6610	9169	3220		
9135	7086	9170	3693		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
9201	3.9638350	9236	3.9654839	9271	3.9671266
9202	8822	9237	5309	9272	1734
9203	9294	9238	5780	9273	2203
9204	9766	9239	6250	9274	2671
9205	3.9640238	9240	6720	9275	3139
9206	0710	9241	3.9657190	9276	3.9673607
9207	1181	9242	7660	9277	4076
9208	1653	9243	8130	9278	4544
9209	2125	9244	8599	9279	5012
9210	2596	9245	9069	9280	5480
9211	3.9643068	9246	9539	9281	3.9675948
9212	3539	9247	3.9660009	9282	6416
9213	4011	9248	0478	9283	6883
9214	4482	9249	0948	9284	7351
9215	4953	9250	1417	9285	7819
9216	3.9645425	9251	3.9661887	9286	3.9678287
9217	5896	9252	2356	9287	8754
9218	6367	9253	2826	9288	9222
9219	6838	9254	3295	9289	9690
9220	7309	9255	3764	9290	3.9680157
9221	3.9647780	9256	3.9664233	9291	0625
9222	8251	9257	4703	9292	1092
9223	8722	9258	5172	9293	1559
9224	9193	9259	5641	9294	2027
9225	9664	9260	6110	9295	2494
9226	3.9650134	9261	3.9666579	9296	3.9682961
9227	0605	9262	7048	9297	3428
9228	1076	9263	7517	9298	3895
9229	1546	9264	7985	9299	4362
9230	2017	9265	8454	9300	4829
9231	3.9652488	9266	3.9668923		
9232	2958	9267	9392		
9233	3428	9268	9860		
9234	3899	9269	3.9670329		
9235	4369	9270	0797		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
9301	3.9685296	9336	3.9701608	9371	3.9717859
9302	5763	9337	2074	9372	8323
9303	6230	9338	2539	9373	8786
9304	6697	9339	3004	9374	9249
9305	7164	9340	3469	9375	9713
9306	3.9687630	9341	3.9703934	9376	3.9720176
9307	8097	9342	4399	9377	0639
9308	8564	9343	4863	9378	1102
9309	9030	9344	5328	9379	1565
9310	9497	9345	5793	9380	2028
9311	9963	9346	3.9706258	9381	3.9722491
9312	3.9690430	9347	6722	9382	2954
9313	0896	9348	7187	9383	3417
9314	1362	9349	7652	9384	3880
9315	1829	9350	8116	9385	4343
9316	3.9692295	9351	3.9708581	9386	3.9724805
9317	2761	9352	9045	9387	5268
9318	3227	9353	9509	9388	5731
9319	3693	9354	9974	9389	6193
9320	4159	9355	3.9710438	9390	6656
9321	3.9694625	9356	0902	9391	3.9727118
9322	5091	9357	1366	9392	7581
9323	5557	9358	1830	9393	8043
9324	6023	9359	2294	9394	8506
9325	6488	9360	2758	9395	8968
9326	3.9696954	9361	3.9713222	9396	3.9729430
9327	7420	9362	3686	9397	9892
9328	7885	9363	4150	9398	3.9730354
9329	8351	9364	4614	9399	0816
9330	8816	9365	5078	9400	1279
9331	3.9699282	9366	3.9715542		
9332	9747	9367	6005		
9333	3.9700213	9368	6469		
9334	0678	9369	6932		
9335	1143	9370	7396		

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
9401	3·9731741	9436	3·9747879	9471	3·9763958
9402	2202	9437	8340	9472	4417
9403	2664	9438	8800	9473	4875
9404	3126	9439	9260	9474	5334
9405	3588	9440	9720	9475	5792
9406	3·9734050	9441	3·9750180	9476	3·9766251
9407	4511	9442	0640	9477	6709
9408	4973	9443	1100	9478	7167
9409	5435	9444	1560	9479	7625
9410	3·9735896	9445	2020	9480	8083
9411	6358	9446	3·9752479	9481	3·9768541
9412	6819	9447	2939	9482	8999
9413	7281	9448	3399	9483	9457
9414	7742	9449	3858	9484	9915
9415	8203	9450	4318	9485	3·9770373
9416	3·9738664	9451	3·9754778	9486	0831
9417	9126	9452	5237	9487	1289
9418	9587	9453	5697	9488	1747
9419	3·9740048	9454	6156	9489	2204
9420	0509	9455	6615	9490	2662
9421	3·9740970	9456	3·9757075	9491	3·9773120
9422	1431	9457	7534	9492	3577
9423	1892	9458	7993	9493	4035
9424	2353	9459	8452	9494	4492
9425	2814	9460	8911	9495	4950
9426	3·9743274	9461	3·9759370	9496	3·9775407
9427	3735	9462	9829	9497	5864
9428	4196	9463	3·9760288	9498	6322
9429	4656	9464	0747	9499	6779
9430	5117	9465	1206	9500	7236
9431	3·9745577	9466	3·9761665		
9432	6038	9467	2124		
9433	6498	9468	2582		
9434	6959	9469	3041		
9435	7419	9470	3500		

Num	Logarithms	Num	Logarithms	Num	Logarithms
9501	3.9777693	9536	3.9793662	9571	3.9809573
9502	8150	9537	4118	9572	3.9810027
9503	8607	9538	4573	9573	0481
9504	9064	9539	5028	9574	0934
9505	9521	9540	5484	9575	1388
9506	9978	9541	3.9795939	9576	3.9811841
9507	3.9780435	9542	6394	9577	2295
9508	0892	9543	6849	9578	2748
9509	1348	9544	7304	9579	3202
9510	1805	9545	7759	9580	3655
9511	3.9782262	9546	3.9798214	9581	3.9814108
9512	2718	9547	8669	9582	4562
9513	3175	9548	9124	9583	5015
9514	3631	9549	9579	9584	5468
9515	4088	9550	3.9800034	9585	5921
9516	3.9784544	9551	0488	9586	3.9816374
9517	5001	9552	0943	9587	6827
9518	5457	9553	1398	9588	7280
9519	5913	9554	1852	9589	7733
9520	6369	9555	2307	9590	8186
9521	3.9786826	9556	3.9802761	9591	3.9818639
9522	7282	9557	3216	9592	9092
9523	7738	9558	3670	9593	9544
9524	8194	9559	4125	9594	9997
9525	8650	9560	4579	9595	3.9820450
9526	3.9789106	9561	3.9805033	9596	0902
9527	9562	9562	5487	9597	1355
9528	3.9790017	9563	5942	9598	1807
9529	0473	9564	6396	9599	2260
9530	0929	9565	6850	9600	2712
9531	3.9791385	9566	3.9807304		
9532	1840	9567	7758		
9533	2296	9568	8212		
9534	2751	9569	8666		
9535	3207	9570	9119		

Num	Logarithms	Num	Logarithms	Num	Logarithms
9601	3.9823165	9636	3.9838968	9671	3.9854714
9602	3617	9637	9419	9672	5163
9603	4069	9638	9869	9673	5612
9604	4522	9639	3.9840320	9674	6061
9605	4974	9640	0770	9675	6510
9606	3.9825426	9641	3.9841221	9676	3.9856959
9607	5878	9642	1671	9677	7407
9608	6330	9643	2122	9678	7856
9609	6782	9644	2572	9679	8305
9610	7234	9645	3022	9680	8754
9611	3.9827686	9646	3.9843473	9681	3.9859202
9612	8138	9647	3923	9682	9651
9613	8589	9648	4373	9683	3.9860099
9614	9041	9649	4823	9684	0548
9615	9493	9650	5273	9685	0996
9616	9945	9651	3.9845723	9686	3.9861445
9617	3.9830396	9652	6173	9687	1893
9618	0848	9653	6623	9688	2341
9619	1299	9654	7073	9689	2790
9620	1751	9655	7523	9690	3238
9621	3.9832202	9656	3.9847973	9691	3.9863686
9622	2654	9657	8422	9692	4134
9623	3105	9658	8872	9693	4582
9624	3556	9659	9322	9694	5030
9625	4007	9660	9771	9695	5478
9626	3.9834459	9661	3.9850221	9696	3.9865926
9627	4910	9662	0670	9697	6374
9628	5361	9663	1120	9698	6822
9629	5812	9664	1569	9699	7270
9630	6263	9665	2019	9700	7717
9631	3.9836714	9666	3.9852468		
9632	7165	9667	2917		
9633	7616	9668	3366		
9634	8066	9669	3816		
9635	8517	9670	4265		

Num	Logarithms	Num	Logarithms	Num	Logarithms
9701	3.9868165	9736	3.9883806	9771	3.9899390
9702	8613	9737	4252	9772	9835
9703	9060	9738	4698	9773	3.9900279
9704	9508	9739	5144	9774	0723
9705	9955	9740	5590	9775	1168
9706	3.9870403	9741	3.9886035	9776	3.9901612
9707	0850	9742	6481	9777	2056
9708	1298	9743	6927	9778	2500
9709	1745	9744	7373	9779	2944
9710	2192	9745	7818	9780	3389
9711	3.9872640	9746	3.9888264	9781	3.9903833
9712	3087	9747	8710	9782	4277
9713	3534	9748	9155	9783	4721
9714	3981	9749	9601	9784	5164
9715	4428	9750	3.9890046	9785	5608
9716	3.9874875	9751	0492	9786	3 9906052
9717	5322	9752	0937	9787	6496
9718	5769	9753	1382	9788	6940
9719	6216	9754	1828	9789	7383
9720	6663	9755	2273	9790	7827
9721	3.9877109	9756	3.9892718	9791	3.9908270
9722	7556	9757	3163	9792	8714
9723	8003	9758	3608	9793	9158
9724	8449	9759	4053	9794	9601
9725	8896	9760	4498	9795	3.9910044
9726	3.9879343	9761	3.9894943	9796	0488
9727	9789	9762	5388	9797	0931
9728	3.9880236	9763	5833	9798	1374
9729	0682	9764	6278	9799	1818
9730	1128	9765	6722	9800	2261
9731	3.9881575	9766	3.9897167		
9732	2021	9767	7612		
9733	2467	9768	8056		
9734	2913	9769	8501		
9735	3360	9770	8946		

Num	Logarithms	Num	Logarithms	Num	Logarithms
9801	3.9912704	9836	3.9928185	9871	3.9943612
9802	3147	9837	8627	9872	4051
9803	3590	9838	9068	9873	4491
9804	4033	9839	9510	9874	4931
9805	4476	9840	9951	9875	5371
9806	3.9914919	9841	3.9930392	9876	3.9945811
9807	5362	9842	0834	9877	6251
9808	5805	9843	1275	9878	6690
9809	6247	9844	1716	9879	7130
9810	6690	9845	2157	9880	7569
9811	3.9917133	9846	3.9932598	9881	3.9948009
9812	7575	9847	3039	9882	8448
9813	8018	9848	3480	9883	8888
9814	8461	9849	3921	9884	8327
9815	8903	9850	4362	9885	9766
9816	3.9919345	9851	3.9934803	9886	3.9950206
9817	9788	9852	5244	9887	0645
9818	3.9920230	9853	5685	9888	1085
9819	0673	9854	6126	9889	1524
9820	1115	9855	6566	9890	1963
9821	3.9921557	9856	3.9937007	9891	3.9952402
9822	1999	9857	7448	9892	2841
9823	2441	9858	7888	9893	3280
9824	2884	9859	8329	9894	3719
9825	3326	9860	8769	9895	4158
9826	3.9923768	9861	3.9939210	9896	3.9954597
9827	4210	9862	9650	9897	5036
9828	4651	9863	3.9940090	9898	5474
9829	5093	9864	0531	9899	5913
9830	5535	9865	0971	9900	6352
9831	3.9925977	9866	3.9941411		
9832	6419	9867	1851		
9833	6860	9868	2291		
9834	7302	9869	2731		
9835	7744	9870	3172		

A TABLE of LOGARITHMS.

Num	*Logarithms*	*Num*	*Logarithms*	*Num*	*Logarithms*
9901	3·9956791	9936	3·9972116	9971	3·9987387
9902	7229	9937	2553	9972	7823
9903	7668	9938	2990	9973	8258
9904	8106	9939	3427	9974	8694
9905	8545	9940	3864	9975	9129
9906	3·9958983	9941	3·9974301	9976	9564
9907	9422	9942	4738	9977	3·9990000
9908	9860	9943	5174	9978	0435
9909	3·9960298	9944	5611	9979	0870
9910	0737	9945	6048	9980	1305
9911	3·9961175	9946	3·9976485	9981	3·9991741
9912	1613	9947	6921	9982	2176
9913	2051	9948	7358	9983	2611
9914	2489	9949	7794	9984	3046
9915	2927	9950	8231	9985	3481
9916	3·9963365	9951	3·9978667	9986	3·9993916
9917	3803	9952	9104	9987	4350
9918	4241	9953	9540	9988	4785
9919	4679	9954	9976	9989	5220
9920	5117	9955	3·9980413	9990	5655
9921	3·9965554	9956	0849	9991	3·9996090
9922	5992	9957	1285	9992	6524
9923	6430	9958	1721	9993	6959
9924	6868	9959	2157	9994	7393
9925	7305	9960	2593	9995	7828
9926	3·9967743	9961	3·9983029	9996	3·9998262
9927	8180	9962	3465	9997	8697
9928	8618	9963	3901	9998	9131
9929	9055	9964	4337	9999	9566
9930	9492	9965	4773	10000	4.0000000
9931	9930	9966	3·9985209		
9932	3·9970367	9967	5645		
9933	0804	9968	6080		
9934	1242	9969	6516		
9935	1679	9970	6952		

The End of the Logarithms.